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ELEMENTARY MECHANICS
HYDROSTATICS
PNEUMATICS
HYDRAULICS
ELEMENTARY CHEMISTRY
HEAT

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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

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NOTE.—This volume is made up of a number of separate parts, or as indicated by their titles, and the page numbers of each usually begin with this list of contents the titles of the parts are given in the order in which they in the book, and under each title is a full synopsis of the subjects treated.

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HEAT, PART 1

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ELEMENTARY MECHANICS

(PART 1)

MATTER: ITS FORMS AND PROPERTIES

GENERAL AND SPECIFIC PROPERTIES

1. Matter is anything that occupies space; it is likewise anything that can be acted on by a force or that can be put in motion.

2. A **body** is a limited portion of matter.

3. A body, for instance a drop of water, may be divided and subdivided until each particle is so small that it can only be seen by the most powerful microscope, but each particle will still be a body and have the nature of the original body, that is, it will still be water in the example here chosen. When the division has been carried on until the particles have become so small that another division will change their nature, the particles are called *molecules*.

4. A **molecule** is the smallest body or portion of matter that can exist as such without changing its nature. It has been calculated that the diameter of a molecule is larger than $\frac{1}{1250000000}$ inch, and smaller than $\frac{1}{5000000000}$ inch. Molecules of a body are thought to be separated from one another by distances, which though smaller than we can detect or even imagine, are appreciable when compared with the size of the molecules. The molecules of a body are not at rest relative to each other, but are continually vibrating and rotating; this motion is the cause of the heat in the body.

5. If three parts of finely ground sulphur and two parts of finely ground iron are thoroughly mixed together, a mixture may be obtained so thorough that the naked eye may not detect the particles of sulphur from those of iron, but a powerful microscope will show them apart. If, however, this mixture be heated, it will commence to glow and afterwards no microscope however powerful will show distinct particles of sulphur and iron. A chemical combination has taken place and a new substance, ferric sulphide, has formed. This substance has properties that differ from those of the two substances composing it; nevertheless, every particle of the new substance, every molecule of it, contains the original two substances in the proportion of three to two. Conversely, molecules can be divided by chemical means into smaller parts that will not, however, possess, as a rule, the properties of the original body; these are called *atoms*.

6. An **atom** is the smallest portion of matter that can enter into a chemical combination.

If a molecule of water be divided chemically, it will yield two atoms of hydrogen gas and one of oxygen gas. If a molecule of sulphuric acid be divided, it will yield two atoms of hydrogen, one of sulphur, and four of oxygen. These atoms do not, however, exist by themselves, but immediately form into molecules of hydrogen, sulphur, and oxygen, respectively.

7. Matter exists in three conditions or forms: *solid*, *liquid*, and *gaseous*.

8. A **solid body** is one whose molecules change their relative positions with great difficulty; as iron, wood, stone, etc.

9. A **liquid body** is one whose molecules tend to change their relative positions easily. Liquids readily adapt themselves to the vessel that contains them and their upper surface always tends to become perfectly level. Water, mercury, molasses, etc. are liquids.

10. A **gaseous body,** or **gas,** is one whose molecules are at relatively great distances from each other, so that

they are not held near each other by mutual attraction. The molecules are, therefore, free to separate as far as the enclosure containing the gas will permit, and a given quantity of gas will, therefore, always fill the vessel containing it no matter how large the vessel may be. oxygen, hydrogen, etc. are gases.

11. Gaseous bodies are sometimes called **aeriform** (a like) bodies; they are divided into two classes: *vapors* and the so-called *permanent gases*.

12. A **permanent gas** is one which remains a gas at ordinary temperatures and pressures.

13. A **vapor** is a body which, at ordinary temperatures, is a liquid or a solid, but when heat is applied becomes a gas. Some examples are: water vapor (steam), ammonia vapor, etc.

14. One body may exist in all three states; for example, mercury, which at ordinary temperatures is liquid, becomes a solid (freezes) at about 39° F. below zero and a vapor (gas) at about 675° F. above zero. By means of great cold and pressure, all gases, except several only recently discovered, have been converted into liquids and afterwards solidified.

By means of heat, all solids have been liquified and a great many vaporized. It is probable that if we had the means of producing sufficiently great extremes of heat and cold, all solids might be converted into gases and all gases into solids.

15. Every portion of matter possesses certain *properties*. These properties are divided into two classes: *general* and *special*.

16. **General properties** of matter are those which are common to all bodies. They are as follows: *extension*, *impenetrability*, *weight*, *indestructibility*, *inertia*, *mobility*, *divisibility*, *porosity*, *compressibility*, *expansibility*, and *elasticity*.

17. **Special properties** are those which are not possessed by all bodies. Some of the most important are as follows: *hardness*, *tenacity*, *brittleness*, *malleability*, and *ductility*.

18. Extension is the property of occupying space. Since all bodies must occupy space, it follows that extension is a general property.

19. Impenetrability is that general property of matter by virtue of which no two bodies can occupy exactly the same space at the same time.

20. Weight is that general property of matter by virtue of which it is attracted by the earth. All bodies have weight. In former times it was supposed that gases had no weight, since, if unconfined, they tend to move away from the earth; but, nevertheless, a closed vessel filled with gas weighs more than the same vessel when the gas is exhausted from it.

21. Inertia is that general property of matter by virtue of which it retains its state of rest or of uniform rectilinear (straight line) motion so long as no external force acts to change that state.

22. Mobility is that general property of matter by which its position can be changed. Any body can be moved if acted on by a force sufficiently large.

23. Divisibility is that general property of matter by virtue of which a body may be separated into parts.

24. Porosity is that general property of matter which indicates that there is a space between the molecules of a body. Salt may be dissolved in water, and, unless too much salt is added, the water will occupy no more space than it did before. This does not prove that water is penetrable, for the molecules of salt occupy the space that the molecules of water do not. Water has been forced through iron by pressure, thus proving that iron is porous.

25. Compressibility is that general property of matter by virtue of which the molecules of a body may be crowded nearer together, so as to occupy a smaller space.

26. Expansibility is that general property of matter by virtue of which the molecules of a body may be forced apart, so as to occupy a greater space.

27. **Elasticity** is that general property of matter by virtue of which if a body be distorted, within certain limits, it will resume its original form when the distorting force is removed. Glass, ivory, and steel are very elastic.

28. **Indestructibility** is that general property of matter by virtue of which it can never be destroyed. A body may undergo thousands of changes—be resolved into its molecules and its molecules into atoms, which may unite with other atoms to form other molecules and bodies, which may be entirely different from the original body—but the same number of atoms remains. The whole number of atoms in the universe is exactly the same now as it was millions of years ago and will always be the same. Indestructibility is, therefore, a property of all matter.

29. **Hardness** is a special property of matter by virtue of which solid bodies may be scratched or abraded by others when rubbed against them; thus, iron scratches lead, glass scratches iron, and the diamond scratches glass. Of two bodies, the one that scratches the other is the harder one, that is, possesses greater hardness. Fluids and gases do not possess hardness. The diamond is the hardest substance known.

30. **Tenacity** is that special property of matter by virtue of which some solid bodies resist a force tending to pull them apart. Most metals are tenacious, some more than others; steel is very tenacious.

31. **Brittleness** is that special property of matter by virtue of which some solid bodies are easily broken. Glass, crockery, bricks, etc. are brittle.

32. **Malleability** is that special property of matter that renders some solid bodies capable of being hammered or rolled into various shapes, as bars or sheets; gold is the most malleable of all substances known.

33. **Ductility** is that special property of matter that renders some solid bodies capable of being extended by drawing, as for instance of being drawn into wire; platinum is the most ductile of known substances.

MOTION AND REST

34. Motion is defined as change of position. A body is said to be in motion when it changes its position relative to surrounding objects.

35. Rest is the absence of motion; that is, the permanence of position of a body in relation to surrounding objects.

If a stone is rolled down hill, it is in motion in relation to the hill. If a person is in a railway train and walks in the opposite direction from that in which the train is moving and with the same speed, he will be in motion as regards the train, but at rest with respect to the earth, since, until he gets to the end of the train, he will be directly over the spot at which he was when he started to walk. Using the earth's surface as a standard, a body is said to be at rest when its position on the earth's surface remains unchanged, and it is said to be in motion when its position on the earth's surface changes.

36. The **path** of a body in motion is the successive positions that the body occupies. Each point of a moving body describes a line; hence, the path of a moving point is a line.

Bodies are conceived as being made up of a very large number of extremely small bodies having the same properties (except size) as the body composed of them. These small bodies are called *particles* or *points;* they are not mathematical points, which have no dimensions whatever, but nevertheless are considered as being too small to be measured. In order to determine the motion of a moving body, it is customary to consider the motion of one of its points; the one usually selected—and the one always meant, unless otherwise especially stated—is the center of the body. As the body moves through space, the center occupies an indefinite number of points, which make up a line that is called the path of the body, and the body is then said to *describe a path.*

37. The **direction** of the motion of a point is the straight line between one position and the next one succeeding. The

direction may be always the same, in which case all the succeeding positions of the moving point lie in a straight line; the point describes a rectilinear path and the body has **rectilinear motion.** After a moving point has occupied a series of positions in a straight line, the direction of the motion may change, the point occupying successively a series of positions in another straight line; the path made up of several straight lines will be a broken line. The direction of the motion may change at each position of the moving point immediately succeeding the previous one; the path will then be a curved line, and the motion is then called **curvilinear motion.**

38. **Velocity** is the rate at which a moving body travels along its path. When it so moves that equal parts of the path are traversed in equal times, the velocity is said to be **constant** (or uniform) and the motion is said to be **uniform;** in all other cases, velocity and motion are **variable.**

If the flywheel of an engine keeps up a constant speed of a certain number of revolutions per minute, the velocity of any point on the wheel is uniform. A railway train having a constant speed of 40 miles per hour moves 40 miles every hour, or $\frac{40}{60} = \frac{2}{3}$ of a mile every minute; and, since equal spaces are passed over in equal times, the velocity is uniform.

39. Unless stated otherwise, the space passed over will be the length of the path of the body, and will be measured in feet and decimals of a foot, and, unless otherwise stated, the time will be measured in seconds. When these units are used, the **unit of velocity** will be *1 foot per second.*

FORCE

40. It is seen from the preceding that a body may be at rest; that it may be in motion; that the motion may be changed either in direction or as regards velocity. To bring a body from a state of rest into a state of motion, *force* must be applied; a force must be used to push, pull, or lift a stone that rests on the ground if it is desired to move

it. To bring a moving body to rest, force must be applied; the brake of a moving car must be set if it is desired to stop it. To change the velocity of motion, force must be applied; the steam pressure must be increased to increase the speed of an engine. To change the direction of motion, force must be applied; a pull on the rudder of a boat is necessary in order to change the course of the boat. In the first case, force produces motion; in the second, it destroys it; in the third, it increases it; in the fourth, it changes its direction or, in other words, imparts another and additional motion to the body.

41. **Force,** therefore, is that which produces or destroys or tends to produce or destroy motion.

42. It ought always to be remembered that every force, whether small or large, requires time to put a body in motion, bring it to rest, or to change the motion of a body as regards velocity or direction. For instance, if one pushes at the crank of a heavy grindstone, it will at first turn but slowly and only gradually acquires the speed desired.

43. Forces are called by various names, according to the effects that they produce on a body, as *attraction*, *repulsion*, *cohesion*, *adhesion*, *accelerating force*, *retarding force*, *resisting force*, etc., but all are equivalent to a push or a pull according to the direction in which they act on a body. That the effect of a force on a body may be compared with another force, it is necessary that three conditions be fulfilled in regard to both forces; they are as follows:

1. *The point of application, or point at which the force acts on the body, must be known.*

2. *The direction of the force, or, what is the same thing, the straight line along which the force tends to move the point of application, must be known; this line is called the line of action of the force.*

3. *The magnitude or value of the force, when compared with a given standard, must be known.*

44. **Measure of Force.**—Forces are measured by comparing them with the most prevalent force. the earth's

attraction or gravity, which is measured by weight; hence, all forces are measurable by weight. The unit of magnitude of forces is generally taken as 1 pound, and all forces are thus spoken of as a certain number of pounds.

45. Representation of Force.—A force may be represented by a line; thus, in Fig. 1, let O be the *point of application* of the force; let the length of the line OA represent its *magnitude;* and let the arrowhead indicate the *direction* in which the force acts, then the line OA fulfils the three conditions (see Art. 43) and the force is fully represented.

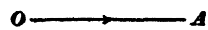


FIG. 1

THE THREE LAWS OF MOTION

46. The fundamental principles of the relations between force and motion were first stated by Sir Isaac Newton; they are called **Newton's Three Laws of Motion,** and are as follows:

1. *All bodies continue in a state of rest, or of uniform motion in a straight line, unless acted on by some external force that compels a change.*

2. *Every motion, or change of motion, is proportional to the acting force and takes place in the direction of the straight line along which the force acts.*

3. *To every action there is always opposed an equal and opposite reaction.*

From the *first law*, it is inferred that a body once set in motion by any force, no matter how small, will move forever in a straight line and always with the same velocity, unless acted on by some other force that compels a change. It is not possible to completely verify this law, on account of the earth's attraction for all bodies, but, from astronomical observations, we are certain that the law is true. This law is often called the **law of inertia.** (See Art. 21.)

The *second law* states the relation between the force acting on a body and the effect produced. Suppose that a man pushing a heavy wagon is able to bring it up to a certain

speed in, say, 10 seconds; then two men, each exerting the same force and pushing together, will be able to bring it up to double the speed in the same time.

Another consequence of the second law is that, if two or more forces act on a body, their final effect on the body will be in proportion to their magnitude and to the directions in which they act. Thus, if the wind is blowing due west with a velocity of 50 miles per hour and a ball is thrown due north with the same velocity, or 50 miles per hour, the wind will carry the ball just as far west as the force of the throw will carry it north, and the combined effect will be to cause it to move northwest. The amount of departure from due north will be proportional to the force of the wind and independent of the velocity due to the force of the throw.

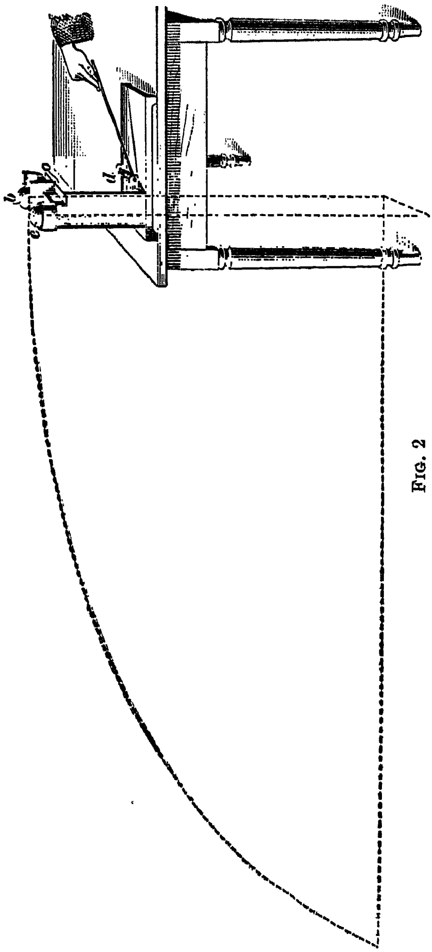


FIG. 2

In Fig. 2, a ball *e* is supported in a cup, the bottom of which is attached to the lever *o* in such a manner that a movement of *o* will swing the bottom horizontally and allow the ball to drop. Another ball *b* rests in a horizontal groove that is provided with a slit in the bottom. A swinging arm

is actuated by the spring *d* in such a manner that, when drawn back, as shown, and then released, it will strike the lever *o* and the ball *b* at the same time. This gives *b* an impulse in a horizontal direction and swings *o* so as to allow the ball *e* to fall. On trying the experiment, it is found that *b* follows a path shown by the curved dotted line, and reaches the floor at the same instant as *e*, which drops vertically. This shows that the force which gave the first ball its horizontal movement had no effect on the vertical force which compelled both balls to fall to the floor, the vertical force producing the same effect as if the horizontal force had not acted. The second law may also be stated as follows:

A force has the same effect in producing motion, whether it acts on a body at rest or in motion and whether it acts alone or with other forces.

The *third law* states that forces in nature occur in pairs, never singly. Thus far a single force has been mentioned

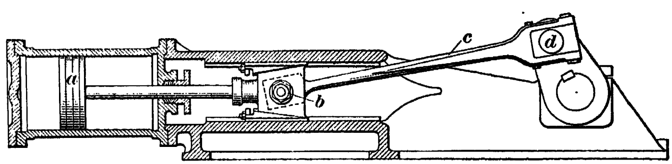


FIG. 3

as acting on a body, but in reality there are always two forces concerned in any manifestation of force. Take the case of a book lying on a table. The weight of the book causes it to exert a pressure on the table; hence, when the table is the body, the downward pressure of the book is one of the forces acting on the table. But taking the book as the body, the table exerts on it an upward pressure just equal to the weight of the book. The reaction of the table on the book is equal and opposite to the action of the book on the table.

As another example of this law take the case of the steam-engine mechanism, Fig. 3. The steam presses to the right on the piston *a* and the piston pushes to the left on the steam. The crosshead pin *b* pushes against the connecting-rod *c*

and the rod pushes back against the pin. The cross-head pushes downwards on the guide bar and the bar pushes upwards on the crosshead. The connecting-rod pushes against the crankpin *d* and the pin pushes against the rod. At each point of contact, therefore, there are two forces, the action and the reaction, and these are opposite and equal.

In problems relating to the action of forces it is necessary to be very careful to take the right one of the two forces of each pair. A rule that must be resolutely followed is this:

Rule.—*Take some one body as the body under consideration, and take the forces acting on this body, not those exerted by this body on other bodies.*

Thus, if the connecting-rod *c* is the body, take the pressure of the pin *b* on this rod; but if the crank is the body considered, then take the equal and opposite pressure of the rod on the pin *d*.

A consequence of the third law is that if two bodies press against each other and are at rest, so that there is no friction, *action and reaction, that is, pressure and counter pressure take place in the direction perpendicular to the plane of contact of the two bodies.* This will be reverted to again when dealing with friction.

47. On the three fundamental laws just given the science of *mechanics* rests.

Mechanics is that science which treats of motion and forces. There are four distinct problems the solution of which mechanics affords; they are:

1. The question as to the *equivalence of forces*, or set of forces; that is, the establishment of the conditions under which two forces or two sets of forces acting on the same body are equivalent, in other words produce the same effect.

2. The question as to the *equilibrium of forces*, that is, the establishment of the conditions under which various forces acting on the same body are in equilibrium. Forces are in **equilibrium** when their joint action does not change the motion of the body; when, in particular, the body is at rest and is acted on by the forces, it remains at rest.

3. The investigation of all the possible *motions of bodies without regard to the forces producing them.*

4. The investigation of the *motions of bodies as dependent on the forces acting on them.*

According to these four problems, the science of mechanics has been subdivided as follows: That part of mechanics which treats of motion without considering the forces, problem (3) is called **kinematics**, while the other three problems, in which forces are considered, are treated by that part of mechanics called **dynamics**, which is again subdivided into two parts; viz., **statics**, which treats of problems (1) and (2), that is, of the equivalence and equilibrium of forces, and **kinetics**, which treats of problem (4). Another subdivision divides the subject into *statics* and *dynamics* only, the one comprising problems (1) and (2), and the other problems (3) and (4); that is, this divison of the subject does not class statics under dynamics.

48. In the preceding definitions, the bodies are assumed to be rigid solids; the effects of forces on gaseous and liquid bodies are studied in the following division of mechanics:

49. Pneumatics treats of the laws of the pressure and of the movement of air and other gaseous bodies.

50. Hydrostatics treats of the equilibrium of liquids.

51. Hydrokinetics (also called hydraulics and hydrodynamics) treats of liquids in motion, and the effects that they may produce.

52. Thermodynamics treats of the mechanical effects of heat on bodies.

COMPOSITION AND RESOLUTION OF FORCES

COMPOSITION OF FORCES

53. To **compose** two forces means to find a third force that will be equivalent to the two given forces, that is, will have the same effect as the two given forces combined. The third force found by composition is called the **resultant.** The process of finding the resultant is called **composition of forces.**

54. Forces Having Same Line of Action.—If two forces having the same point of application have the same direction, one tends to increase the effect of the other. Their resultant is equal to their sum and has the same direction. It is represented by a line having the combined length

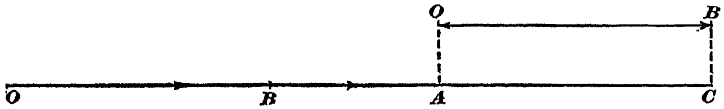


FIG. 4

of the given forces. Thus, in Fig. 4, the resultant of the forces OA and OB is equal to $OC = OA + OB$.

55. If two forces having the same point of application act in exactly opposite directions, one tends to decrease the effect of the other. Their resultant is equal to their difference and has the direction of the larger one of the two given forces. It is represented by a line having a length equal to the difference of the lengths representing the given forces and whose arrowhead points in the direction of the greater force. Thus, in Fig. 5, the resultant of the forces OA and OB is equal to $OC = OA - OB$.

56. If the forces acting in one direction, say to the right, are called *positive* and those in the opposite direction *negative*, the statements in Arts. **54** and **55** may be combined into one

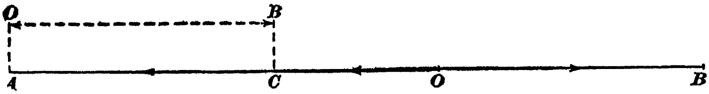


FIG. 5

viz.: the resultant of two forces that have the same point of application and line of action is equal to their algebraic sum.

57. Forces Having Different Lines of Action.—If two forces have the same point of application but different lines of action, their resultant is represented by the diagonal of a parallelogram, whose two sides joining at one end of the diagonal represent the given forces. Thus, in Fig. 6, the resultant of the forces OA and OB is represented by the diagonal OC of the parallelogram $OACB$.

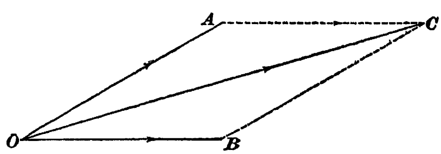


FIG. 6

58. The proof of the statements in Arts. **54, 55,** and **57** follows from the second law of motion. Since the effect of a force is proportional to the magnitude of the force, the length of the line representing the magnitude of the force also represents, to a certain scale, the effect that the force produces, and thus also the velocity that it can give to the body in a certain time; it also represents the length of the path that the body travels in a certain time under the influence of the force.

According to the second law of motion the final effect of two forces on a body will be the same as if one acted after the other. Thus, in the case of two forces having the same direction, Fig. 4, the body will move to A a distance equal to OA under the action of the force OA and from A to C a

total distance equal to OB under the action of the two forces OA and OB.

In the case of two forces acting in opposite directions, Fig. 5, the body will move to A a distance equal to OA under the action of the force OA, and from A in opposite direction to C a distance equal to OB under the action of the force OB.

In case of two forces having different lines of action, Fig. 6, the body will move to A a distance equal to OA under the action of the force OA, and from A to C in the direction of—that is, parallel to—OB a distance equal to OB under the action of the force OB. Join B and C, then BC is parallel to OA, and $OACB$ is a parallelogram called the **parallelogram of forces.**

The final effect in all three cases is a motion of the body from O to C, which is measured by the length of this line, representing to a certain scale the magnitude of a force that would have the same effect as the two given forces combined, that is, their resultant.

The body under the combined and simultaneous action of the two given forces does not, of course, actually travel from O to A first and then from A to C, but from O to C direct. To prove this, imagine that the two forces in Fig. 6 act alternately, first OA for a brief period of time, then OB for an equally brief period of time, next again OA, then OB, etc. Evidently the path of the body would be the zigzag line $O–a–c–a'–c'–a''–c''–a'''–C$, Fig. 7. Next, imagine the brief periods of the time to be chosen smaller and smaller until at last they act at the same time, then the zigzag line will reduce to a straight line OC, the diagonal of the parallelogram $OACB$, that is, the body will move along this diagonal; in other words, this diagonal is the line of action of the resultant.

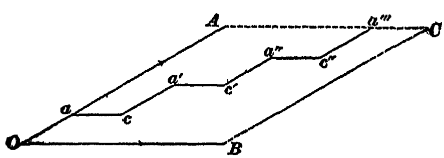


FIG. 7

EXAMPLE.—If two forces act on a body at a common point, both acting away from the body, and the angle between them is 80°, what

is the value of the resultant, the magnitude of the two forces being 60 pounds and 90 pounds, respectively?

SOLUTION.—Draw an indefinite line OB, Fig. 8; by means of a protractor, lay off the angle of 80° and draw line OA. Suppose a line 1 in. long be taken to represent 10 lb.; then measure off $OA = 60 \div 10 = 6$ in. and $OB = 90 \div 10 = 9$ in., and the lengths of these lines will represent 60 lb. and 90 lb., respectively, to a scale of 10 lb. = 1 in. Through A, draw AC parallel to OB, and through B draw BC parallel to OA, intersecting at C. Then draw OC, and OC will be the resultant; its direction is toward the point C, as shown by the arrow. By measurement, OC is found to be 11.7 in. long; hence, $11.7 \times 10 = 117$ lb. Ans.

CAUTION—In solving problems by this method, use as large a scale as possible; more accurate results are then obtained. Be very careful also in laying out angles by means of the protractor. Use a very sharp pencil in marking.

59. The example just given might also have been solved by the **triangle of forces.** Since the line AC, Figs. 6 and 8, is equal and parallel to the line OB, it represents in magnitude and direction the same force that OB represents. Hence, to find the resultant of two forces we may proceed thus: Lay off OA to represent one of the forces and from the end A lay off AC parallel to OB, to represent the other; then joining the starting point O with the end point C the line OC is obtained, which represents the resultant in magnitude and direction. Notice that the direction of OC is *opposed* to that of OA and AC, by which is meant that, starting from the point O where the triangle was begun and tracing the two lines representing the given forces, the pencil will have the same general direction around the triangle, as if passing around a circle, from left to right, or from right to left, but that the closing line or resultant must have an opposite direction, that is, the two arrowheads must point toward the point of intersection of the resultant and the last side.

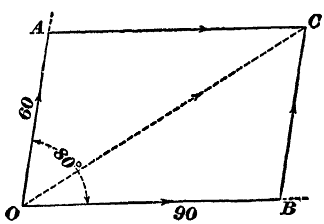


FIG. 8

60. When three or more forces act on a body at a given point, their resultant can be found by first composing two

forces, composing their resultant with the third, and so on. This holds good no matter if the forces all act in the same plane or in different planes. In this treatise on mechanics *all forces will be considered as acting in the same plane.*

EXAMPLE.—Find the resultant of all the forces acting on the point O, Fig. 9, the length of the lines being proportional to the magnitude of the forces.

SOLUTION.—Draw OE parallel and equal to AO, and EF parallel and equal to BO, then OF is the resultant of these two forces and its direction is from O to F, opposed to OE and EF. Treat OF as if OE and EF did not exist, and draw FG parallel and equal to OC; OG will be the resultant of OF and FG; but OF is the resultant of OE and EF, hence, OG is the resultant OE, EF, and FG, and

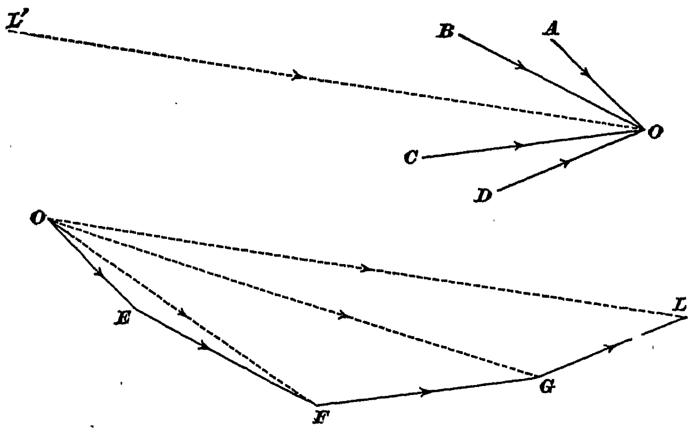


FIG. 9

likewise of AO, BO, and CO. The line FG, parallel to CO, could not be drawn from the point O to the right of OE for in that case it would be opposed in direction to OF; but FG must have the same direction as OF, in order that the resultant may be opposed to both OF and FG. For the same reason, draw GL parallel and equal to DO. Join O and L, and OL will be the resultant of all the forces AO, and BO, CO and DO (both in magnitude and direction), acting at the point O. If $L'O$ were drawn parallel and equal to OL and having the same direction, it would represent the effect produced on the body by the combined action of the forces AO, BO, CO, and DO.

61. Polygon of Forces.—It will be noticed that OE, EF, FG, GL, and LO, Fig. 9, are sides of a polygon $OEFGL$, in which OL the resultant is the closing side,

and that its direction is opposed to that of all the other sides. It is, therefore, not necessary to compose first two forces and their resultant with a third and so on by the method of the triangle of forces, but the resultant of any number of forces having the same point of application may be found by drawing the polygon direct; the process is then called the **polygon of forces.** The polygon so drawn is called the **force polygon.**

EXAMPLE.—If five forces act on a body at angles of 60°, 120°, 180°, 240°, and 270° toward the same point, and their respective magnitudes are 60, 40, 30, 25, and 20 pounds, find the magnitude and direction of their resultant by the method of polygon of forces.*

SOLUTION.—From a common point O, Fig. 10, draw the lines of action of the forces, making the given angles with a horizontal line

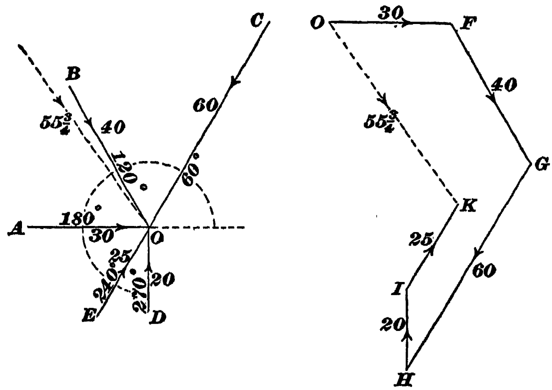


FIG. 10

through O and mark them as acting toward O by means of arrowheads, as shown. Now, choose some convenient scale, such that the whole figure may be drawn in a space of the required size on the drawing. Choose one of the forces, as $A\,O$, and draw $O\,F$ parallel to it, and equal in length to 30 lb. on the scale; it must also act in the same direction as $O\,A$. Indicate this by an arrowhead. At F, draw $F\,G$ parallel to one of the other forces, as $B\,O$, which equals 40 lb. In a similar manner, draw $G\,H$, $H\,I$, and $I\,K$ parallel to $C\,O$, $D\,O$, and $E\,O$, and equal to 60 lb., 20 lb., and 25 lb., respectively. Join O and K

*All angles are measured from a horizontal line, in a direction opposite to the movement of the hands of a watch (from around the circle to the left), from 1° or less up to 360°.

by OK, and OK will be the resultant of the combined action of the five forces; its direction is opposite to that of the other forces around the polygon $OFGHIK$ and its magnitude is $55\frac{3}{4}$ lb. Ans.

62. If the resultant OK, Fig. 10, were to act alone on the body in the direction shown by the arrowhead, with a force of $55\frac{3}{4}$ pounds, it would produce exactly the same effect on a body as the combined action of the five forces.

If OF, FG, GH, HI, and IK represent the distances and directions that the forces would move the body, if acting separately, OK is the direction and distance of movement of the body when all the forces act together. The arrowhead indicating the direction in which the force acts should be placed on each line.

In selecting the scale, it will be well to bear in mind the following considerations: The size of the drawing will depend entirely on the desired accuracy of the results. If the shortest line of the polygon is 10 inches long and a scale divided into hundredths of an inch is used, four significant figures can be laid off exactly. For suppose that it were required to lay off a line to represent 15,462 pounds; by choosing a scale of 1,000 pounds = 1 inch, the length of the line would be 15.462 inches. An ordinary scale cannot be read to thousandths and, hence, the length laid off would be 15.46 inches. It is not usually practicable to work closer than to three significant figures, and no greater accuracy than this will be required in connection with this subject. Referring to Fig. 10, the smallest force is 20 pounds; with a scale of 10 pounds = 1 inch, the line HI will be 2 inches long and HG will be $60 \div 10 = 6$ inches long. If the polygon were drawn accurately, the resultant could easily be measured to three significant figures. If a scale divided into hundredths is not at hand, one divided into sixteenths or thirty-seconds—better into sixty-fourths—may be used. If divided only into sixteenths, with a little practice, sixty-fourths can be laid off by eye. With such a scale, the shortest line on the diagram must not be less than $100 \div 64 = 1\frac{9}{16}$ inches long in order to measure correct to three significant figures. To lay off 15,462 pounds with this scale,

select a scale for the diagram of 6,400 pounds = 1 inch; then the length of the line to be measured is 15,462 ÷ 6,400 $= 2\frac{26.62}{64}$ inches, or say $2\frac{27}{64}$ inches. That this is correct to three figures is readily seen by multiplying the length by the scale. Thus, $2\frac{27}{64} \times 6{,}400 = 15{,}500$ pounds. But 15,462 pounds expressed to three significant figures is 15,500 pounds also.

The number 6,400 was arrived at as follows: It is readily seen that 15,462 divided by 64 or 640 gives results much larger than $1\frac{9}{16} = 1.5625$ inches, but if divided by 6,400 the result is a single figure in the integral part of the quotient. When the measuring scale is divided into sixteenths, thirty-seconds, etc., it is most convenient to use for the scale of forces 1, 10, 100, etc. times 64 or .1, .01, etc. times 64, the object being to get one figure in the integral part of the quotient obtained by dividing the smallest force by the scale, provided that the quotient is not less than $1\frac{9}{16}$. In Fig. 11, the smallest force is 20 pounds; hence, the scale should be 6.4 pounds = 1 inch, in order to obtain three significant figures in the result. The lengths of the various lines will then be $20 \div 6.4 = 3\frac{.8}{6.4} = 3\frac{8}{64} = 3\frac{1}{8}$ inches; $30 \div 6.4 = 4\frac{11}{16}$ inches; $29 \div 6.4 = 4\frac{17}{32}$ inches: $37 \div 6.4 = 5\frac{25}{32}$; $40 \div 6.4 = 6\frac{1}{4}$ inches; and $25 \div 6.4 = 3\frac{29}{32}$ inches. The same results may also be obtained as follows: $20 \div 6.4 = 200 \div 64 = 3\frac{1}{8}$; $30 \div 6.4 = 300 \div 64 = 4\frac{11}{16}$, etc. This same scale would also be used in drawing Fig. 10.

It is more convenient in every way to use a measuring scale divided into hundredths as the same accuracy can be obtained with a shorter line; thus, with such a measuring scale and a scale of forces of 10 pounds = 1 inch, the lengths of the lines in Fig. 11 would be 2 inches, 3 inches, 2.9 inches, 3.7 inches, 4 inches, and 2.5 inches.

It is not necessary to take the forces in order, as shown in Fig. 11; any force may be laid off at any time. Advantage is taken of this fact to reduce the area covered by the diagram.

It may happen that a scale chosen in accordance with the foregoing will produce a diagram inconveniently large; in such cases, multiply the scale by 2, 3, 4, etc. up to 9, until the shortest line is reduced to the length desired, which must not be less than 1 inch, when the measuring scale is divided into hundredths; or less than $1\frac{9}{16}$ inches, when the measuring scale is divided into sixteenths, thirty-seconds, etc. For example, if the smallest force to be laid off is 5,800 pounds,

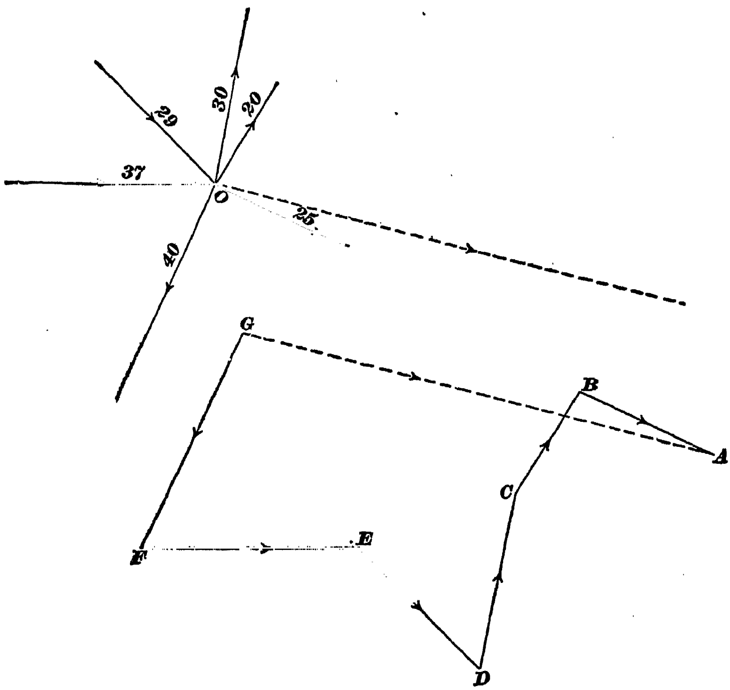


FIG. 11

the scale selected may be either 5,000 pounds = 1 inch or 5×640 pounds = 3,200 pounds = 1 inch; in one case, the length of the shortest line is $5,800 \div 5,000 = 1.16$ inches and in the other case, $5,800 \div 3,200 = 1\frac{13}{16}$ inches.

To obtain the best results, a hard, sharp-pointed pencil should be used and the utmost care must be taken to measure the lengths exactly and to draw the lines correctly in position.

The larger the scale used, the more accurately can these results be accomplished.

EXAMPLE.—Find the resultant of the forces shown in Fig. 11. Notice that, while in the previous examples all the given forces acted toward the common point of application, in this example some act toward and some away from the common point of application. This is to be carefully considered in placing the arrowheads in the force polygon.

SOLUTION.—Take any convenient point G, and draw a line GF, parallel to one of the forces, say the one marked *40*, making it equal in length to 40 pounds on the scale, and indicate its direction by the arrowhead. Take some other force, the one marked *37* will be convenient; the line FE represents this force. From the point E, draw a line parallel to some other force, say the one marked *29*, and make it equal in magnitude and direction to it. So continue with the remaining forces, taking care that the general direction around the polygon is not changed. The last force drawn in the figure is BA, representing the force marked *25*. Join the points G and A; then GA is the resultant of all the forces shown in the figure. Its direction is from G to A opposed to the general direction of the others around the polygon. It does not matter in what order the different forces are taken, the resultant will be the same in magnitude and direction, if the work is done correctly.

63. From what has been stated before, it is seen that any number of forces acting on a body at the same point, or having their lines of action pass through the same point, can be replaced by a single force (resultant), whose line of action shall pass through that point. The various methods so far described for finding the resultant of forces by drawing are called **graphical methods.**

64. Methods of Calculation.—The resultant may, however, be calculated. In the case of two forces having the same line of action, the resultant is the sum or difference of the forces. In the case of two forces having different lines of action, not only the magnitude of the resultant must be calculated but also the angles that it forms with the original forces, the angles that these form with each other being given. This requires the application of the principles of trigonometry. The calculation of the resultant will here be considered only in connection with those cases where the given forces act at right

angles to each other, it being then more convenient, as a rule, to perform the calculation than to apply the graphic method. In this case, the parallelogram of forces is a rectangle $AOBC$, Fig. 12, the triangle of forces, OAC or OBC, is a right triangle, and the magnitude of the resultant is found by the geometric principle that the square of the hypotenuse is equal to the sum of the squares of the sides. The angle AOC giving the direction of the resultant is found by the trigonometric rule, that *in a right triangle, the tangent of either of the two oblique angles is equal to the side opposite divided by the side adjacent.*

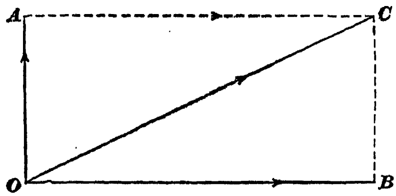


FIG. 12

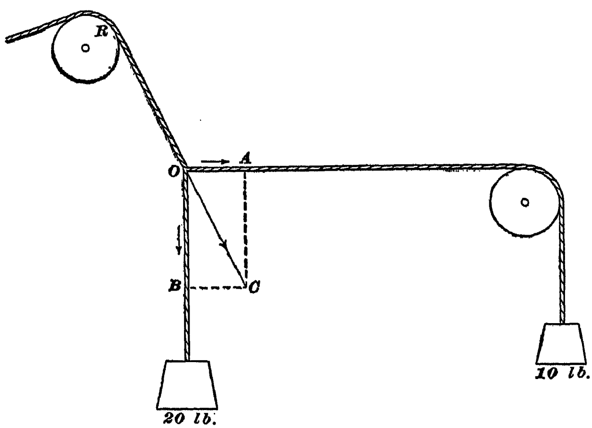


FIG. 13

Expressed as formulas, referring to Fig. 12,

$$\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2$$

$$OC = \sqrt{\overline{OA}^2 + \overline{OB}^2} \qquad (1)$$

$$\left.\begin{aligned} \tan AOC &= \frac{OB}{OA} \\ \tan COB &= \frac{OA}{OB} \end{aligned}\right\} \qquad (2)$$

EXAMPLE.—From a drum R, Fig. 13, is suspended by a rope a weight of 20 pounds. Another rope is attached at some point O of the first rope and led over a pulley to another weight of 10 pounds; the pull on this rope being horizontal, what is the pull on the part of the first rope above the point O, and what angle will it form with a plumb-line from O?

SOLUTION.—Lay off on the horizontal line OA = 10 lb. and on the vertical line OB = 20 lb.; complete the parallelogram—in this case a rectangle—$OACB$ and draw the diagonal OC. Then OC represents the magnitude and direction of the pull in the part of the rope above O. In the right triangle OAC, the sides are $OA = 10$, $AC = 20$, therefore OC, the hypotenuse, is $\sqrt{10^2 + 20^2} = \sqrt{100 + 400} = \sqrt{500} = 22.36 +$ lb. The tangent of the angle BOC is equal to $\frac{\text{side opposite}}{\text{side adjacent}} = \frac{10}{20} = \frac{1}{2}$, which by a table of tangents, is 26° 33′ 53.5″.

RESOLUTION OF FORCES

65. To **resolve** a force into two others means to find two forces, the resultant of which is the force given. The two forces so found are called **components** of the given force.

The resolution of a force into two components is the reverse of finding the resultant of two forces.

Thus, to resolve a force into two forces, a parallelogram is constructed whose diagonal represents the given force and whose two sides, concurring at one end of the diagonal representing the point of application, are the components. For example, in Fig. 14, let OA be the force to be replaced by two components having the lines of action OB and OC, respectively. Construct the parallelogram of forces and so find the magnitude of OB and OC. Instead of constructing the parallelogram $OBAC$, it is sufficient to construct the triangle of forces OBA or OCA.

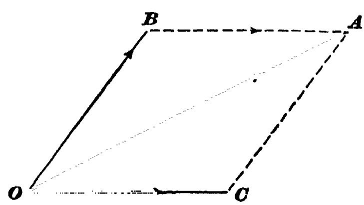
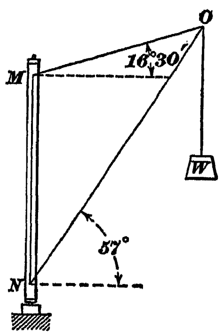


FIG. 14

EXAMPLE.—From a wall crane, Fig. 15, is hung a load W of 3,000 pounds, which exerts a pull in the direction of the piece OM and a push in the piece ON; what are the magnitudes of these two forces?

Solution.—Draw a vertical line OA representing the weight $W = 3,000$ to a certain scale. Next draw, through O, a line parallel to the piece MO making an angle AOB of $90° + 16° 30' = 106° 30'$ with OA; and through A a line parallel to the piece NO, making an angle OAB of $90° - 57° = 33°$ with OA. The last two lines intersect at B. Mark OB and BA with arrowheads opposite in general direction to OA; then these lines represent the forces in the pieces MO, and NO, respectively, in magnitude and direction, as will be understood by completing the force parallelogram $OBAC$. The line



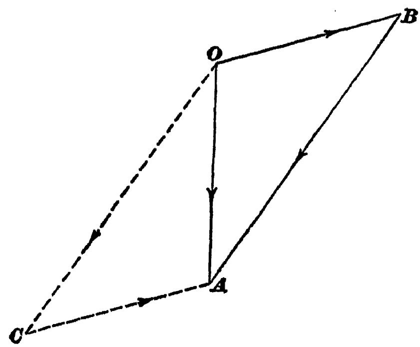


Fig. 15

parallel to MO could also have been drawn through A and that parallel to NO through O, with the same result, the triangle of forces being then the other half of the parallelogram, care being taken in placing the arrowheads correctly. On measuring the lines OB and BA with the same scale as OA, they will be found to represent 2,520 and 4,430 lb., respectively. By trigonometry, the line OB is found to represent 2,515.8 lb. and AB 4,429 lb. Ans.

66. The problem of the resolution of a force treated in the previous article is, in many cases, simplified in practice by the condition that the components shall act at right angles to each other. Only the direction of one of the components, that is, the angle it forms with the given force, need then be known; the parallelogram of forces becomes a rectangle and the triangle of forces a right triangle. In this case, the forces are very easily and

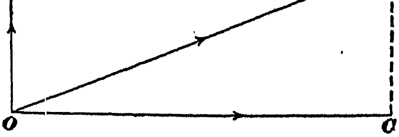


Fig. 16

conveniently calculated by the trigonometric rules for the solution of right triangles, that *either of the sides is equal to the hypotenuse multiplied by the sine of the angle opposite*, or *either of the sides is equal to the hypotenuse multiplied by the cosine of the angle adjacent*, or if only one angle is considered *the side opposite the angle is equal to the hypotenuse multiplied by the sine and the side adjacent is equal to the hypotenuse multiplied by the cosine.*

Expressed as formulas, referring to Fig. 16, these rules become

$$OB = OA \cos BOA = OA \sin COA$$
$$OC = OA \sin BOA = OA \cos COA$$

EXAMPLE.—A body weighing 200 pounds rests on an inclined plane whose angle of inclination to the horizontal is 18°; the weight tends to move the body downwards on the plane and also causes pressure between the body and the plane. What is the magnitude of the component tending to move the body and what is the pressure perpendicular to the plane?

SOLUTION.—Let LMN, Fig. 17, be the plane, the angle M being 18°, and let W be the body, weighing 200 lb. To solve graphically,

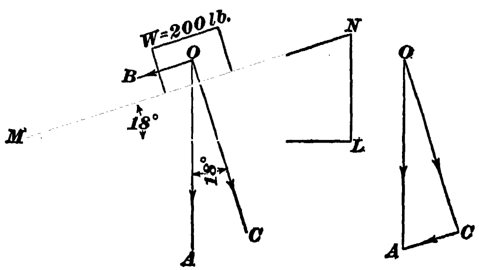


FIG. 17

draw a vertical line OA = 200 lb., to represent the magnitude of the weight. Through A, draw AC parallel to MN; and through O, draw OC perpendicular to MN, the two lines intersecting at C, measure the components CA and OC. By the formula, the calculation is performed thus: Since OA is the direction of the given force perpendicular to ML and OC, the direction of the one component is perpendicular to MN, the angle enclosed between OA and OC is also 18°, and since the angle C is a right angle the components are:

$OC = 200 \times \cos 18° = 200 \times .95106 = 190.212$ lb., and $OB = 200 \times \sin 18° = 200 \times .30902 = 61.804$ lb.

Force parallel to the plane is 61.8 lb. Ans.

Force perpendicular to the plane is 190.2 lb. Ans.

67. Composition of Forces by Means of Resolution.—The resolution of a force into two components acting at right angles to each other affords a ready means for composing any number of forces acting at a common point, by calculation. Let F_1, F_2, F_3, F_4, F_5, Fig. 18, be five

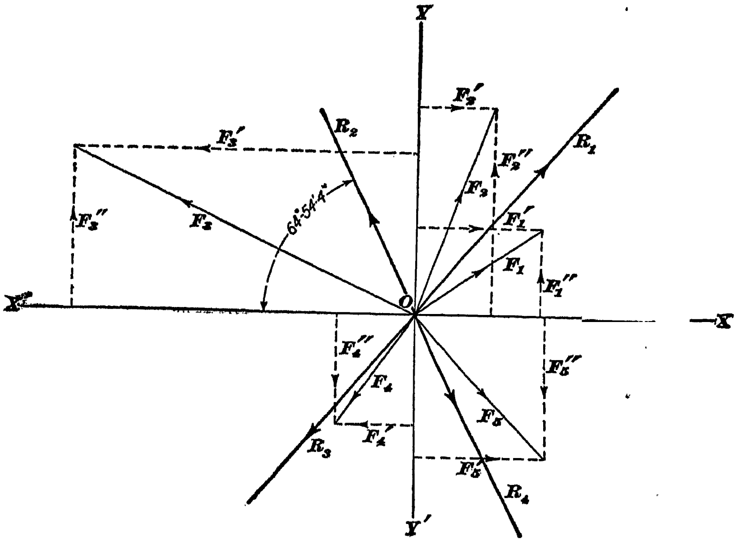


FIG. 18

forces having the common point of application O; through O, let any two lines XX' and YY' be drawn at right angles to each other. Each of the forces may be resolved into two components parallel to XX' and YY', respectively, which according to Art. 66 will be:

$$\left.\begin{array}{l} F_1' = F_1 \cos F_1 OX \\ F_2' = F_2 \cos F_2 OX \\ F_3' = F_3 \cos F_3 OX' \\ F_4' = F_4 \cos F_4 OX' \\ F_5' = F_5 \cos F_5 OX \end{array}\right\} \text{With their lines of action, coinciding with } X'X.$$

$$\left.\begin{array}{l} F_1'' = F_1 \cos F_1 OY \\ F_2'' = F_2 \cos F_2 OY \\ F_3'' = F_3 \cos F_3 OY \\ F_4'' = F_4 \cos F_4 OY' \\ F_5'' = F_5 \cos F_5 OY' \end{array}\right\} \text{With their lines of action, coinciding with } YY'.$$

In the first set of forces having the common line of action $X'X$, there are some that act from O toward X and others that act from O toward X', that is, in the opposite direction. Let us consider the former positive and the latter negative. The algebraic sum of all the components having the line of action $X'X$ is the resultant of the horizontal components; likewise, the algebraic sum of all the components having the line of action YY' is the resultant of the vertical components, calling those from O to Y positive and these from O to Y' negative. Call the resultants thus obtained X and Y, which, according to Art. **64,** can be composed into a resultant by formula **1,** Art. **64.** Then, terming the resultant of all the forces R:

$$R = \sqrt{X^2 + Y^2} \qquad (1)$$

The acute angle that the resultant R makes with $X'X$ is found lastly by formula **2,** Art. **64,** which must now be written:

$$\tan ROX = \frac{Y}{X} \qquad (2)$$

If X and Y are both positive or both negative, the resultant will lie in the quadrant XOY or $X'OY'$, respectively, corresponding to R_1 or R_3; if X is negative and Y positive or X is positive and Y negative, the resultant will lie in the quadrant YOX', or $Y'OX$, respectively, corresponding to R_2 or R_4. Evidently the angles ROY and ROX are to be laid off from OY, OY', OX, OX' according to which of these directions X or Y have been found to have.

EXAMPLE.—In Fig. 18, let $F_1 = 19$ pounds, $F_2 = 28$ pounds, $F_3 = 48$ pounds, $F_4 = 17$ pounds, $F_5 = 24$ pounds, angle $F_1OX = 35°$, angle $F_2OX = 70°$, angle $F_3OX' = 25°$, angle $F_4OX' = 55°$, angle $F_5OX = 48°$; then $F_1OY = 55°$, $F_2OY = 20°$, $F_3OY = 65°$, $F_4OY = 35°$, $F_5OY' = 42°$. Find the resultant of the forces.

SOLUTION.—The algebraic sum of the horizontal components is

$$X = 19 \times \cos 35° + 28 \times \cos 70° - 48 \times \cos 25° - 17 \times \cos 55° + 24 \times \cos 48° = 19 \times .81915 + 28 \times .34202 - 48 \times .90631 - 17 \times .57358 + 24 \times .66913 = 15.564 + 9.5766 - 43.503 - 9.7509 + 16.059 = -12.054,$$ using the first five significant figures only.

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The algebraic sum of the vertical components is

$Y = 19 \times \cos 55° + 28 \times \cos 20° + 48 \cos 65° - 17 \times \cos 35° - 24 \times \cos 42° = 19 \times .57358 + 28 \times .93969 + 48 \times .42262 - 17 \times .81915 - 24 \times .74314 = 10.898 + 26.311 + 20.286 - 13.926 - 17.835 = 25.734$ using but five significant figures.

X being negative and Y positive, the final resultant lies in the quadrant YOX'. Its magnitude is from formula **1**, $R_2 = \sqrt{12.054^2 + 25.734^2} = \sqrt{145.30 + 662.24} = \sqrt{807.54} = 28.42$ lb., and the tangent of the angle it forms with the line OX', from formula **2**, $\tan R_2OX' = \frac{25.734}{12.054} = 2.13489$; and the angle itself, from a table of tangents, $R_2OX' = 64° 54' 4''$. Ans.

EQUILIBRIUM OF FORCES ACTING AT A POINT

68. Definition.—The forces acting on a body are said to be in **equilibrium** when the motion of the body is unchanged by the action of the forces. In particular, if a body is at rest and is acted on by forces, and then remains at rest the forces are in equilibrium.

69. Condition of Equilibrium.—As has been shown, several forces having the same point of application can be replaced by a single force, the resultant. This single force acting on the body, according to Newton's second law, must cause a change of motion. If, therefore, there is to be no change of motion, the magnitude of the resultant must be zero. The condition of equilibrium, therefore, is the following: *Forces acting at the same point are in equilibrium when the resultant of the forces is zero.*

It follows at once that two forces having the same line of action are in equilibrium when they are equal in magnitude and opposite in direction, for their difference, which is their resultant, is then zero.

70. Graphic Condition.—When the forces acting at a point are laid off in succession, as in Fig. 10, from O to K, the single line OK drawn from the starting point O to the final point K represents the resultant. The resultant is zero if K coincides with O, that is, if the force polygon is closed. Hence the following statements:

If a system of forces having the same point of application is in equilibrium, the force polygon for the forces must close.

Conversely, *if the force polygon is closed, the system of forces is in equilibrium.*

Thus, in the case of three forces in equilibrium, they must be such in magnitude and direction that a triangle can be formed of the lines representing them; in the case of four forces, a quadrangle; and so on. Suppose the three forces OA, OB, OC, Fig. 19 (a), to be in equilibrium, placing them

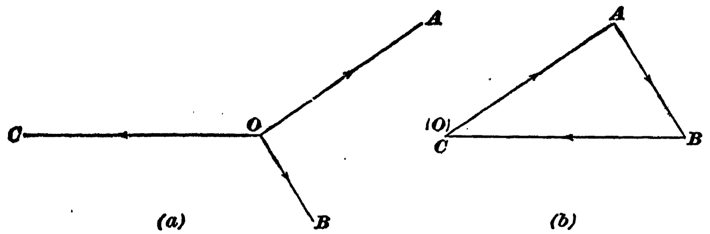


FIG. 19

end for end they will form the triangle ABC, Fig. 19 (b), $C(O)A$ representing the force OA, AB the force OB, BC the force OC. Notice that C and O are the same point, also, that in the force polygon representing three forces in equilibrium the arrowheads all have the same general direction.

71. Algebraic Condition.—The algebraic sum of the horizontal components must be zero, and the algebraic sum of the vertical components must be zero. The resultant of a system of forces acting on a point is, according to Art. **67**, equal to $R = \sqrt{X^2 + Y^2}$ in which X is the resultant—that is, the algebraic sum—of all the horizontal, and Y is the resultant—that is, the algebraic sum—of all the vertical components. If R is to be zero, X and Y must also be zero. Thus we have:

$$X = F_1 \cos A_1 + F_2 \cos A_2 + F_3 \cos A_3 + \ldots = 0$$
$$Y = F_1 \sin A_1 + F_2 \sin A_2 + F_3 \sin A_3 + \ldots = 0$$

if A_1, A_2, A_3 are the angles that the forces form with the horizontal.

72. Let OA and OB, Fig. 20, be two forces and OC their resultant. A force OC' equal and opposite in direction

to $O\,C$ will evidently hold $O\,C$ in equilibrium, but as $O\,C$ is the resultant of $O\,A$ and $O\,B$, it will hold these in equilibrium. A force that holds other forces in equilibrium is called the **equilibrant** of the forces and is equal and opposite in direction to their resultant.

When a system of forces is in equilibrium each force may be regarded as the equilibrant of the others, also any two of three forces in equilibrium may be regarded as the components of a force equal and opposite of the third. To find the equilibrant of two forces is therefore to find their resultant and to reverse its direction. To find two forces that are held in equilibrium with a given force is to find two components of this force and to reverse their direction. The same methods, therefore, apply here as in the composition and

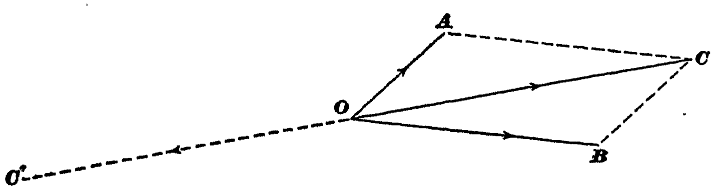


FIG. 20

resolution of forces, with the addition that the forces given and required should have all the same general directions around the force polygon.

To make this more clear, take the example in Art. **65.** The example may be stated thus: From a wall crane, Fig. 15, is hung a weight W. This weight, a pull in the piece $M\,O$, and a push in the piece $N\,O$ are three forces acting on the point O; evidently the forces are in equilibrium for the point O remains at rest. The solution is identically the same as in Art. **65,** only the forces $O\,B$ and $B\,A$ must be marked with arrowheads opposite to those shown in Fig. 15, that is, in the same general direction with $O\,A$, for $B\,O$ and $A\,B$ are now forces acting on the point O, which are the pull and push, respectively, of the pieces $M\,O$ and $N\,O$ on O, not the pull and push exerted by W on these pieces.

EXAMPLE.—An engine crosshead, shown in Fig. 21, is acted on by three forces: (1) the steam pressure P transmitted by the piston

rod; (2) the push T of the connecting-rod; (3) an upward pressure R of the lower guide. If $P = 15{,}000$ pounds, find the other forces, assuming the three to be in equilibrium.

GRAPHIC SOLUTION.—Lay off $C(O)A$ parallel to P and make the length represent 15,000 lb. to some scale. Then through A draw AB parallel to R and through $C(O)$ draw BC parallel to T. The lengths AB and BC represent the magnitudes of R and T to the same scale that CA represents the magnitude of P.

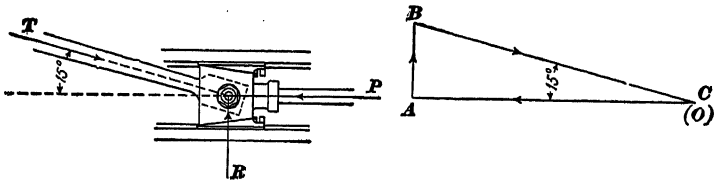


FIG. 21

SOLUTION BY CALCULATION.—The triangle ABC is a right triangle; therefore, the hypotenuse BC is equal to the side AC divided by the cosine of the adjacent angle BOA, or $BC = \frac{15{,}000}{\cos 15°} = \frac{15{,}000}{.96593}$ $= 15{,}529$ lb. to five significant figures. Ans.

The side AB is equal to the other side AC multiplied by the tangent of the angle ACB opposite the first side, or $AB = 15{,}000 \times \tan 15° = 15{,}000 \times .26795 = 4{,}019.3$ lb. to five significant figures. Ans.

EXAMPLES FOR PRACTICE

1. A weight W of 600 pounds hangs from a beam, as shown in Fig. 22; find the pull in the tie-rod and the horizontal thrust along the beam. Solve graphically, and by trigonometry.

Ans. $\begin{cases} \text{Thrust on beam } 1{,}500 \text{ lb.} \\ \text{Pull on rod } 1{,}615.6 \text{ lb.} \end{cases}$

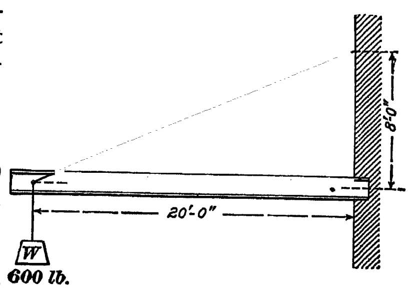


FIG. 22

2. A roller weighing 2,000 pounds rests on an inclined plane; the plane makes an angle of 24° with the horizontal, as shown in Fig. 23. (a) What force F is required to keep the roller from moving down the plane? (b) Find the perpendicular pressure of the roller on the plane. Solve graphically and by trigonometry.

Ans. $\begin{cases} (a) \ 813.48 \text{ lb.} \\ (b) \ 1{,}827.1 \text{ lb.} \end{cases}$

3. A man pulls by a rope on a hook a, Fig. 24, screwed into the wall with a force of 60 pounds; the angle that the rope makes with the wall is 50°. What is the horizontal force tending to pull the hook out of the wall

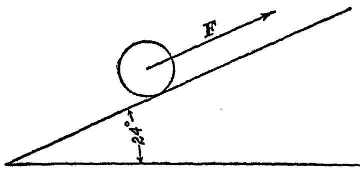


FIG. 23

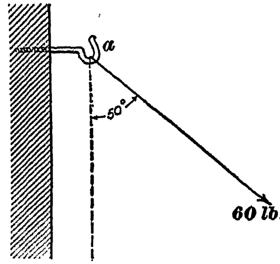


FIG. 24

and what is the vertical force tending to bend it? Solve graphically and by trigonometry.

Ans. $\begin{cases} \text{Horizontal force pulling hook out 45.96 lb.} \\ \text{Vertical force bending hook 38.57 lb.} \end{cases}$

4. Fig. 25 represents a togglejoint press; the pull on the bar B is 50 pounds. What vertical pressure is exerted on the platen P and horizontal pressure on guide G? Solve graphically by resolution or equilibrium, considering first point a, then point b.

Ans. $\begin{cases} \text{Pressure in both toggles } T \text{ and } T = 144 \text{ lb., nearly} \\ \text{Pressure on platen} = 142 \text{ lb., nearly} \\ \text{Pressure on guide} = 25 \text{ lb., nearly} \end{cases}$

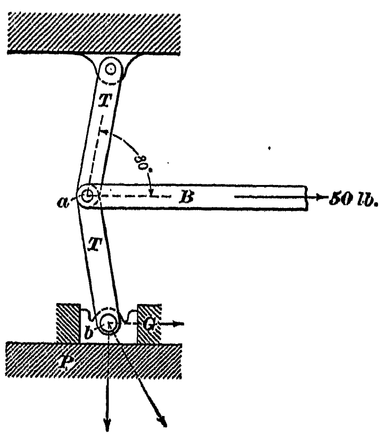


FIG. 25

5. Find the resultant R of the forces shown in Fig. 26.

Ans. $\begin{cases} R = 32.24 \text{ lb., nearly} \\ \text{Angle } \alpha = 2° \, 38', \text{ nearly} \end{cases}$

6. The push of the connecting-rod against the crosshead pin (see Fig. 21) is 8,750 pounds; when the rod makes an angle of 8° 30′ with

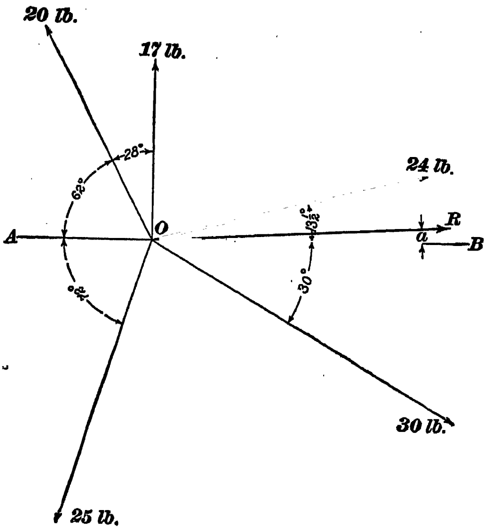


FIG. 26

the line of stroke, find the pressure between crosshead and guide. Solve by trigonometry. Ans. 1,293.3 lb.

ELEMENTARY MECHANICS
(PART 2)

MOMENTS AND COUPLES

MOMENTS

1. Fig. 1 shows a beam, of uniform cross-section, balanced on a so-called knife edge. It is evident that if a force P be applied at some point a, it will cause the beam to turn, or rotate, about the point o. It is further evident that the greater the distance between o and a the greater will be the effect of P in producing rotation. If P is applied at b, $a\,b$ being one-half of $o\,a$, the effect produced by P will be only one-half as much as when P is applied at a. If P is applied at o, it will act directly through the beam on the knife edge and will not tend to rotate the beam at all. If P is applied at some point c to the right of o, it will tend to rotate the beam in the opposite direction. Assuming that $o\,a$ is perpendicular to the direction $d\,a$ of the force P, the product $P \times o\,a$ is called the **moment** of P about o.

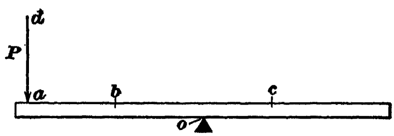


FIG. 1

2. If from any point O, Fig. 2, a perpendicular is drawn to the line of action of force, the product of the magnitude of the force and the length of the perpendicular is called the **moment of the force** about the point O. Thus, in the figure, the moment of force F'' about the point O is $F'' \times O\,A$;

the moment of the force F' about O is $F' \times OB$; and the moment of F''' about O is $F''' \times OC$.

The point O from which the perpendiculars are drawn is the point about which the rotation is assumed to take place, and is called the **center** (or **origin**) **of moments.**

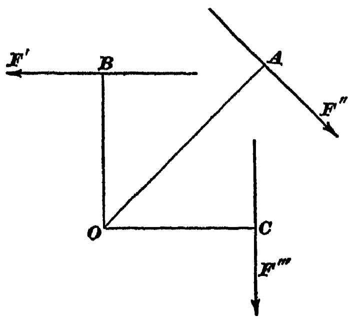


FIG. 2

3. Referring to Fig. 3, it will be seen that the force P tends to turn the beam in the direction of the arrow A, or in a direction opposite to the hands of a watch; the movement so produced is termed **left-hand rotation.** The force Q tends to turn the beam in the direction of the arrow B, or in the same direction as the hands of a watch; the movement so produced is termed **right-hand rotation.** For convenience, any moment that tends to produce left-hand rotation will hereafter be considered positive (+), while a moment tending to produce right-hand rotation will be considered negative (−); hence, the moment of P in Fig. 3 is $+P \times oa$ and of Q is $-Q \times ob$.

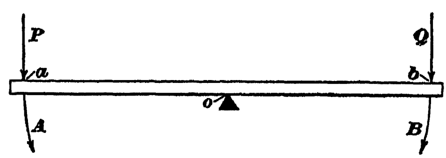


FIG. 3

4. A little consideration will show that in order to prevent the beam from rotating, the moment of P about o must equal the moment of Q about o; in other words, the algebraic sum of the moments of P and Q about o must equal zero, that is, $+P \times oa + (-Q \times ob) = P \times oa - Q \times ob = 0$. Similarly, if there are any number of forces between P and Q, *the algebraic sum of the moments of all the forces must equal zero to produce equilibrium.*

EXAMPLE.—In Fig. 4, o is the center of moments, F_1 is 14 pounds and is 12 inches from o, F_2 is 8 pounds and is 7 inches from o; both forces act in the directions indicated by the arrowheads. What must

be the magnitude and sign of F_3 acting 10 inches to the left of o to produce equilibrium?

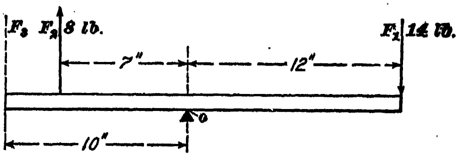


FIG. 4

SOLUTION.—Since the algebraic sum of the moments must equal zero, and since F_1 and F_2 both tend to produce right-hand rotation and are therefore negative, the equation stating the condition becomes

$$-14 \times 12 - 8 \times 7 + F_3 \times 10 = 0$$

whence, $$F_3 = \frac{14 \times 12 + 8 \times 7}{10} = +22.4 \text{ lb. Ans.}$$

REMARK.—The plus sign of the third term of the foregoing equation indicates addition only; it does not show whether the sign of the term is positive or negative. Written out in full with all the signs, the equation is $(-14 \times 12) + (-8 \times 7) + (?F_3 \times 10) = 0$, the interrogation point indicating that the sign of the term is not known. Had F_3 been given as found above and F_2 been unknown, the equation of condition would have been $22.4 \times 10 + F_2 \times 7 - 14 \times 12 = 0$; from which $F_2 = \dfrac{-22.4 \times 10 + 14 \times 12}{7} = -8$ pounds.

5. By means of the principle stated in the last article, the magnitude of any force required to produce equilibrium among a system of parallel forces may be found; or, if the magnitude is known, the distance of the force from any assumed center of moments may be found.

It should be noted that the effect of the forces acting as shown in Fig. 4 is to produce a pressure on the support o. The amount of this pressure is determined by calling the forces acting downwards positive and those acting upwards negative, and adding algebraically. It will be observed that the pressure produced by the forces results in a reaction at o, an upward force against the bottom of the beam. Representing this reaction by R, it is evident that in order to produce equilibrium *the algebraic sum of all the vertical forces acting on the beam must be zero.* In other words

$$F_1 + F_3 + F_2 + R = 14 + 22.4 - 8 + R = 0$$

from which, $$R = -28.4 \text{ pounds}$$

The plus sign is used before R because its real sign is not supposed to be known; it is also used before F_2 to indicate addition only.

6. By means of the two principles just given, the magnitude and position of the resultant of any number of parallel forces may be found. For example, referring to Fig. 5, the sum of the reactions R_1 and R_2 is evidently equal to the sum of the forces F_1, F_2, F_3, F_4 acting on the beam; i. e., $R_1 + R_2 = 40 + 50 + 60 + 40 = 190$ pounds $=$ the resultant of all the parallel forces acting downwards.

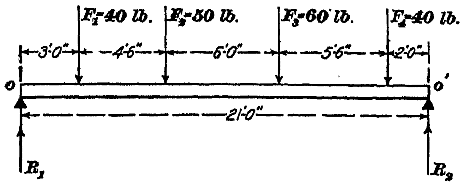


FIG. 5

Taking o for the center of moments, the distance from o to the line of action of R_1 is zero, and

$$R_1 \times 0 - 40 \times 3 - 50 \times 7\tfrac{1}{2} - 60 \times 13\tfrac{1}{2} - 40 \times 19 + R_2 \times 21 = 0$$

from which, $R_2 = 98\tfrac{1}{3}$ pounds

and $R_1 = 190 - 98\tfrac{1}{3} = 91\tfrac{2}{3}$ pounds

Now, assuming that the four forces are replaced by their resultant, 190 pounds, situated at a distance x from the center of moments o,

$$R_1 \times 0 - 190 \times x + 98\tfrac{1}{3} \times 21 = 0$$

from which, $x = 10.87 -$ feet.

7. R_1 might also have been found by taking the center of moments at o', a point on the line of reaction R_2, since the equation of condition would then have been

$$-R_1 \times 21 + 40 \times 18 + 50 \times 13\tfrac{1}{2} + 60 \times 7\tfrac{1}{2} + 40 \times 2 + R_2 \times 0 = 0$$

from which, $R_1 = 91\tfrac{2}{3}$ pounds

This forms a means of checking the work independently.

In making calculations of this kind, it is best to take the center of moments on one of the unknown forces, since its moment then becomes zero; it is not, however, necessary to do this; since if some other point is taken as the center of

moments, the value of $R_1 - R_2$ can be found; and as $R_1 + R_2$ is known, R_1 and R_2 are readily found. For example, suppose that the center of the beam, Fig. 5, is taken as the center of moments; then

$$R_2 \times 10\tfrac{1}{2} - 40 \times 8\tfrac{1}{2} - 60 \times 3 + 50 \times 3 + 40 \times 7\tfrac{1}{2} - R_1 \times 10\tfrac{1}{2} = 0$$

from which, $R_1 - R_2 = -6\tfrac{2}{3}$

But, $R_1 + R_2 = 190$

Hence, $2R_1 = 183\tfrac{1}{3}$, or $R_1 = 91\tfrac{2}{3}$ pounds

and $2R_2 = 196\tfrac{2}{3}$, or $R_2 = 98\tfrac{1}{3}$ pounds

It will be noticed that the operation is shortened and simplified by taking the center of moments on one of the reactions.

COUPLES

8. Two equal parallel forces acting in opposite directions constitute a **couple**. In Fig. 6, suppose that the body A is resting on a flat horizontal surface and that it is acted on by two forces P and Q, which are equal and parallel, but point in opposite directions, as indicated by the arrowheads; then, by definition, these forces constitute a couple.

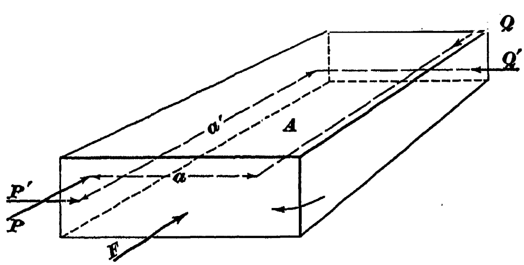


FIG. 6

9. An examination of Fig. 6 shows that the result of the action of the forces P and Q, assuming that A is free to move, is to cause A to rotate; furthermore, this is the only effect that is produced by a couple.

A single force cannot produce equilibrium in a body acted on by a single couple. For, suppose, any force F to act on the body; the only effect produced is to cause the body to

move in the direction of the force F, the couple P, Q making it rotate as before. The only way in which the body can be brought to rest, assuming the forces P and Q to act constantly, is for another couple having an equal rotation effect to act on the body in a contrary direction.

10. The moment of a couple is equal to one of the forces multiplied by the perpendicular distance between the forces. In Fig. 7 let $+P$ and $-P$ be two equal, parallel, and opposite forces, and let o be any point, assumed as the center of moments. Then, the moment of $+P$ is $P \times o\,b$ and of $-P$, $-P \times o\,a$; hence, the moment of the couple, which is the sum of the moments of the two forces composing it, is

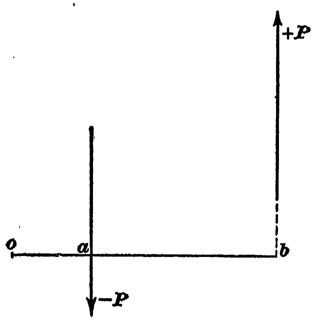


FIG. 7

$$P \times ob - P \times oa = P(ob - oa)$$

But, $o\,b = o\,a + a\,b$; hence,

$$P(ob - oa) = P(oa + ab - oa) = P \times ab$$

Since the center of moments may be taken anywhere with the same numerical result, it follows that the moment of a couple is equal to the product of one of the forces and the perpendicular distance between the forces.

11. Returning to Fig. 6, the body A will be in equilibrium when the moment of the couple $P\,Q$ equals the moment of the couple $P'\,Q'$, the two couples acting in the same plane or in parallel planes, that is, when $P a = P' a'$. Furthermore, it does not matter where either couple acts on the body (all the forces lying in the same plane or in parallel planes); all that is necessary is that their moments be equal and that one couple tends to rotate the body in a direction opposite that of the other.

EXAMPLES FOR PRACTICE

1. Three parallel forces act as shown in Fig. 4. F_2 is 48 pounds and its distance from o is $17\frac{1}{2}$ inches; F_1 is 31 pounds and its distance from o is 13 inches; F_3 is 64 pounds. Assuming that F_1 and F_3 act downwards and F_2 upwards, how far from o must F_3 be to produce equilibrium? Ans. $19\frac{27}{64}$ in.

2. In example 1, assume that F_2 also acts downwards; how far from o must F_3 be to produce equilibrium?

Ans. $6\frac{53}{64}$ in. to the right of o; or, acting upwards, $6\frac{53}{64}$ in. to the left of o.

3. Referring to Fig. 5, suppose that the beam is 19 feet long; that the distance between o and $F_1 = 120$ pounds is 2 feet 5 inches; between F_1 and $F_2 = 96$ pounds is 3 feet 9 inches; between F_2 and $F_3 = 130$ pounds is 4 feet 10 inches; and between F_3 and $F_4 = 160$ pounds is 5 feet 3 inches. What are the reactions R_1 and R_2, the resultant R, and the distance of R from o?

$$\text{Ans.}\begin{cases} R_1 = 247.47+ \text{ lb.} \\ R_2 = 258.53- \text{ lb.} \\ R = 506 \text{ lb.} \\ \text{Distance from } o \text{ to } R = 9 \text{ ft. } 8\frac{1}{2} - \text{ in.} \end{cases}$$

4. In Fig. 6, suppose $P = Q = 42$ pounds, $P' = Q'$, $a = 15$ inches, and $a' = 27$ inches, what must be the value of $P' = Q'$ to keep the block from turning? Ans. $23\frac{1}{3}$ lb.

CENTER OF GRAVITY

PRELIMINARY REMARKS

12. Definition of Center of Gravity.—A flat, thin plate is placed on a knife edge, as in Fig. 8, and shifted until it balances, the line in contact with the edge being ab. Suppose cd and ef are other lines in the plate such that the plate will balance when these lines are in contact with the knife edge. It is found that all such lines pass through one point G; this point is called the **center of gravity** of the plate.

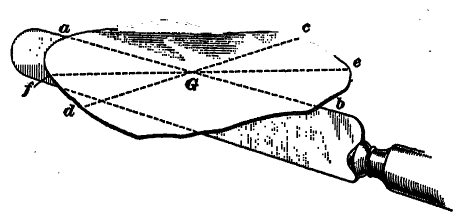


FIG. 8

Suppose that a solid body of any form be suspended from the point A by a cord as c, Fig. 9. Let AB be an imaginary line passing through the body as a prolongation of the cord c; that is, c and AB are in the same vertical line. Suppose that when the body is suspended at C and at F, the lines CD and EF are, respectively, in the same vertical line as the cord. All these lines pass through one point G whatever the point of attachment; therefore, the point G lies in line with the supporting cord c; this point is called the center of gravity of the body.

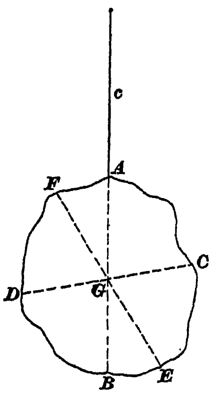


FIG. 9

13. Suppose the thin plate, Fig. 8, to be held in a horizontal position. The weight of each particle of the plate, that is, the force of gravity on each particle, is a downward force. The weights of all the particles, therefore, form a system of parallel forces and these forces have a single resultant. The point where the line of action of this resultant pierces the plate is the center of gravity G.

Likewise the weights of all the particles of the body in Fig. 9 are a system of parallel forces, the resultant of which must be equal and opposite to the upward pull of the cord c. The prolongation of the cord is, therefore, the line of action of this resultant. In whatever position the body may be, there is one point through which the line of action of the resultant of the parallel weights of the particles passes; this point is G the center of gravity.

The center of gravity of a body is that point at which the body may be balanced, or it is the point at which the whole weight of a body may be considered as concentrated.

In a moving body, the line described by its center of gravity is always taken as the path of the body. In finding the distance that the body has moved, the distance that the center of gravity has moved is taken.

14. The definition of the center of gravity of a body may be applied to a system of bodies, if they are considered as being connected at their centers of gravity.

If w and W, Fig. 10, be two bodies of known weights, their center of gravity will be at some point C between them. The point C may be readily determined by applying the principle of moments.

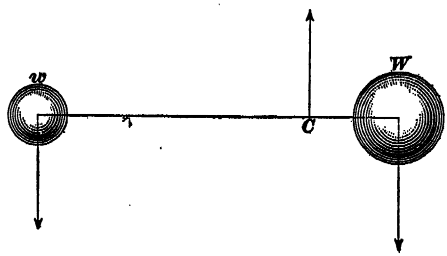


FIG. 10

In Fig. 11, suppose that w and W represent forces w and W which equals the weights w and W, Fig. 10, and $a\,b$ is the distance between them. Then, in order that there may be equilibrium, the force R having the same line of action as the resultant of w and W, but acting in the opposite direction, must pass through the center of gravity of the two bodies. Evidently C must lie in the line $a\,b$ joining the centers of w and W. Pass a line $c\,e$ through b perpendicular to W and to $a\,b$, and take b as the

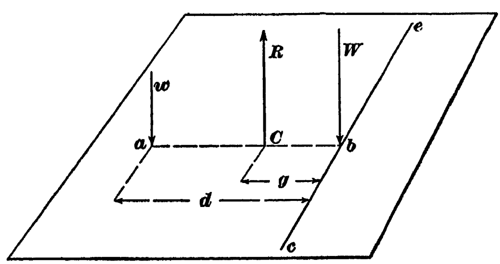


FIG. 11

center of moments. Represent the distances $a\,b$ and $C\,b$ by d and g, respectively; then, since $R = W + w$,

$$w \times d - R \times g = 0, \text{ or } g = \frac{w\,d}{R} = \frac{w\,d}{W + w}$$

EXAMPLE.—In Fig. 10, $w = 10$ pounds, $W = 30$ pounds, and the distance between their centers of gravity is 36 inches; where is the center of gravity of both bodies situated?

SOLUTION.—Applying the formula just given,

$$g = WC = \frac{wd}{W + w} = \frac{10 \times 36}{30 + 10} = 9 \text{ in., and } wC = 36 - 9 = 27 \text{ in. Ans.}$$

15. It is very easy to extend this principle to the finding of the center of gravity of any number of bodies when their weights and the distances apart of their centers of gravity are known, by first finding the center of gravity of two of the bodies as W_1 and W_4, in Fig. 12 at C_1. Assume that the weight of both bodies is concentrated at C_1 and find the center of gravity of this combined weight at C_1 and of W_2 to be at C_2; then find the center of gravity of the combined weights of W_1, W_4, and W_2 (concentrated at C_2) and W_3 to be at C; and C will be the center of gravity of the four bodies. A better way is to proceed as shown in Fig. 13.

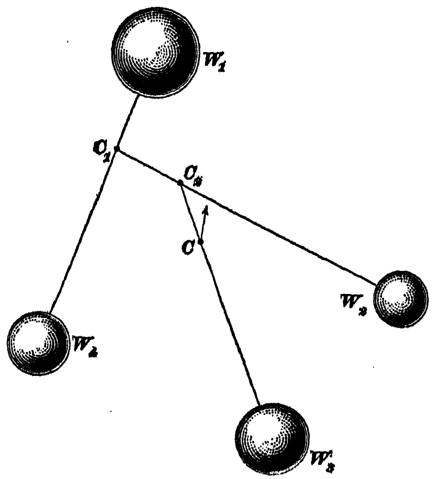


FIG. 12

16. In Fig. 13, let W_1, W_2, etc. be the position and magnitude of the four weights shown in Fig. 12, with reference to a plane perpendicular to the lines of action, and which is therefore horizontal. The lengths of the lines from the points W_1, W_2, etc. (where the lines touch the plane) to their upper extremities represent the weights of the bodies. Draw a line YY_1 through W_4 and W_3; draw another line XX_1 perpendicular to YY_1 and passing through W_3; measure the distances of W_1 and W_2 from YY_1 and the distances of W_1, W_2, and W_4 from XX_1. Then the distance of the center of gravity G from YY_1 may be determined by taking the moments of the forces about YY_1 as an axis, and the distance of G from XX_1 by taking the moments about XX_1 as an axis. YY_1 and XX_1

are called the **axes of reference.** Lay off these distances and draw lines through the points so laid off parallel to the axes; the point of intersection of these lines will be the center of gravity G.

Thus, in the figure (remembering that the force necessary to produce equilibrium acts upwards and is equal to the sum of all the forces; i. e., the weight of the four bodies),

$$W_1 x_1 + W_2 x_2 - (W_1 + W_2 + W_3 + W_4)\, x = 0 \qquad \textbf{(1)}$$

and

$$W_1 y_1 + W_2 y_2 + W_4 y_4 - (W_1 + W_2 + W_3 + W_4)\, y = 0 \qquad \textbf{(2)}$$

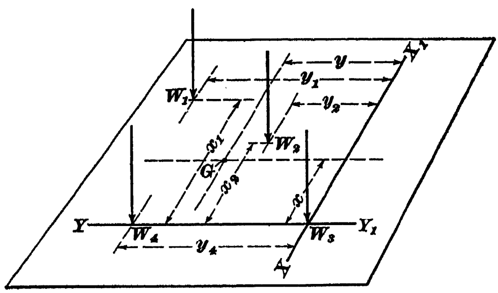


FIG. 13

EXAMPLE.—Suppose $W_1 = 60$ pounds, $W_2 = 18$ pounds, $W_3 = 32$ pounds, $W_4 = 20$ pounds; $x_1 = 15$ inches, $x_2 = 13$ inches, $y_1 = 14$ inches, $y_2 = 5\frac{1}{2}$ inches, and $y_4 = 12$ inches; what is the position of G with reference to XX_1 and YY_1?

SOLUTION.—Substituting in formula **1**,

$$x = \frac{60 \times 15 + 18 \times 13}{60 + 18 + 32 + 20} = \frac{1{,}134}{130} = 8.723+ \text{ in.}$$

$$y = \frac{60 \times 14 + 18 \times 5\frac{1}{2} + 20 \times 12}{60 + 18 + 32 + 20} = \frac{1{,}179}{130} = 9.069+ \text{ in.}$$

A more convenient way of arranging the work is as follows:

$60 \times 15 =$	900	$60 \times 14 =$	840
$18 \times 13 =$	234	$18 \times 5\frac{1}{2} =$	99
$32 \times 0 =$	0	$32 \times 0 =$	0
$20 \times 0 =$	0	$20 \times 12 =$	240
sum 130	1,134	*sum* 130	1,179

$1{,}134 \div 130 = 8.723+$ in. $\quad 1{,}179 \div 130 = 9.069+$ in.

Here each force is multiplied by its distance from the axes and the sum of the products is divided by the sum of the forces. The distance of G from YY_1 is 8.723 in., and from XX_1 is 9.069 in. Ans.

SYMMETRICAL AND IRREGULAR FIGURES

17. A figure is said to be **symmetrical** with respect to a line (called an **axis of symmetry**) when, if the figure be folded on this line one part coincides with the other in every respect. Thus, if a circle be folded on one of its diameters, the two semicircles will coincide; see (*a*), Fig. 14. If an isosceles triangle be folded on a line passing through the vertex perpendicular to the base, the two parts will coincide; see (*b*), Fig. 14. A rectangle may be folded on a line passing through the center and parallel to the base or perpendicular to the base; see (*c*), Fig. 14.

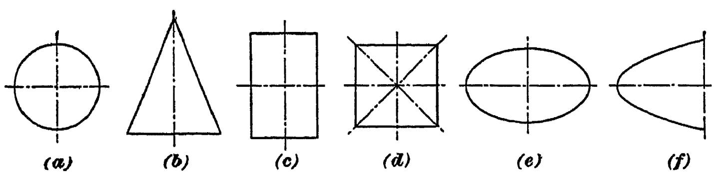


FIG. 14

A figure is said to be symmetrical with respect to one axis when there is only one line on which it can be folded, as the isosceles triangle (*b*) and the parabola (*f*), Fig. 14; the rectangle and ellipse—see (*c*) and (*e*), Fig. 14—have two axes of symmetry; the square—see (*d*), Fig. 14—has four axes of symmetry; the circle has an infinite number of axes of symmetry.

18. Whenever a figure has an axis of symmetry, the center of gravity lies on that axis; if the figure has more than one axis of symmetry, the center of gravity lies at the point of intersection of any two of the axes. Hence, when finding the center of gravity of a figure, first notice whether it has any axis of symmetry.

Whenever a figure has an axis of symmetry only one axis of reference is necessary, when applying the method described in Art. **16,** which should probably be taken at right angles to the axis of symmetry.

CENTER OF GRAVITY OF PLANE FIGURES

LINES AND PERIMETERS

19. Straight Line.—The center of gravity of a *straight line* is at its middle point, since the line is symmetrical with respect to an axis passing through the middle point.

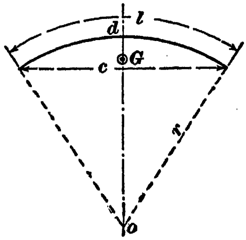


FIG. 15

20. Circular Arc.—The center of gravity of an *arc of a circle* lies on a radius bisecting the arc and at a distance from the center equal to the chord times the radius divided by the length of the arc. Thus, in Fig. 15,

$$oG = \frac{cr}{l}$$

in which c = chord of arc;
r = radius;
l = length of arc.

For a semicircle, $c = 2r$, $l = \pi r$, and

$$oG = \frac{2r \times r}{\pi r} = \frac{2}{\pi} r = .6366 r$$

It will be noticed that the center of gravity of an arc does not lie on the arc. This means that if a straight rigid piece, having no weight, were attached to the arc at d, the middle point, and laid in the direction do, the whole being suspended from G on a knife edge, a point, or a pivot, there would be equilibrium. In other words, if the arc were made of fine wire, there would be no tendency for the ends to fall downwards.

21. Broken Line.—The center of gravity of a broken line is readily found by applying the principle explained in Art. **16**, substituting for forces the lengths of the lines. Thus, let $abcd$, Fig. 16, be a broken line, and e, f, and g the middle points of the three lines composing it. Draw the two axes XX' and YY' at right angles to each other.

Represent $e e'$, $f f'$, $g g'$ by x', x'', x''', respectively, and $e e''$, $f f''$, $g g''$, by y', y'', y''', respectively. Let x represent the distance of the center of gravity of the line $a b c d$ from $Y Y'$

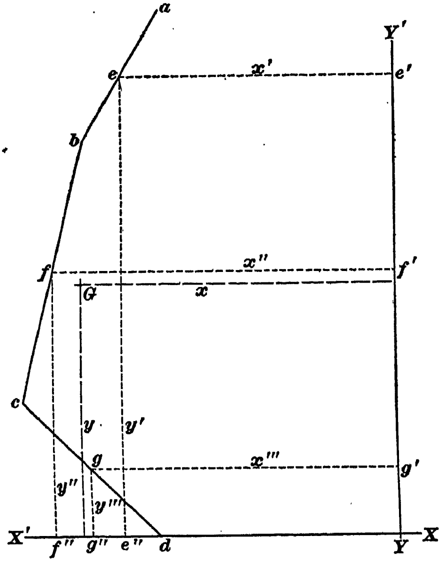


FIG. 16

and y its distance from $X X'$; then, letting l', l'', and l''' represent the respective lengths $a b$, $b c$, $c d$,

$$(l' + l'' + l''')\, x = l'\, x' + l''\, x'' + l'''\, x'''$$

or

$$x = \frac{l'\, x' + l''\, x'' + l'''\, x'''}{l' + l'' + l'''}$$

and

$$y = \frac{l'\, y' + l''\, y'' + l'''\, y'''}{l' + l'' + l'''}$$

EXAMPLE.—Suppose, in Fig. 16, $a b = 4$ inches, $b c = 7$ inches, $c d = 5$ inches, $x' = 7\frac{1}{8}$ inches, $x'' = 9$ inches, $x''' = 8$ inches, $y' = 12$ inches, $y'' = 6\frac{29}{32}$ inches, and $y''' = 1\frac{3}{4}$ inches, where is the center of gravity with respect to the axes $X X'$ and $Y Y'$?

SOLUTION.—

$$\begin{array}{ll} 4 \times 7\frac{1}{8} = 28\frac{1}{2} & \qquad 4 \times 12 = 48 \\ 7 \times 9 = 63 & \qquad 7 \times 6\frac{29}{32} = 48\frac{11}{32} \\ 5 \times 8 = 40 & \qquad 5 \times 1\frac{3}{4} = 8\frac{3}{4} \\ \hline 16 \quad 131\frac{1}{2} & \qquad 16 \quad 105\frac{3}{32} \end{array}$$

$x = 131\frac{1}{2} \div 16 = 8\frac{7}{32}$ in. $\quad y = 105\frac{3}{32} \div 16 = 6\frac{9}{16}$ in., nearly

Therefore, $x = 8\frac{7}{32}$ in. and $y = 6\frac{9}{16}$ in., nearly. Ans.

22. Perimeter of Regular Polygons.—Every regular polygon has as many axes of symmetry as it has sides and the center of gravity is located at their point of intersection; this point is also the center of the inscribed and circumscribed circles and is the geometrical center.

CENTER OF GRAVITY OF PLANE SURFACES

23. Symmetrical Figures.—By center of gravity of a plane surface or area is meant the center of gravity of a very thin, flat plate having the given outline and area.

If a figure is symmetrical about one axis, the center of gravity must lie in that axis; and if symmetrical also about a second axis the center of gravity lies also on this axis and therefore at the intersection of the two. For example, a circle is symmetrical about a diameter; hence, the intersection of two diameters, that is, the center of the circle, is the center of gravity.

The center of gravity of any regular polygon is the center of the circumscribed circle. See Art. 22.

24. The Triangle.—In the triangle ABC, Fig. 17, let AE be drawn from the vertex A to the middle point E of the opposite side BC. We may conceive the triangle made up of little strips parallel to BC. The middle point of each strip lies on AE; each strip will therefore balance on a knife edge lying along AE. Consequently, the triangle as a whole will balance on a knife edge lying along AE, and therefore the center of gravity G lies somewhere in AE. The same reasoning shows that the center of gravity lies in the line CF or BH drawn from the vertex C to the middle point F of the side AB or from B to the middle point H of the side AC; hence, it lies at the intersection of AE, BH, and CF.

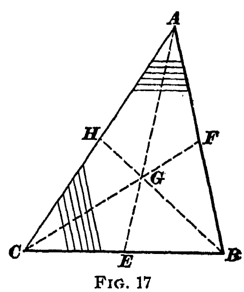


FIG. 17

It is proved in geometry that $GE = \frac{1}{3} AE$, $GF = \frac{1}{3} CF$, and $GH = \frac{1}{3} BH$; hence, instead of drawing the second

line CF or BH, G may be located by measuring from E a distance equal to $\frac{1}{3}$ EA.

25. Sector of Circle.—In the circular sector $AEBO$, Fig. 18, OE bisects the angle AOB. The center of gravity G lies on OE. If m denotes the angle AOE, expressed in degrees, and r the radius OA, the distance OG is

$$OG = 120\frac{r \sin m}{\pi m} = \frac{38.197\, r \sin m}{m} \qquad (1)$$

For the semicircle, Fig. 19,

$$m = AOE = 90°, \sin m = 1, \text{ and}$$

$$OG = 120\frac{r \times 1}{\pi \times 90} = \frac{4r}{3\pi} = .42441\, r \qquad (2)$$

26. Segment of Circle.—The center of gravity G' of the segment AEB, Fig. 18 (the shaded area), lies also on OE. Let c equal length of chord AB, and A equal area of segment AEB; the distance OG' is given by the formula

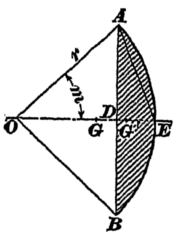


Fig. 18

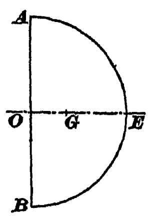


Fig. 19

$$OG' = \frac{c^3}{12A}$$

27. The length of the arc of a sector or segment is easily calculated by trigonometry. Let l = the length of the arc; c = the chord; r = the radius; h = the height = DE, Fig. 18; and m = number of degrees in the angle AOE = $\frac{1}{2}$ angle AOB, Fig. 18. If only the chord and height of the arc are given, calculate the radius by the formula

$$r = \frac{c^2 + 4h^2}{8h} \qquad (1)$$

Next find the number of degrees subtended by the arc by calculating the sine of one-half the arc and then finding one-half the angle from a table of sines.

$$\sin m = \frac{c}{2r} = \frac{4ch}{c^2 + 4h^2} \qquad (2)$$

Then, $$l = \frac{\pi r m}{90} = .034907\, r m \qquad (3)$$

28. The Quadrilateral and Trapezoid.—Let $ABCD$, Fig. 20, be a quadrilateral. To find its center of gravity G, draw AC dividing it into two triangles ABC and ACD. Locate the centers of gravity of the triangles, on the lines AE and CF, as in Art. **24**; these are G_1 and G_2. Find the area A_1 and A_2 of the triangles and locate G between G_1 and G_2 by means of the formula in Art. **14**, substituting areas for weights.

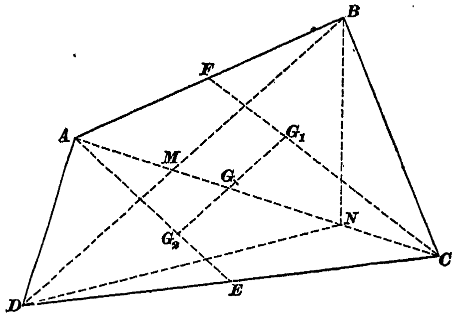


FIG. 20

The following is a convenient *graphical* construction: Draw the second diagonal BD intersecting the first in M; lay off $CN = AM$, join N to points B and D, and find the center of gravity of the triangle BDN. The center of gravity of the triangle BDN is also the center of gravity of the quadrilateral.

If the quadrilateral is a parallelogram, the intersection of the two diagonals is the center of gravity.

Let E and F be the middle points of the parallel sides of the trapezoid $ABCD$, Fig. 21. Evidently, as explained in Art. **24**, the figure would balance on a knife edge lying along EF; therefore, the center of gravity G lies in EF. Draw the diagonal AC and locate G_1 and G_2, the centers of gravity of the triangles ABC and ACD; then G also lies on line G_1G_2, and therefore at the intersection of EF and G_1G_2.

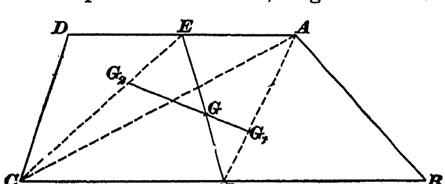


FIG. 21

29. Center of Gravity of Any Plane Figure With Regular Outline.—To find the center of gravity of a plane

figure that may be divided up into triangles, parallelograms, circles, ellipses, etc., find the area and center of gravity of each part separately and combine the centers of gravity thus found, as in the case of more than two bodies whose weights are known, except that the area of each part is used instead of its weight. This method is illustrated by the following examples:

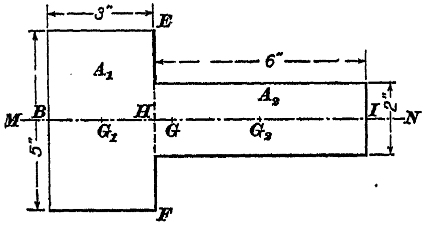


FIG. 22

EXAMPLE 1.—Find the center of gravity of the area shown in Fig. 22.

SOLUTION.—The figure being symmetrical about the horizontal axis MN, its center of gravity lies on MN. Let the figure be divided into two rectangles. The area A_1 of the first is $3 \times 5 = 15$ sq. in. and its center of gravity G_1 lies midway between B and H, or $1\frac{1}{2}$ in. to the left of EF; the area A_2 of the other rectangle is $2 \times 6 = 12$ sq. in. and its center of gravity G_2 lies midway between H and I or $6 \div 2 = 3$ in. to the right of EF. The distance between G_1 and G_2 is therefore $1\frac{1}{2} + 3 = 4\frac{1}{2}$ in. Taking G_1 as the center of moments,

$$A_1 \times 0 + A_2 \times 4.5 = (A_1 + A_2) \times x$$

from which

$$x = \frac{12 \times 4.5}{27} = 2 \text{ in.}$$

G therefore lies 2 in. to the right of G_1 or $\frac{1}{2}$ in. to the right of EF. Ans.

EXAMPLE 2.—Find the center of gravity of the area shown in Fig. 23 consisting of a rectangle and a semicircle.

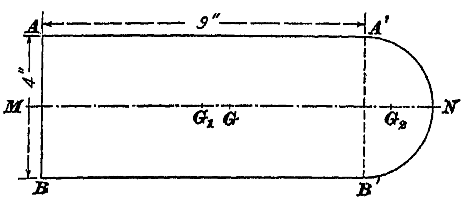


FIG. 23

SOLUTION.—The center of gravity G_1 of the rectangle $A\,A'\,B'\,B$ is in the axis of symmetry MN and $4\frac{1}{2}$ in. to the left of $A'B'$. The center of gravity G_2 of the semicircle is $.42441\,r = .42441 \times 2 = .84882$ in. to the right of $A'B'$; hence, the distance G_1G_2 is $4.5 + .84882 = 5.34882$ in. The area of the rectangle is $4 \times 9 = 36$ sq. in.; that of the semicircle is $\frac{1}{2} \times .7854 \times 4^2 = 6.2832$ sq. in.; and their sum is 42.2832 sq. in. Taking the center of moments at G_2,

$$36 \times 5.34882 + 6.2832 \times 0 = 42.2832 \times x$$

whence, $$x = G_2 G = \frac{36 \times 5.34882}{42.2832} = 4.554 \text{ in.}$$

Hence, G lies $5.349 + 4.5 - 4.554 = 5.295$ in. to the right of $A B$ on the line $M N$. Ans.

30. In problems in strength of materials and machine design, it is frequently necessary to find the center of gravity of a section. Such sections are usually symmetrical about some axis and are frequently hollow. The method employed to find the center of gravity in such cases is similar to that explained in Art. **16**.

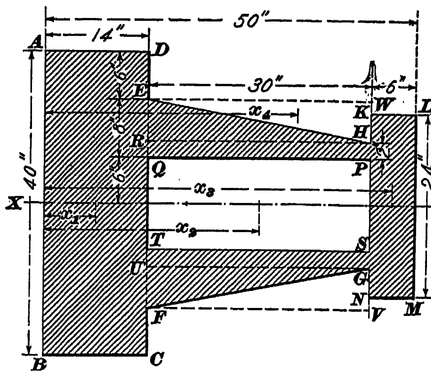


Fig. 24

Example.—The cross-section of the frame of a machine has the form and dimensions shown in Fig. 24; find the center of gravity of the section, which is symmetrical about the line $X X$.

First Solution.—The section may be divided into separate areas as follows:

	Area in Square Inches	Distance of Center of Gravity From $A B$, in Inches
Rectangle, $A B C D$ . .	$14 \times 40 = 560$	$14 \div 2 = 7$
Rectangle, $K L M N$.	$6 \times 24 = 144$	$14 + 30 + \frac{6}{2} = 47$
Rectangle, $H P Q R$ . .	$2 \times 30 = 60$	$14 + \frac{30}{2} = 29$
Rectangle, $G S T U$ . .	$2 \times 30 = 60$	$14 + \frac{30}{2} = 29$
Triangle, $E H R$ . .	$6 \times 30 \div 2 = 90$	$14 + \frac{30}{3} = 24$
Triangle, $F G U$ . .	$6 \times 30 \div 2 = 90$	$14 + \frac{30}{3} = 24$

The area of each part may be considered as a force acting at the center of gravity of that part. Take $A B$ as the axis of reference (see last paragraph Art. **18**). The area multiplied by the distance of the center of gravity from $A B$ gives the moment of that area about $A B$; and from the equation of moments, the moment of the whole section

about AB is equal to the sum of the moments of the parts about AB. The moments are as follows:

	Area	Arm	Moment
	560	7	3920
	144	47	6768
	60	29	1740
	60	29	1740
	90	24	2160
	90	24	2160
total	1004		18488

The area of the section 1,004 sq. in. has a moment 18,488 about AB; hence, the arm is $18,488 \div 1,004 = 18.414$ in.; that is, the center of gravity lies 18.414 in. from AB and in the axis of symmetry XX. Ans.

Second Solution.—Draw EW and FV perpendicular to KN, thus forming a rectangle $EWVF$. Then, area of whole figure $\times x$ $= ABCD \times x_1 + EFVW \times x_2 + KNML \times x_3 - 2 \times EHW \times x_4$ $- QPST \times x_2$.

$$\begin{aligned}
(40 \times 14) \times 7 &= \quad 3920 \\
(30 \times 28) \times 29 &= \quad 24360 \\
(6 \times 24) \times 47 &= \quad 6768 \\
\frac{-2(30 \times 6)}{2} \times 34 &= -\ 6120 \\
-(12 \times 30) \times 29 &= -10440 \\
\hline
1004 \times x &= \quad 18488 \\
x &= \quad 18.414 \text{ in.} \quad \text{Ans.}
\end{aligned}$$

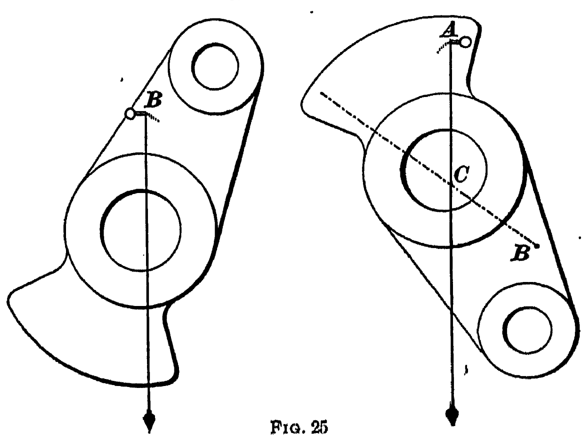


Fig. 25

31. The center of gravity of an area of irregular outline may be found as shown in Fig. 25. Draw the outline on

stiff paper or cardboard and cut out with knife or scissors. Suspend it at some point B so that it will move freely, drop a plumb-line as shown, and mark its direction; then suspend the card from another point A, and drop a plumb-line; the point of intersection C of the two lines will be the center of gravity. By exercising reasonable care this method gives very accurate results.

CENTER OF GRAVITY OF SOLIDS

32. Geometrical Solids.—In the case of solids that have geometrical centers, as the sphere, cylinder, etc., the geometrical center is also the center of gravity.

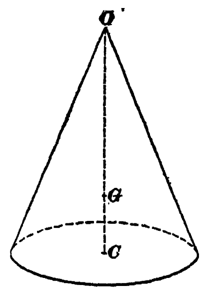


FIG. 26

33. Cone or Pyramid.—In the case of a cone or pyramid, the center of gravity lies on the line joining the vertex to the center of gravity of the base and at a distance from the base equal to one-fourth the length of the joining line; that is, if O is the vertex, Fig. 26, C the center of gravity of the base, and G the center of gravity of the cone or pyramid, G lies on the line OC and

$$CG = \tfrac{1}{4}\,CO$$

34. Irregular Solids.—In a body free to move, the center of gravity will lie in a vertical plumb-line drawn through the point of support. Therefore, to find the position of the center of gravity of an irregular solid, as the crank, Fig. 25, suspend it at some point B so that it will move freely, and proceed exactly as described in Art. **31**. Since the center of gravity depends wholly on the shape and weight of a body, it may be without the body, as in the case of a circular ring, whose center of gravity is at the center of the circumference of the ring.

STABLE AND UNSTABLE EQUILIBRIUM

35. A body is in **stable equilibrium** when, if slightly displaced from its position of rest, it tends to return to that position; for example, a cube, a cone resting on its base, a pendulum, etc.

If a body is in stable equilibrium, its *center of gravity is raised when the body is displaced.*

36. A body is in **unstable equilibrium** when, if slightly displaced from its position of rest, it tends to fall farther from that position; for example, a cone standing on its point, an egg balanced on its end, etc.

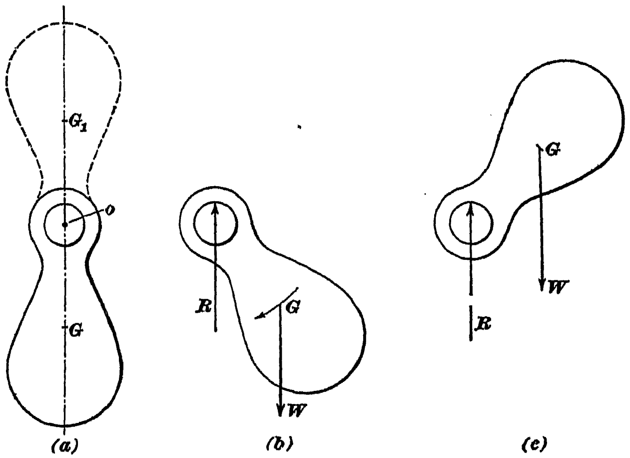


FIG. 27

Any movement, however slight, lowers the center of gravity when the body is in unstable equilibrium.

37. A body is in **neutral equilibrium** when, after being slightly displaced, it has no tendency either to return to its original position or to move farther from that position; for example, a sphere of uniform density on a horizontal plane, a cone resting on its side.

38. The difference between stable and unstable equilibrium is shown in Fig. 27. A body, see Fig. 27 (*a*), is

supported by a pin and the center of gravity G is in the same vertical line with the center o of the pin. The reaction R of the pin against the surrounding eye is equal and opposite to the weight W and the two forces have the same line of action.

Suppose the mass to be in the dotted position, so that its center of gravity G_1 is directly above o; then, as before, the upward reaction balances the downward weight, and in both cases equilibrium exists. Suppose, now, that the body is displaced, as shown at (b). The equal and opposite forces R and W are not now in the same straight line, but form a couple that tends to turn the body clockwise and bring G back under o. On the other hand, suppose the body to be displaced from the dotted position, as shown at (c). R and W form a couple that tends to rotate the body clockwise or away from its first position.

In the first case in which the center of gravity is below the axis of the supporting pin, the equilibrium is stable. When, as in the second case, the center of gravity is above the supporting pin, the equilibrium is unstable.

39. If a body rests on a horizontal surface, as in Fig. 28, the pressure between the surface and the body will be so distributed that the resultant upward reaction R of the surface will lie in the same line with the center of gravity G. Then the weight W acting through G is balanced by the reaction R. But the pressure cannot be distributed so that R can lie to the right of the edge E; hence, if the vertical through G passes to the right of E, the forces W and R will form a couple causing rotation about E; that is, the body falls.

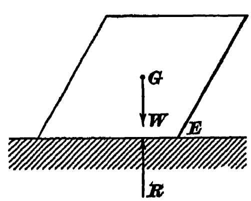


FIG. 28

EXAMPLES FOR PRACTICE

1. Find the perpendicular distance between the center of gravity and the longer side of a triangle whose sides are 7 feet, 10 feet, and 15 feet long. Solve graphically. Ans. 1.31 ft.

2. A rectangle 2 feet long and 1 foot wide has equal weights of 50 pounds each suspended from two of its diagonally opposite corners; a weight of 60 pounds and another of 80 pounds are suspended from the other two corners. Supposing the rectangle to be without weight, where is the center of the gravity of the four weights considered as one body?

Ans. On the diagonal joining the 60-lb. and 80-lb. weights, 1.118 in. from the center.

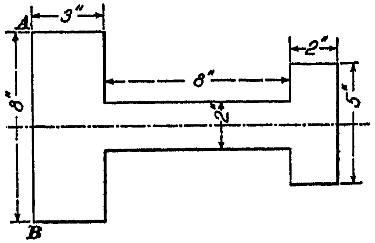


FIG. 29

3. Find the distance of the center of gravity of the section shown in Fig. 29 from the side AB. Ans. 5.36 in.

4. Find the magnitude and line of action of the resultant of two parallel forces of 30 and 17 pounds, respectively, whose lines of action are 24 inches apart, when both act downwards.

Ans. 47 lb. downwards, 8.68 in. from the 30 lb. force.

5. A bar of iron of uniform cross-section is 30 inches long and weighs 18 pounds; from one end A is hung a weight of 12 pounds; how far from the other end B must the bar be supported to just balance?

Ans. 21 in

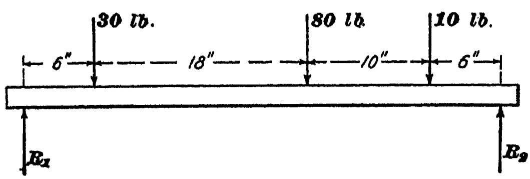


FIG. 30

6. A beam is loaded as shown in Fig. 30 and is supported at the two ends; find the upward pressures R_1 and R_2 of the supports, neglecting the weight of the beam. Ans. $\begin{cases} R_1 = 59 \text{ lb.} \\ R_2 = 61 \text{ lb.} \end{cases}$

ELEMENTARY MECHANICS

(PART 3)

SIMPLE MACHINES

DEFINITIONS

1. A **machine** is a contrivance for changing the rate of motion of a body or for keeping a body in motion against a resistance. A machine is a *simple* one when it contains but one moving part.

The motion of a body can be altered only by the application of a force, and a force can be changed with regard to one, two, or all three of its elements; that is, its point of application, its line of action, its magnitude. In this change consists the utility of a machine, inasmuch as most forces of nature could be applied directly only to a very limited degree. Thus the force of the wind or that of a stream of water cannot directly be made to grind grain; it requires a mill to so change the force as to make it act on the grain. A large rock that a man with the force of his muscles alone cannot move, can be moved by means of a lever, for by the help of this machine the force of the muscles may be many times multiplied and at the same time made to act on a point away from the human arm.

2. Kinds of Simple Machines.—All simple machines are based either on the *lever* or the *inclined plane*. Modifications of these two fundamental contrivances are the *pulley*, the *wheel and axle*, the *wedge*, and the *screw*. In the lever, the moving body has a rotary motion; in the inclined plane, it has a straight-line motion.

THE LEVER

3. A **lever** is a rigid body, usually bar-shaped, capable of being turned about a fixed pin, pivot, or point, as in Figs. 1 to 7, and acted on by at least three forces.

4. One of the forces W, which is assumed to be overcome, as for instance, lifted, is called the **load**, or **weight**;

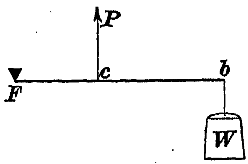


FIG. 1

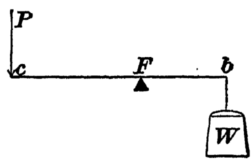


FIG. 2

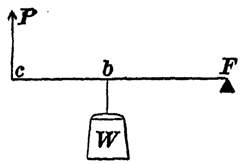


FIG. 3

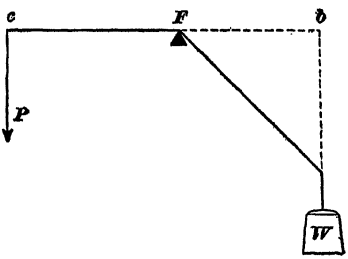


FIG. 4

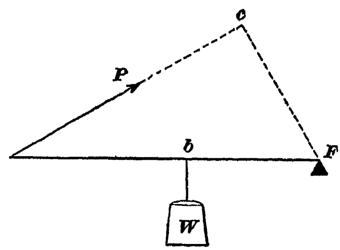


FIG. 5

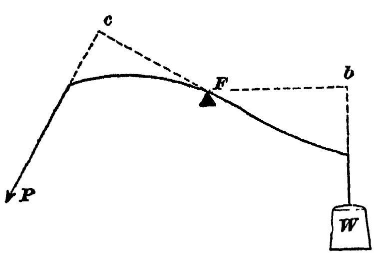


FIG. 6

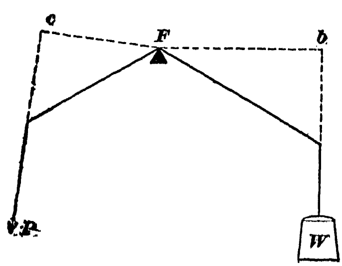


FIG. 7

the second force P, which is supposed to overcome the load, is called the **power**; and the third is the **reaction** of the **fulcrum**, as the point or pivot F is called.

5. If the lever is considered to be without weight, it is called a *mathematical lever*, otherwise a *physical lever*.

The perpendicular distances from the fulcrum to the lines of action of the weight (or load) and power are called the **arms of the lever**, and are referred to as the **weight** (or **load**) **arm** and **power arm**, respectively.

6. If the lines of action of load and power are parallel, so that the arms of the lever form a straight line, the lever is a **straight lever.** See Figs. 1, 2, 3, and 4. If the lines of action of load and power form an angle with one another, the lever is a **bent lever.** See Figs. 5, 6, and 7. Notice that the shape of the body or bar of which the physical lever may be made does not enter into this consideration but only the direction of the forces. Thus the lever in Fig. 4 is a straight lever, though the lever is represented by a broken line, and the lever in Fig. 5 is a bent lever, though the bar is straight.

7. Levers are divided into three kinds or orders according to the relative positions of the power, load, and fulcrum.

When the fulcrum lies between the points of application of the power and load, the lever is one of the **first order.** See Figs. 2, 4, 6, and 7.

When the weight (or load) acts on a point between the fulcrum and the point of application of the power, the lever is one of the **second order,** Figs. 3 and 5.

When the power acts on a point between the fulcrum and the weight, the lever is one of the **third order.** See Fig. 1.

8. Equilibrium of Forces on the Lever.—In order that the forces may be in equilibrium, the negative moments must be equal to the positive moments; and if the fulcrum is taken as the center of moments, the moment of the reaction at F is zero and drops out, so that we have only the moments of W and P, whose algebraic sum must be zero, or, in other words, which must be equal numerically. The direction of the moments of load and power must, therefore, always be opposite to each other. For all three classes, $P \times Fc - W \times Fb = 0$, from which,

$$P \times Fc = W \times Fb$$

or in words, *when a lever is in equilibrium, the power multiplied by the power arm equals the load multiplied by the load arm.*

Taking the two products as the four terms of proportion we have

$$P : W = Fb : Fc$$

or, in words, *when a lever is in equilibrium, the power is to the load as the load arm is to the power arm*, or *the power and load are inversely as their arms.*

EXAMPLE.—If the load arm of a lever is 6 inches long and the power arm is 4 feet long, how great a load can be raised by a force of 20 pounds at the end of the power arm?

SOLUTION.— 4 ft. = 48 in. Hence, $20 \times 48 = W \times 6$, or $W = 160$ lb. Ans.

9. Furthermore, in case of equilibrium, the reaction at the fulcrum must be the equilibrant of the load and power; that is, the reaction of the fulcrum is equal and opposite to the resultant of *load and power* so that in a straight lever of the first order, the reaction is equal to the sum of the load and power and has a direction parallel and opposite to these forces. In the case of a straight lever of the second and third orders, the magnitude of the reaction is equal to the difference of the load and power and has a direction parallel and opposite to the larger of the two. All this follows from what has previously been stated of parallel forces. In the case of a bent lever, the reaction is found by means of the force triangle.

EXAMPLE 1.—What is the magnitude and direction of the reaction of the fulcrum in the example of the preceding article if the lever is a straight one?

SOLUTION.—The lever may be one of the first order, as shown in Fig. 2; then, the reaction is equal to $160 + 20 = 180$ lb., parallel and opposite to load and power, that is, the reaction is upwards. The lever may also be one of the second order, as shown in Fig. 3; then, the reaction is equal to $160 - 20 = 140$ lb., parallel and opposite to the load, the greater of the two forces, which acts downwards; hence, the reaction is upwards. Ans.

EXAMPLE 2.—(*a*) What force must be applied at a distance of 6 inches from the fulcrum in a straight lever of the third order,

Fig. 1, to raise a load of 180 pounds, the load arm being 4 feet long? (*b*) What is the magnitude and direction of the reaction?

SOLUTION.—(*a*)

$$180 \times 48 = P \times 6;\ P = \frac{180 \times 48}{6} = 1{,}440 \text{ lb. Ans.}$$

(*b*) 1,440 − 180 = 1,260 lb., parallel and opposite to the greater force, that is, the reaction is downwards. Ans.

EXAMPLE 3.—Let the load on the bent lever, Fig. 5, be 100 pounds, the load arm Fb 14 inches, the power arm Fc 16 inches, and the angle cFb between the arms 60°; (*a*) what is the power? (*b*) what is the reaction?

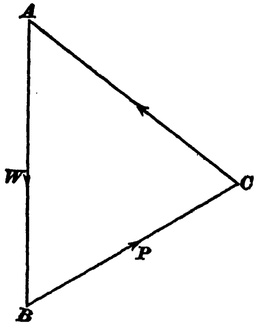


FIG. 8

SOLUTION.—(*a*)

$$P : 100 = 14 : 16;\ P = \frac{100 \times 14}{16}$$
$$= 87.5 \text{ lb. Ans.}$$

(*b*) For the reaction, draw the force triangle ABC, Fig. 8. AB, a vertical line, represents the load, or W = 100 lb.; BC parallel to P = 87.5; then CA, equal in magnitude and direction to the reaction, will be found to be 94.373 lb. and the angle ACB = 66° 35′. Ans.

NOTE.—It is evident that the reaction has a horizontal component, and that the end F of the lever must be resisted by a horizontal force equal to the horizontal component of BC, acting from F toward b, to keep the lever from sliding off the fulcrum.

10. If F, Fig. 9, be taken as the center of a circle, and arcs be described through a and b, it will be seen that, if the weight arm is moved through a certain angle bFb_1, the power arm will move through an equal angle aFa_1; also, that the vertical distance that W moves is Fb multiplied by the sine of this angle, and the vertical distance that P moves is Fa multiplied by the sine of the same angle. From the similar triangles b_1Fb_2 and a_1Fa_2, $\frac{Fb_1}{b_1b_2} = \frac{Fa_1}{a_1a_2}$, or $Fb_1 \times a_1a_2 = Fa_1 \times b_1b_2$. From this, it is seen that the power arm is

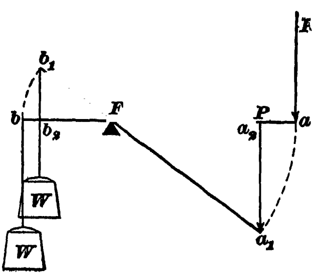


FIG. 9

proportional to the distance through which the power moves, and the weight arm is proportional to the distance through which the weight moves.

Hence, instead of writing $P \times Fa = W \times Fb$, it might have been written $P \times$ distance through which P moves $= W \times$ distance through which W moves. It must be carefully noted that the distances the load and power move are the lengths of the load arm and power arm, respectively, multiplied by the sine of the angle moved through. That is, for any lever:

The power multiplied by the vertical distance through which it moves equals the weight multiplied by the vertical distance through which it moves.

Written in the form of a proportion, the rule becomes

$P : W =$ distance through which load moves : distance through which power moves

or, for any lever:

Load and power are inversely proportional to the distances through which they move.

EXAMPLE.—(*a*) What is the ratio between the power and the weight in the example of Art. 8? (*b*) If P moves 24 inches, how far does W move? (*c*) What is the ratio between the two distances?

SOLUTION.—(*a*) $20 : 160 = 1 : 8$; that is, the weight moved is 8 times the power. Ans.

(*b*) $20 \times 24 = 160 \times x$. $x = \frac{480}{160} = 3$ in., the distance that W moves. Ans.

(*c*) $3 : 24 = 1 : 8$, or the ratio is $1 : 8$. Ans.

11. Suppose a certain load W to be raised by a lever a certain distance, say 1 foot. Let the length of the load arm be fixed; then it is evident, from the preceding article, that the distance through which the power must move is the greater

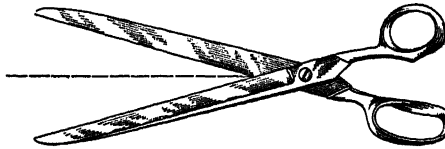


FIG. 10

the smaller is the power available. Now, suppose that the power is capable of moving just so many inches per second,

it will require more time to raise the load the required distance, the smaller the power and vice versa; or, in other words, by sacrificing time power is saved; by exerting power, time is saved. From this spring two uses of the lever: When it is the

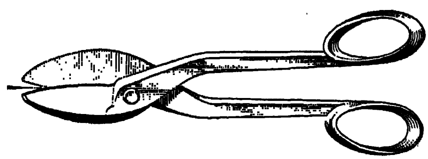


FIG. 11

object to save time rather than power, levers are used in which the power arm is shorter than the load arm; when it is desirable rather to have as small a power as possible sacrificing speed, then levers with long power arms and short load arms are used. Striking examples are a paper shears and a sheet-metal shears, Figs. 10 and 11. Both are examples of levers of the first order. In the

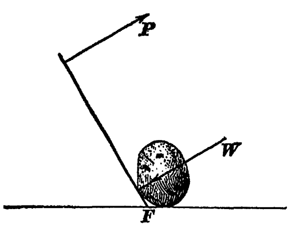


FIG. 12

former, the force needed to cut the paper is small, and the human hand is capable of exerting an excess of it; this excess is employed in cutting the paper quickly, by making the handles short and the blades long. When using the metal shears, however, great force is needed to cut the metal and thus the handles are made long and the blades short, cutting slowly, but with much force. Levers of the third order are always

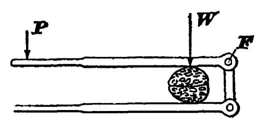


FIG. 13

such that the power is greater than the load. In those of the second order, the load is always greater than the power. An example of a lever of the second order is the crowbar, Fig. 12, and the nut cracker, Fig. 13. An example of a lever of the third order is the trip hammer, Fig. 14. In each of these F is the fulcrum, W the load, and P the power.

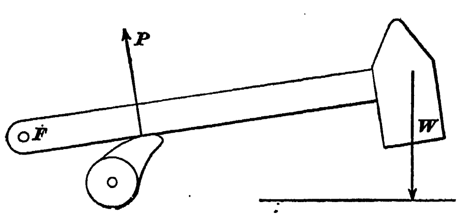


FIG. 14

THE PULLEY

12. A **pulley** is a wheel turning on an axle, over which a cord, chain, or band is passed in order to transmit the force through the cord, chain, or band.

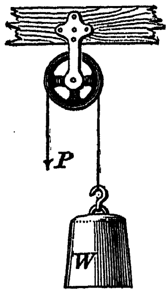


FIG. 15

13. The frame that supports the axle of the pulley is called the **block.**

14. A **fixed pulley** is one whose block is not movable, as in Fig. 15. The pulley is a modification of the lever. For if the upper and lower parts be removed, leaving only a horizontal strip, the force and load will be acting in the same manner as before; the lever arms will be equal to the horizontal radii, and the lever will be one of the first order, with the fulcrum at the center. Since the arms are equal, the power must be equal to the load, and since the power and load are equal, the distances they move simultaneously are

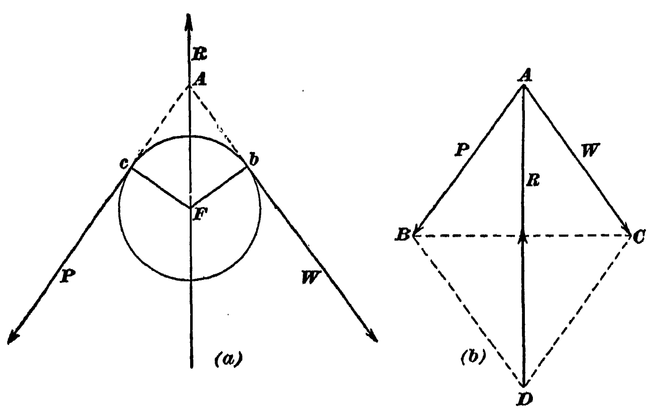


FIG. 16

also equal. The fixed pulley serves only to change the *direction of a force;* it does not affect the magnitude of the force. If the two ends of the cords are parallel, the pulley is equivalent to a straight lever, and the reaction of the axle

is evidently $P + W = 2\ W$. If the ends of the cord are not parallel, as in Fig. 16 (*a*), the pulley is equivalent to a bent lever $c F b$, and the reaction R passes through F and is equal and opposite to the resultant of P and W, and since P and W are equal, the reaction is equal to the diagonal $A D$ of a rhombus $A C D B$, Fig. 16 (*b*), the sides $A B$ and $A C$ of which represent P and W.

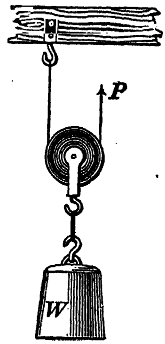


FIG. 17

15. **A movable pulley** is one whose block is movable, as in Fig. 17. One end of the cord is fastened to the beam and the weight W is suspended from the pulley, the other end of the cord being drawn up by the application of a force P.

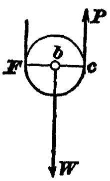


FIG. 18

The movable pulley may be regarded as a lever of the second order, as indicated in Fig. 18, the power arm $c F$ being equal to the diameter of the pulley, the load arm $b F$ being equal to the radius of the pulley, and the fulcrum F being the point of tangency of the fixed rope end. The power arm being twice as long as the load arm, the power P necessary to lift the load W is but half the load, but it moves a distance twice as great as that which the load moves. The latter statement also follows directly from observation, for it is evident that to move W, say, 1 inch, 1 inch must be taken up in both parts of the rope; that is, 2 inches at the loose or power end. It is thus seen that the movable pulley serves to *change the magnitude of a force*. The condition would remain the same if the free end of the cord were passed over a fixed pulley, as in Fig. 19, in which case the fixed pulley merely changes the direction in which P acts; this combination of a fixed and a movable pulley changes the magnitude of the force P as well as its direction.

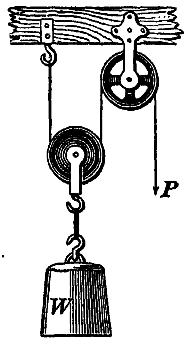


FIG. 19

THE WHEEL AND AXLE

16. The **wheel and axle** consists of *two cylinders of different diameters, rigidly connected*, so that they turn together about a common axis, as in Fig. 20 (*a*). This machine may be considered as a lever of the first order bFc when the power acts downwards, as in Fig. 20 (*b*), and as a lever of the second order cbF if the power acts upwards, as in Fig. 20 (*c*). The reaction of the fulcrum is equal to the sum of load and power, $P+W$, in the one case, and equal to their difference, $W-P$, in the second case. This reaction resolves into two parallel components one each at the two journals F_1 and F_2, Fig. 20 (*d*). The components are found in the same manner as the reactions in Art. 5, *Elementary Mechanics*, Part 2. As in the lever, the forces are inversely as their arms as well as the distances through which they move; that is,

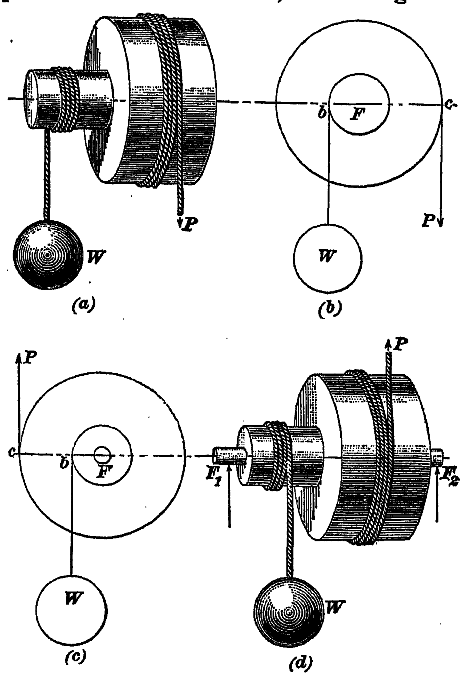


FIG. 20

$$P : W = Fb : Fc = \text{distance through which load moves} : \text{distance through which power moves}$$

It is not necessary that an entire wheel be used; an arm, projection, radius, or anything that the power causes to revolve in a circle, may be considered as the wheel. Consequently, if it is desired to hoist a weight W with a windlass, Fig. 21, the force P is applied to the handle of the crank, and

the distance between the center line of the crank-handle and the axis of the drum corresponds to the *radius* of the wheel.

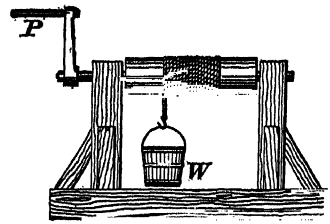


FIG. 21

EXAMPLE.—If the distance between the center line of the handle and the axis of the drum, in Fig. 21, is 18 inches, and the diameter of the drum is 6 inches, what force will be required at P to raise a load of 300 pounds?

SOLUTION.—The radius of the drum is $6 \div 2 = 3$ in.; hence,

$$P \times 18 = 300 \times 3; \text{ whence, } P = 50 \text{ lb. Ans.}$$

Or, the diameter of the circle described by the handle is $18 \times 2 = 36$ in.; hence, $P \times 36 = 300 \times 6$; from which $P = 50$ lb. Ans.

EXAMPLES FOR PRACTICE

1. The lever of a safety valve is of the form shown in Fig. 1, where the force is applied at a point between the fulcrum and the weight lifted. If the distance from the fulcrum to the valve is $5\frac{1}{2}$ inches and from the fulcrum to the weight is 42 inches, what total force is necessary to raise the valve, the weight being 78 pounds and the weight of valve and lever being neglected? Ans. 595.64 lb.

2. If the distance from the fulcrum to the point at which a force of 135 pounds is applied to a lever is 4 feet, and the distance from the fulcrum to the weight is $1\frac{1}{2}$ inches, how great a weight will the force lift? Ans. 4,320 lb.

3. A windlass is used to hoist a weight. If the diameter of the drum on which the rope winds is 4 inches, and the distance from the center of the handle to the axis of the drum is 14 inches, how great a weight can a force of 32 pounds applied to the handle raise? Ans. 224 lb.

THE INCLINED PLANE

17. **An inclined plane** is a slope or a flat surface making an angle with another flat surface, usually horizontal.

Three cases may arise in practice with the inclined plane:

I. When the power acts parallel to the plane, as in Fig. 22.

II. When the power acts parallel to the base, as in Fig. 23.

III. When the power acts at any angle to the plane or the base, as in Fig. 24.

18. Case I.—Referring to Fig. 22, $a\,b$ is the plane and W is a body resting on it; it is supposed, of course, that there is no friction between W and the plane. The load W acts vertically downwards, and if the surface $a\,b$ were horizontal, it would press against this surface with a force equal to the weight of W and the *direction* of the pressure would be perpendicular to $a\,b$. As soon, however, as the end b is raised, the perpendicular pressure against $a\,b$ is lessened and there is a tendency for the body W to slide along $a\,b$ toward a. Moreover, the greater the angle $b\,a\,c$, the less will be the perpendicular pressure against $a\,b$, and the greater will be the effect of the weight in urging the body downwards. To ascertain the effects produced by W, draw $A\,C$ vertical, make its length represent the weight W, and mark it as acting from A to C. Resolve $A\,C$ into two components, the first acting along $A\,D$, parallel to the plane $a\,b$, and the second acting along $A\,B$, perpendicular to the plane. Complete the parallelogram (rectangle) by drawing $C\,B$ and $C\,D$ from C, parallel to $A\,D$ and $A\,B$, respectively. Then $A\,D$ (or $B\,C$) represents the effect of W in urging the body down the plane and $A\,B$ represents the perpendicular pressure against the plane. It is evident that, in order that there shall be equilibrium, i. e., that the body shall be at rest, the force (component) $A\,D$ must be opposed by an equal and opposite force $A\,D'$, i. e., by a force having a component parallel to $a\,b$ and equal to $A\,D'$, acting from A toward D'.

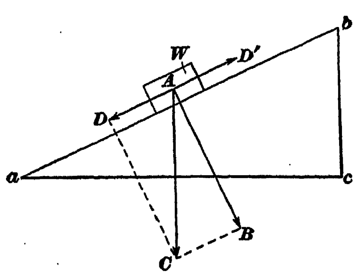


FIG. 22

In Case I, the power acts parallel to $a\,b$, and, hence, is represented by $A\,D'$. The value of $A\,D'$, which equals $C\,B$, acting from C to B, is readily found, as follows: Since $A\,C$ is perpendicular to $a\,c$ and $A\,B$ is perpendicular to $a\,b$, angle A = angle a; and since the angles B and C are right angles the right triangles $A\,B\,C$ and $a\,c\,b$ are similar. Therefore, $A\,C : a\,b = C\,B : b\,c$, or since $A\,C$ represents W and $C\,B$

represents P, $W : ab = P : bc$; whence, $W \times \overline{bc} = P \times \overline{ab}$. But ab represents the *length* of the plane and bc represents the *height* of the plane; consequently, expressed in words:

The load multiplied by the height of the inclined plane equals the power multiplied by the length of the plane. Hence, if the length ab is 40 feet, and the height cb is 20 feet, $W \times 20 = P \times 40$, or a force of 1 pound in the direction AD' will balance a weight of 2 pounds.

In the triangle ABC, P (or CB) $= AC \sin A = W \sin a$ and AB (the perpendicular pressure) $= AC \cos A = W \cos a$, a being the angle bac.

EXAMPLE 1.—What force acting parallel to a plane whose length is 18 inches and height $1\frac{1}{2}$ inches will balance a weight of 4,500 pounds?

SOLUTION.— $W \times$ height of plane $= P \times$ length of plane; hence, $4{,}500 \times 1\frac{1}{2} = P \times 18$, or $P = 375$ lb. Ans.

EXAMPLE 2.—The angle that a plane makes with its base is 11° 24′. (*a*) What force acting parallel to the plane will balance a weight of 4,500 pounds? (*b*) What is the perpendicular pressure against the plane?

SOLUTION.— (*a*) $P = W \sin a = 4{,}500 \times \sin 11° 24' = 4{,}500 \times .19766 = 889.47$ lb. Ans.

(*b*) Perpendicular pressure $= W \cos a = 4{,}500 \times \cos 11° 24' = 4{,}500 \times .98027 = 4{,}411.215$ lb. Ans.

19. The object of the inclined plane is to raise the load a distance bc, the height of the plane. To do this directly, a force would be needed equal to W, as for instance by means of the fixed pulley. The distances through which W and P moved would then be equal. By means of the inclined plane, the load is lifted vertically, that is, in the direction of the line of action of the load a distance equal to bc, while the power moves in the direction of *its* line of action (parallel to the plane, in this case) a distance equal to ab. The equation $W \times bc = P \times ab$, may then be interpreted as follows:

The load multiplied by the distance through which it moves (in the direction of its line of action, that is, vertically) is equal to the power multiplied by the distance through which it moves. Or, what is the same thing, the load and power are inversely as the distances through which they move. It may here be stated

that this law is perfectly general for all machines, no matter what means are employed to raise the load. Thus, the smaller the power, the larger is the distance it must move, or gain in power is obtained at the expense of speed. As the length of the plane is always longer than the base, there is a gain of power always in the arrangement shown in Fig. 22.

20. Case II.—In Case II, the power acts parallel to the base, or horizontally. Referring to Fig. 23, AD' represents, as in Fig. 22, the force, equal and opposite to AD, acting parallel to the plane, that will produce equilibrium. If the force acting on the body has the direction AE, parallel to the base, it must have a component parallel and equal to AD'. Hence, resolve AD' into two components AE parallel to ac and $D'E$ perpendicular to AD'. It is at once evident that the force AE thus determined can be resolved into the forces AD' and $D'E$; in other words, AE represents, to the same scale as was used to measure AD', the magnitude of the force required to produce equilibrium when acting parallel to the base.

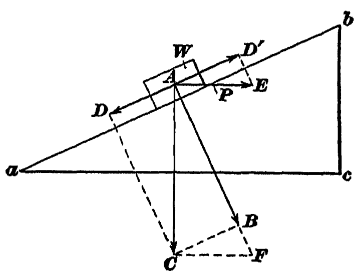


FIG. 23

Draw from C, the horizontal line CF, and produce AB to F. Then the triangles CBF and $AD'E$ are equal, since their sides are parallel and $AD' = CB$. Hence, the power AE might have been more readily found by drawing CF parallel to the base, and then drawing AF perpendicular to the plane; CF then represents the power necessary to produce equilibrium and AF represents the perpendicular pressure against the plane. It will be noted that AF is longer than AC; hence, the perpendicular pressure in this case is *greater* than the weight W. This is correct since the perpendicular pressure due to the weight only is AB, while the force AE has a component $D'E$ (acting from D' to E) which is equal to BF, making the total perpendicular pressure $AB + BF = AF$.

Since the two right triangles ACF and acb are similar, $AC:ac = CF:cb$, or $W:ac = P:cb$, and

$$W \times cb = P \times ac$$

Expressed in words: When the power acts parallel to the base, *the load multiplied by the height of the inclined plane equals the power multiplied by the length of the base.* If the length of the base is 40 feet, and the height of the inclined plane is 20 feet, $W \times 20 = P \times 40$, and a force of 1 pound acting in the direction of P will balance a weight of 2 pounds resting on the plane.

Since $CF = W \tan A$ and the angle A = angle a,

$$P = W \tan a$$

Also, AF = perpendicular pressure against the plane $= \dfrac{W}{\cos a}$.

Supposing that the power P, Fig. 23, is kept acting always parallel to the base, then, while the load ascends from the level ac to b, or through the height cb, P will move a distance outwards to the right, equal to the length of the base ac. Considering the lengths bc and ac as the distances the forces P and W move in the direction of their lines of action, the law stated in Art. **19** holds here also. But as the height of the inclined plane may be smaller or larger than the length of the base, there is a gain in power only as long as the height is smaller than the base; the power is equal to the load when the angle bac is 45°, because then height and base are equal; when the angle is larger than 45°, the power is larger than the load and there is a loss in power.

EXAMPLE 1.—In example 1, Art. **18**, what force acting parallel to the base would be required to balance the load?

SOLUTION.—Length of base $= \sqrt{18^2 - 1.5^2} = 17.937$ in.; hence, $4{,}500 \times 1.5 = P \times 17.937$ and,

$$P = 376.32 \text{ lb. Ans.}$$

EXAMPLE 2.—In example 2, Art. **18**, (*a*) what force acting parallel to the base would be required to balance the load? (*b*) What would be the perpendicular pressure against this plane?

SOLUTION.—(*a*) Applying formula given above,
$P = W \tan a = 4{,}500 \times \tan 11° 24' = 4{,}500 \times .20164 = 907.38$ lb. Ans.

(*b*) Perpendicular pressure $= \frac{W}{\cos a} = \frac{4{,}500}{\cos 11° 24'} = \frac{4{,}500}{.98027} = 4{,}590.6$ lb. Ans.

21. Case III.—In Case III, the line of direction in which the power acts makes an angle with the plane $a\,b$, as shown in Fig. 24. Assuming the pulley N to be fixed, the power varies for every position of the load W. As before, $A\,D'$ represents the force acting parallel to the plane that would balance the load and equals $C\,B$. Now to find the magnitude of a force acting in the direction $A\,E$ that will have a component along $A\,D'$ and equal $A\,D'$, draw $D'\,E$ from D', perpendicular to $A\,D'$ and consequently perpendicular also to the plane. Also, draw $C\,F$ parallel to $A\,E$. Then, $C\,F = A\,E = P$, the force required to balance the load, when the load is in the position shown, and $A\,F =$ the perpendicular pressure against the plane; this latter is less than $A\,B$, the perpendicular pressure due to the load, since $A\,E = P$ has a component $D'\,E$ parallel to $A\,B$ acting from D' toward E. Hence, all that was necessary to do in order to find P was to lay off $A\,C$ equal to W, draw $C\,F$ parallel to $A\,E$, and $A\,F$ perpendicular to $a\,b$. For any other position of W as W' draw $C\,F'$ parallel to $A'\,E'$, then $C\,F' = P$ and $A\,F' =$ the perpendicular pressure against the plane.

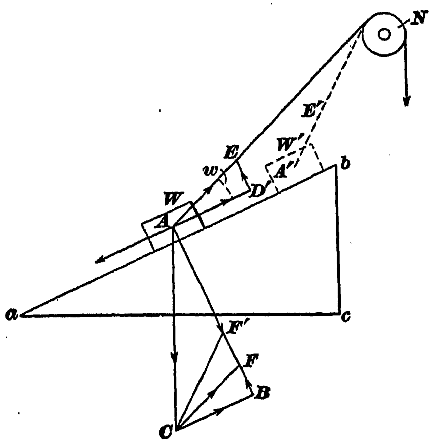


Fig. 24

Let a represent the angle $b\,a\,c$ and w represent the angle that the direction of the force P makes with the plane (this angle is $E\,A\,D' = F\,C\,B$, when the body is in the position W); then, in the triangle $F\,B\,C$,

$$C\,F = \frac{C\,B}{\cos w} = P = \frac{W \sin a}{\cos w} \qquad (1)$$

$$AF = AB - BF = W\cos a - \frac{W\sin a}{\cos w} \times \sin w$$
$$= W(\cos a - \sin a \tan w) \qquad (2)$$

EXAMPLE.—A piece of rock weighing 3,000 pounds is held on an inclined plane by means of a prop, as shown in Fig. 25. The prop makes an angle of 60° with the base of the plane. The plane $a\,b$ makes an angle with the base $a\,c$ of 70°. (*a*) What pressure has the prop to sustain? (*b*) What is the perpendicular pressure against the plane?

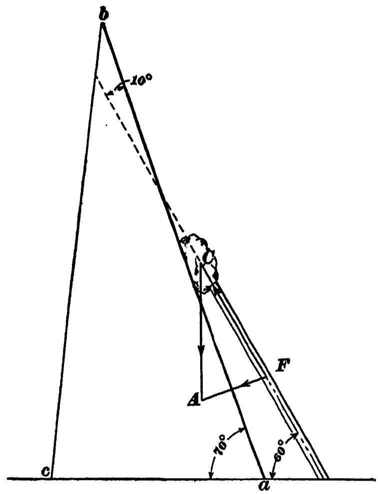


FIG. 25

SOLUTION.—Draw CA vertical (and consequently perpendicular to the base); draw CF in the direction of the center line of the prop, i. e., at an angle of 60° to the base and AF perpendicular to the plane $a\,b$. FC then represents P and FA the perpendicular pressure against the plane. Referring to Fig. 24, the triangle CFA corresponds to the triangle CFA, Fig. 25, with the exception that the arrowheads point in exactly opposite directions. Since CA and AF are perpendicular to $c\,a$ and $b\,a$, respectively, angle A = angle a = 70° = a, formulas **1** and **2** of Art. **21.** The angle between the direction CF of the center line of the prop and $a\,b$ is 10°, but since the direction of FC is exactly opposite to CF in Fig. 24 (because it acts *toward* C, instead of *away* from C as in Fig. 24), it is necessary to substitute 180° − 10° = 170° for w in formulas **1** and **2** of Art. **21.** Substituting in the formulas,

(*a*) $P = \dfrac{3{,}000 \sin 70°}{\cos 170°} = \dfrac{3{,}000 \times .93969}{-.98481} = -\ 2{,}862.6$ lb. Ans.

(*b*) $FA = W\ (\cos 70° - \sin 70° \times \tan 170°)$
$= 3{,}000\ (.34202 + .93969 \times .17633) = 1{,}523.2$ lb. Ans.

The negative sign in the answer to (*a*) refers only to the direction in which the force P acts.

22. If it were desired to find the values of the forces graphically, in the last example, all that would be necessary would be to draw a vertical line $A\,C$, Fig. 26, and make its length equal 3,000 pounds to some convenient scale.

From C, draw CF making an angle $ACF = 70°$ with AC. From A draw AF making an angle $CAF = 90° - 60° = 30°$ with AC. FC and FA may then be scaled.

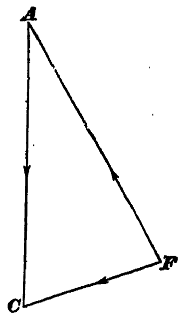


FIG. 26

EXAMPLES FOR PRACTICE

1. An inclined plane is 30 feet long and 7 feet high; what force is required to roll a barrel of flour weighing 196 pounds up the plane, the friction being neglected? In this case, the power acts parallel to the base. Ans. 47.031 lb.

2. It is required to pull a wagon up an inclined plane one mile long. The height of the plane being 120 feet, and the weight of the wagon and load 2,816 pounds, what force will be necessary? In this case, the power acts parallel to the plane. Ans. 64 lb.

THE WEDGE

23. The **wedge** is a movable inclined plane. There are two varieties: the **simple wedge,** whose longitudinal section is a right triangle, Fig. 27, and the **double wedge,** whose longitudinal section is an isosceles triangle, Fig. 28. There are two cases to be considered:

I. The load acts freely, perpendicular to the length ab of the wedge.

II. The load is restrained in a guide, acting perpendicular to the base ac.

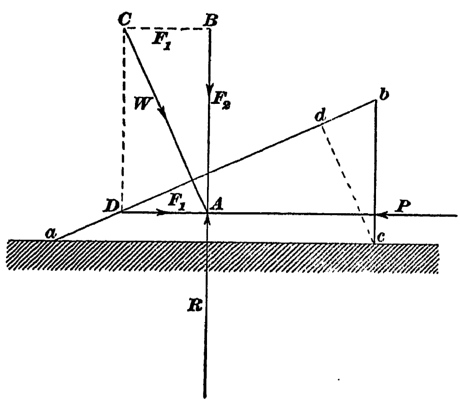


FIG. 27

In both cases, the power acts perpendicular to the height, along or parallel to ca.

24. Case I. Simple Wedge.—See Fig. 27. The three forces *acting on the wedge* are the load W, the reaction R, and the power P. These must be in equilibrium.

Resolving W into two rectangular components F_1 and F_2 in the direction of P and R, equilibrium prevails, when $F_1 = P$ and $F_2 = R$. Since triangle $A\,B\,C$ is similar to triangle $a\,c\,b$,

$$CB : bc = AC : ab$$

But $\quad AC = W, AD = CB = F_1 = P$

and $\quad P : bc = W : ab$

Hence, $\quad P \times ab = W \times bc$

The magnitudes of P and R are, respectively:

$$P = W \sin bac; \; R = W \cos bac$$

25. Double Wedge.—It is evident that in order to produce equilibrium, the wedge $d\,a\,b$, Fig. 28, must be acted on by two symmetrically placed equal forces W and W'; that is, the angle $C\,A\,a$ must equal $C'\,A\,a$. If $a\,c\,b = a\,c\,d = a\,c\,b$ in Fig. 27, and $W = W$ in Fig. 27, then $CB = C'\,B' = C\,B$ in Fig. 27, and $P = C\,B + C'\,B' = 2\;P$ in Fig. 27. If $C'\,A$ be produced to C'', $A\,C''$ being equal to $A\,C' = A\,C = W$ to some convenient scale, $C\,C'' = P$, $C\,A\,C''$ being the triangle of forces. Since the triangles $C\,A\,C''$ and $b\,a\,d$ are similar,

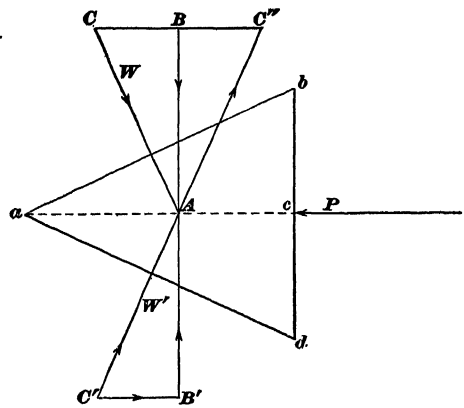


FIG. 28

$$CC'' : bd = CA : ba$$

and since $\quad CC'' = P$ and $CA = W$

$$P : bd = W : ba$$

or $\quad P \times ba = W \times bd$

Expressing in words the results obtained in Arts. **24** and **25**:

The load multiplied by the height of the wedge equals the power multiplied by the length of the wedge.

Referring to Fig. 27, suppose that the wedge is moved in the direction $c\,a$ of the power P. If W act at the end a at the

beginning of the motion, the point of application will be gradually raised above ac until W acts at the point d, at which instant the wedge will have moved the distance ca parallel to the direction of P, cd being parallel to AC. The angle adc is, therefore, a right angle, and since the angle a is common, the two triangles acb and adc are similar; hence,

$$cd : bc = ac : ab$$

which may be written $cd : ac = bc : ab$

The proportion of Art. **24** may be written

$$P : W = bc : ab$$

whence, $$cd : ac = P : W$$

That is, *the load multiplied by the distance through which it moves is equal to the power multiplied by the distance through which it moves.*

26. Case II.—The load acts perpendicular to the base of the wedge. It is evident that if the body W resting on the wedge, Fig. 29, were unrestrained, it would slide down the plane ab of the wedge abc, and there could not, therefore, be equilibrium. A guide M is therefore provided, against which the body strikes, while tending to slide down the plane. This guide exerts a reaction on the body. Let it be called M; it is equal to the power P, but acts in an exactly opposite direction. Let CA represent W and resolve it into two forces CB and BA, the first being drawn parallel to P and the second perpendicular to ab. The force BA tends to push the wedge in the direction BA perpendicular to the side ab of the wedge, while CB represents the force

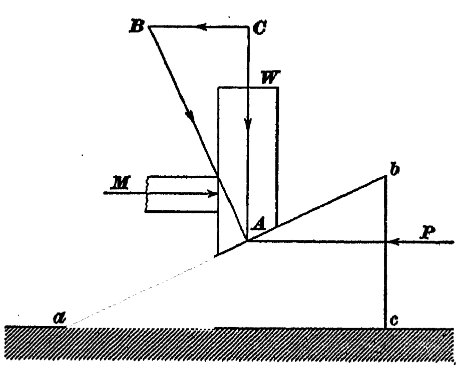


FIG. 29

necessary to keep the wedge from being pushed in the direction BC. Since the triangles ACB and acb are similar,

$$BC : bc = AC : ac$$

or

$$P \times ac = W \times bc$$

In words: *The load multiplied by the height of the wedge is equal to the power multiplied by the base.*

From the above equation,

$$P = W \times \frac{bc}{ac} = W \times \tan a$$

In the right triangle ACB, $BA = \frac{CA}{\cos A} = \frac{CB}{\sin A} = \frac{W}{\cos a}$ $= \frac{P}{\sin a}$, a being the angle bac of the right triangle acb.

27. If the wedge is moved, the load moves the vertical distance bc while the power moves the horizontal distance ac; considering bc and ac as distances moved through, the equation of the preceding article gives again the familiar principle:

The load multiplied by the distance through which it moves equals the power multiplied by the distance through which it moves; or the load and power are inversely as the distance through which they move.

EXAMPLES FOR PRACTICE

1. What force P will be required to produce a pressure of 2,700 pounds in the direction AC, Fig. 27, the length of the wedge along ac being $14\frac{1}{2}$ inches, and the height cb being $1\frac{1}{4}$ inches?

Ans. 231.89 lb.

2. In Fig. 29, the height of the wedge is $2\frac{5}{8}$ inches; and the length ac is 16 inches; what load W will a force P of 800 pounds raise?

Ans. 5,389.5 lb.

THE SCREW

28. A **screw** is a cylinder with a spiral groove winding around its circumference. The spiral line, or helix, that the groove follows may be originated in the following manner: Imagine a right triangle abc, Fig. 30 (a), cut out of a piece of paper and then wound around a cylinder whose circumference is equal in length to the base ac of the triangle, then

the hypotenuse $a\,b$ forms a helix $a\,b$, Fig. 30 (b). Adding other equal triangles $a'\,b'\,c'$, $a''\,b''\,c''$, etc. to the first, the helix is continued. Next imagine a cutting tool to follow the helix, then a groove is formed corresponding to the shape of the cutting edge. The spiral ridges left between the grooves of a screw is in practice called the **thread;** and according to whether the cutting edge of the tool is **V**-shaped, square-shaped, etc., the thread is a **V thread, square thread,** etc.

29. As a thread can be formed on the outside of a solid cylinder, it can be cut also on the inside of a body having a cylindrical hole. Such a body is called a **nut,** and if its thread is cut with a tool of the same form as was used to

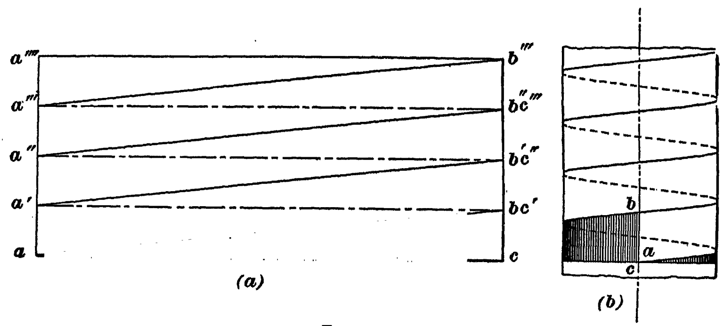


FIG. 30

cut the screw, its ridges will fit into the grooves of the screw, and vice versa.

According to whether the hypotenuse of the triangle $a\,b\,c$ slopes down from the right to the left or from the left to the right, threads are distinguished as *right-handed* and *left-handed*, respectively.

The distance between two successive windings of a screw, measured parallel to the axis, is called the **pitch;** it is equal to the height $a\,b$ of the generating triangle $a\,b\,c$.

30. The screw is, mechanically, an inclined plane on which the nut moves, or vice versa, the nut is an inclined plane on which the screw moves. The inclined plane has a height equal to the pitch $b\,c$, Fig. 30, and a base equal to

the periphery of a circle whose radius is equal to the *mean radius* r of the screw.

Suppose that the screw carries the load and slides on the nut, the latter being held stationary as in Fig. 31.

The load W acts in the direction of the axis of the screw, that is, perpendicular to the base of the inclined plane, the reaction N acts at right angles to the bottom surface of the thread, and the power P horizontally on the periphery of a circle having the mean diameter of the screw.

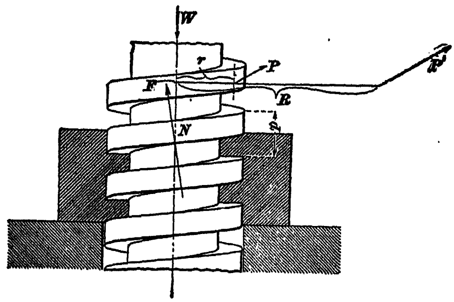


FIG. 31

The conditions are, therefore, the same as in Case II of the inclined plane, Fig. 23, Art. **20,** and

$$P : W = bc : ac$$

Calling the pitch of the screw p $(= bc)$, Fig. 30, and since $ac = 2\pi r$,

$$P : W = p : 2\pi r$$

and

$$W \times p = P \times 2\pi r$$

In words: *The load multiplied by the pitch is equal to the power multiplied by the circumference of the screw.*

By circumference of the screw is, of course, meant the circumference of a circle whose radius is the mean radius of the screw thread.

31. It is evident that the load will be raised the distance p, while the power moves through a distance equal to $2\pi r$, that is, while the screw makes one turn. The law, so far found correct in all other simple machines, also holds good for the screw: *The load multiplied by the distance through which it moves equals the power multiplied by the distance through which it moves.*

Should the power P be applied to a lever arm attached to the screw or nut with the fulcrum at F and the weight at W, as shown in Fig. 32, this law still holds good. In this case,

there is a lever, with its fulcrum F in the axis of the screw, Fig. 31, the power P of the screw constitutes the load of the lever, and the force P' applied at the end of the lever constitutes the power of the lever. Hence, for the lever the power P' multiplied by the distance it moves is $P' \times 2\pi R$, R being the distance PF in Fig. 32, and it equals the load multiplied by the distance through which it moves, or $P \times 2\pi r$. But $P \times 2\pi r = W \times p$; therefore,

$$P' \times 2\pi R = W \times p$$

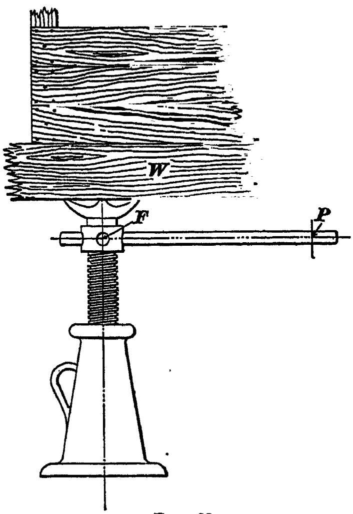


FIG. 32

32. Single-threaded screws of less than 1 inch pitch are generally classified by the number of threads they have in 1 inch of their length. In such cases, *1 inch divided by the number of threads equals the pitch;* thus, the pitch of a screw that has 8 threads per inch is $\frac{1}{8}''$; one of 32 threads per inch is $\frac{1}{32}''$, etc.

EXAMPLE.—It is desired to raise a weight by means of a screw having 5 threads per inch. The force applied is 40 pounds at a distance of 14 inches from the center of the screw; how great a weight can be raised?

SOLUTION.—The radius R of the circumference passed through by the force is 14 in. Therefore, $W \times \frac{1}{5} = 40 \times 2 \times 3.1416 \times 14$, or $W = 17{,}593$ lb. Ans.

COMBINATIONS OF SIMPLE MACHINES

33. The Compound Lever.—A **compound lever** is a series of single levers arranged in such a manner that when a force is applied to the first it is communicated to the second, and from this to the third and so on. Fig. 33 shows a compound lever. It will be seen that when the power is applied to the first lever at P', it will be communicated to

the second lever at P'', from this to the third lever at P''', and thus raise the load W'''.

The weight that the power of the first lever can raise acts as the power of the second lever, and the load that this can raise by means of the second lever acts as the power of the

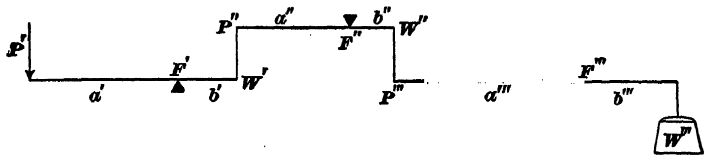


FIG. 33

third lever, and so on, no matter how many single levers make up the compound lever.

Let a', a'', a''' be the power arms of the three levers, b', b'', b''' the load arms, and F', F'', F''' the fulcrums; then for the first of the three levers, according to Art. 8,

$$P' \times a' - W' \times b' = 0$$

or

$$P' = \frac{b'}{a'} \times W' \qquad (1)$$

For the second lever, likewise,

$$P'' = \frac{b''}{a''} \times W'' \qquad (2)$$

But since $P'' = W'$, insert for W' in equation (1) the value for P'' in equation (2) and get,

$$P' = \frac{b'}{a'} \times \frac{b''}{a''} \times W'' \qquad (3)$$

Again, for the third lever,

$$P''' = \frac{b'''}{a'''} \times W''' \qquad (4)$$

and since $P''' = W''$, insert for W'' in equation (3) the value of P''' in equation (4) and get

$$P' = \frac{b'}{a'} \times \frac{b''}{a''} \times \frac{b'''}{a'''} \times W''' \qquad (5)$$

which can be written also thus:

$$P' \times a' \times a'' \times a''' = W''' \times b' \times b'' \times b'''$$

or

$$P' : W''' = b' \times b'' \times b''' : a' \times a'' \times a'''$$

In words: *The continued product of the power and each power arm equals the continued product of the weight and each weight*

arm, or the power is to the load as the product of the load arms is to the product of the power arms.

EXAMPLE.—If, in Fig. 33, the power arms a', a'', a''' are 24 inches, 18 inches, and 30 inches, and load arms b', b'', b''' are 6 inches, 6 inches, and 18 inches, (*a*) how great a force must be applied at the free end P' to raise 1,000 pounds at W'''? (*b*) What is the ratio between P' and W'''?

SOLUTION.—(*a*) $P' \times 24 \times 18 \times 30 = 1{,}000 \times 6 \times 6 \times 18$,

or
$$P' = \frac{\overset{50}{\cancel{1{,}000}} \times \cancel{6} \times \cancel{6} \times \cancel{18}}{\underset{4}{\cancel{24}} \times \cancel{18} \times \underset{5}{\cancel{30}}} = 50 \text{ lb.} \quad \text{Ans.}$$

(*b*) $50 : 1{,}000 = 1 : 20$, or $P' : W''' = 1 : 20$. Ans.

34. It is thus seen that the compound lever is a means for lifting great loads with small forces; this is, however,

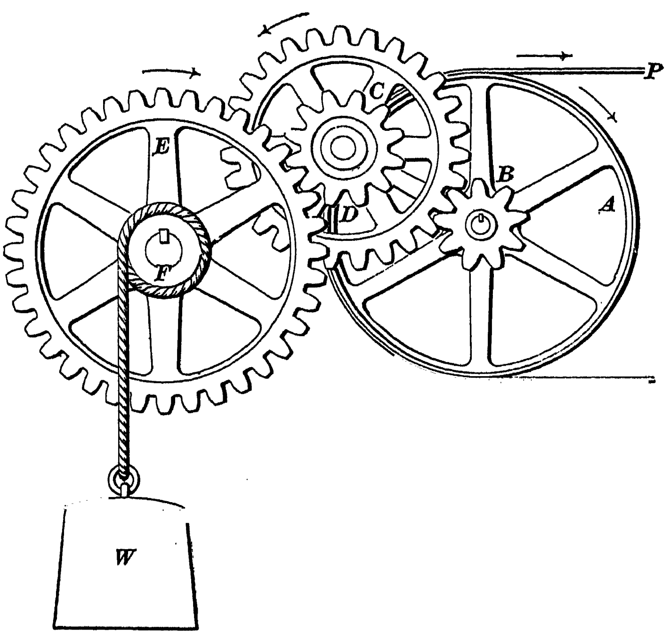


FIG. 34

accomplished through sacrifice of speed, for, as in all other machines, the power and load are inversely as the distances

through which they move. Thus, in the example to move W''' 1 inch, P must move 20 inches.

35. Wheelwork.—A combination of wheels and axles, as in Fig. 34, is called a **train.** The wheel in a train to which motion is imparted from a wheel on another shaft, by such means as a belt or gearing, is called the **driven wheel** or **follower;** the wheel that imparts the motion is called the **driver.**

It will be seen that the wheel and axle bears the same relation to the train that the simple lever does to the compound lever; that is, *the continued product of the power and the radii of the driven wheels equals the continued product of the load, the radius of the drum that moves the load, and the radii of the drivers.*

EXAMPLE.—If the radius of the wheel A, Fig. 34, is 20 inches, of C, 15 inches; and of E, 24 inches; the radius of the drum F is 4 inches; of the pinion D, 5 inches; and of the pinion B, 4 inches; how great a weight W will a force of 1 pound at P raise?

SOLUTION.— $1 \times 20 \times 15 \times 24 = W \times 4 \times 5 \times 4$, or

$$W = \frac{7{,}200}{80} = 90 \text{ lb. Ans.}$$

Hence, also, if W were raised 1 inch, P would move 90 inches, or P would have to move 90 inches to raise W 1 inch.

The law, that *whenever there is a gain in power without a corresponding increase in the initial force, there is a loss in speed,* holds here also.

In the last example, if P were to move the entire 90 inches in 1 second, W would move only 1 inch in 1 second.

36. A combination of pulleys, as shown in Fig. 35, is sometimes used. In this case, there are three movable and three fixed pulleys. The law of equilibrium for any such combination is easily established if one considers: (a) that every one of the pulleys must itself be in equilibrium and (b) that every part of a continuous rope must have the same tension.

In the combination in Fig. 35, the load W hangs on a rope that is composed of six parts a, b, c, d, e, f, not counting

the free end at P, which is a continuation of the part a; each part has the same tension, that is, $\frac{W}{6}$ pounds. Since for the upper pulley the load is this tension $\frac{W}{6}$, the power P must be equal to it. If there were eight pulleys instead of six, the load would have eight parts, and the power necessary to balance the load would be only $\frac{W}{8}$. Hence, the rule:

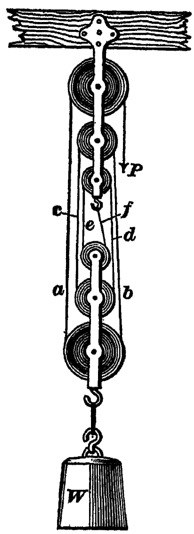


FIG. 35

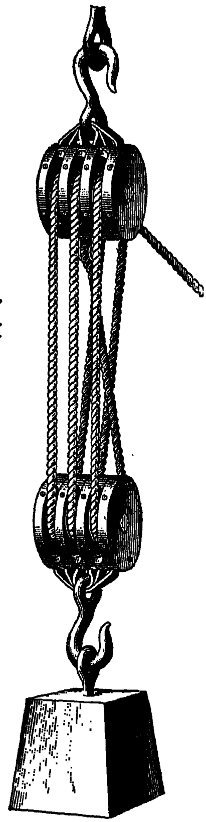


FIG. 36

In any combination of pulleys, where one continuous rope is used, a load on the free end will balance a weight on the movable block as many times as great as the load on the free end as there are parts of the rope supporting the load—not counting the free end.

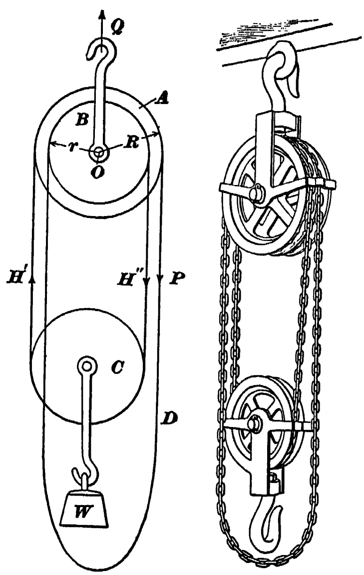
The above law is good, whether the pulleys are side by side, as in the ordinary *block and tackle*, Fig. 36, or whether they are arranged as in Fig. 35.

EXAMPLE.—In a block and tackle having five fixed and five movable pulleys, how great a force must be applied to the free end of the rope to raise 1,250 pounds?

SOLUTION.—Since there are five movable pulleys, there are ten parts of the rope supporting them, and 1 lb. on the free end will balance 10 lb. on the movable block; therefore, the ratio of P to W is 1 : 10, and $P = \frac{1,250}{10} = 125$ lb. Ans.

37. If the movable block, Figs. 35 and 36, be lifted 1 foot, P remaining in the same position, there will be 1 foot of slack in each of the six parts of the rope, or *6 feet* in all. Therefore, P will have to move 6 feet in order to take up this slack, or P moves six times as far as W. Hence, though 1 pound at P will support 6 pounds at W, P must move a distance 6 times as great as the distance through which W moves. Here again is the familiar principle *that the load and the power vary inversely as the distances through which they move.*

38. In Fig. 37 is shown an arrangement called a **differential pulley.** Fig. 37 (a) is a diagrammatic representation of the arrangement, while Fig. 37 (b) shows how the principle is applied in the case of the Weston chain hoist. The upper block contains two pulleys A and B fastened together and having the same axle; R, the radius of A, is slightly larger than r, the radius of B. An endless chain is so wound around these two pulleys as to form two loops. In one of these loops is hung an ordinary loose pulley C, carrying the load W. To that part D of the other loop leading from the large pulley A the power P is applied. The other part of this loop hangs loose. The chain is prevented from sliding by teeth or notches on the circumference of the pulleys A and B with which the chain links mesh. The load is supported by two chain parts H' and H'', so that each carries $\frac{W}{2}$, constituting two forces acting on the body formed by the rigidly connected pulleys A and B. A third force is the



(a) FIG. 37 (b)

power P; a fourth, the reaction Q of the axle. For equilibrium, the positive moments must be equal to the negative ones for any point. Choose point O as the center of moments; then the moment of Q is zero and

$$H' \times R - H'' \times r - P \times R = 0$$

or since $H' = H'' = \frac{W}{2}$

$$\frac{W}{2} \times R - \frac{W}{2} \times r - P \times R = 0$$

which evidently can be written

$$P = \frac{W(R-r)}{2R}, \text{ or } W = \frac{2PR}{R-r}$$

EXAMPLE.—If $R = 7$ inches and $r = 6\frac{1}{2}$ inches, how much weight can be raised at W with a force of 50 pounds at P?

SOLUTION.— $W = \frac{2PR}{R-r} = \frac{2 \times 50 \times 7}{7 - 6\frac{1}{2}} = 1{,}400$ lb. Ans.

39. The equations of the preceding article may also be written in the form of the following proportion:

$$P : W = \frac{R-r}{2} : R$$

Suppose that P moves a distance equal to one whole turn of the pulley A, that is, equal to $2\pi R$, then the part H' of the chain is wound up on A a like amount; at the same time, the part H'' of the chain is let down a distance equal to one turn of pulley B or equal to $2\pi r$, so that the shortening of the whole length of the chain loop $H'H''$ is equal to $2\pi R - 2\pi r$. Of this shortening, both parts H' and H'' partake equally, so that the pulley C moves up just half that amount, or $\frac{2\pi R - 2\pi r}{2} = \pi(R-r)$.

If the power moves in a certain time $2\pi R$ inches or feet, it moves in the 2πth part of that time R inches or feet; likewise if the load moves in that certain time $\pi(R-r)$ inches or feet it moves in the 2πth part just $\frac{\pi(R-r)}{2\pi} = \frac{R-r}{2}$ inches or feet. Comparing these distances moved through with the last two terms of the preceding proportion, we have again

the principle so often repeated that *power and load are inversely proportional to the distances they move through* or that *the power multiplied by the distance through which it moves equals the load multiplied by the distance through which it moves.*

VELOCITY RATIO

40. The ratio of the distance that the power moves to the distance that the load moves on account of the movement of the power is called the **velocity ratio.**

Thus, if the power is moving 12 inches while the weight is moving 1 inch, the velocity ratio is 12 to 1, or 12; that is, P moves 12 times as fast as W.

41. If the velocity ratio is known, the load that any machine can raise equals the *power multiplied by the velocity ratio.* If the velocity ratio is 8.7 to 1, or 8.7, $W = 8.7 \times P$, since $W \times 1 = P \times 8.7$.

NOTE.—In all of the preceding cases, including the last, friction has been neglected; hence, all the results so far arrived at are considerably larger than could be obtained in practice.

EXAMPLES FOR PRACTICE

1. A wedge is caused to move a weight vertically by means of a screw, which pulls the wedge horizontally on its base. If the screw has 5 threads per inch and the handle is 10 inches long, what force will be necessary to apply to the handle to raise a weight of 1,400 pounds, the height of the wedge being 8 inches, and the length, 14 inches?

Ans. 2.546 lb., nearly

2. It is desired to raise a weight of 600 pounds by means of a block and tackle having two fixed and two movable pulleys; what force must be applied at the free end of the rope? Ans. 150 lb.

3. If, in the last example, the free end of the rope be attached to a windlass whose drum is 5 inches in diameter, and which has a handle situated 15 inches from the axis of the drum, what force will be necessary to raise the weight? Ans. 25 lb.

4. In Fig. 34, the radius of the wheel A is 32 inches, of C, 18 inches, and of E, 30 inches; the radius of the drum F is 10 inches, of the pinion D, 4 inches, and of the pinion B, 6 inches. If P moves 4 feet 6 inches, how far will the weight W move? Ans. $\frac{3}{4}$ in.

5. In the last example, how great a force must be applied at P to raise a weight of 2,160 pounds? Ans. 30 lb.

6. In example 3, if the friction be taken at 22% of the load lifted, what force will be necessary? Ans. 30.5 lb.

FRICTION

PRELIMINARY REMARKS

42. Every surface in nature—however smooth or polished it may be to the eye—has minute depressions, so that when two surfaces are in contact these fit into each other more or less closely. Evidently these irregularities cause a resistance to the motion of one surface along the other, and it is to this resistance that the name **friction** is given.

Friction is not a force in the sense that it can produce motion; it acts like a force in opposing the motion of a body, and is *always opposite in direction to the motion of the body.* For example, referring to Fig. 22, if the body W is moving down the plane in the direction $A D$, friction acts in the direction $A D'$; or, if the body is moving up the plane, friction acts in the direction $A D$.

Except in the ideal case of a body falling freely in a vacuum, friction is always present and acting in every case of motion of terrestrial bodies. A ball thrown into the air meets with a resistance caused by the friction between the ball and the air in addition to resistance caused by the displacement of the air. A sled with polished runners sliding on smooth ice encounters friction between the sled and the ice and between the sled and the air.

Friction is both a necessity and a detriment. Without friction it would be impossible for a living being to walk; a nail driven through a board would not hold; no object could rest on any surface that inclined downwards—no matter how slightly. Friction is a detriment in all devices for producing motion of any kind, since it necessitates the

application of a greater force than would be required if friction were absent.

43. Friction is divided into two general classes—according to the relative motion of the bodies in contact—*sliding friction* and *rolling friction*. The simplest case of **sliding friction** is when all points of the moving body describe parallel lines, and the surfaces in contact are planes, as, for example, the crosshead of a steam engine. A practically similar case is the friction between the piston rod and the stuffingbox, the surfaces in this instance being cylindrical instead of plane. Special cases of sliding friction are the friction between the journal of a shaft and its bearing, (*a*) when the axis of the journal is horizontal and (*b*) when the axis is vertical and one of the contact surfaces is the end of the journal.

Rolling friction occurs when a wheel or other body rolls or turns without sliding on the surface with which it comes in contact.

SLIDING FRICTION

44. Friction of Rest and of Motion.—A block W rests on a horizontal plane, Fig. 38, causing a reaction N. A cord attached to the block leads over a pulley and carries a weight at the other end, that portion of the cord between the block and pulley being horizontal. If the weight attached to the end is gradually increased, a value is finally reached at which the block begins to move. Let P

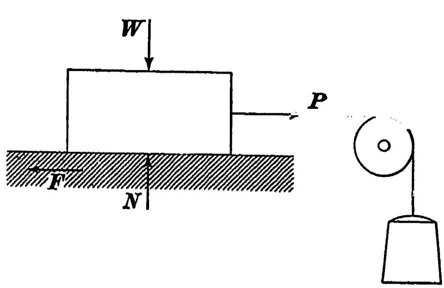


FIG. 38

denote the weight on the cord *just sufficient to cause motion*, then this force is equal and opposite to the friction F between the block and plane, which has a certain value. After the block gets in motion, the force P required to keep it moving

at constant speed is found to have a value different from that required to start the block; therefore, the friction F also has a different value. In the first instance F is called the **friction of rest,** in the second the **friction of motion.**

45. Laws of Sliding Friction.—The following are some of the facts relating to friction that have been established by experiment:

I. *For a given pair of surfaces in contact the sliding friction is proportional to the* **perpendicular pressure** *between the surfaces.*

Thus, if the body in Fig. 38 is a brick and another brick is placed on it, the force P required to start the two is double that required to start one.

II. *The sliding friction is independent of the area of the surface in contact.*

Thus, if the brick is placed on its side instead of its face, the force P required to start it is the same.

This law states that, no matter how small may be the surface that presses against another, if the perpendicular pressure is the same, the friction will be the same. Therefore, large surfaces are used where possible; not to reduce the friction, but to reduce the wear and diminish the liability of heating.

For instance, if the friction between two sliding surfaces is 2,000 pounds and the area of the surface in contact is 80 square inches, the amount of friction for each square inch of surface is $2{,}000 \div 80 = 25$ pounds. If the area of the surface had been 160 square inches, the friction would have been the same—that is, 2,000 pounds—but the friction per square inch would have been $2{,}000 \div 160 = 12\frac{1}{2}$ pounds, just one-half as much as before, and the wear and liability to heat would be one-half as great also.

It is to be mentioned here that this law is absolutely true only for friction of rest; for friction of motion it is practically true for comparatively small pressures per square inch only. For high pressures per square inch, the friction of motion varies somewhat with the pressures, as seen from the tables that follow.

III. *The friction of motion is independent of the speed, that is, the friction is the same at all speeds.*

Recent experiments have shown that this law does not hold for extremely low velocities, and is only approximately true for the higher velocities.

IV. *The friction of rest is usually greater than the friction of motion.*

V. *The friction increases with the roughness of the surfaces and is, therefore, diminished by polishing the surfaces or by interposing between them a lubricant (as oil, lard, plumbago).*

46. Coefficient of Sliding Friction.—Law I says that the friction is proportional to the *perpendicular* pressure between the surfaces, that is, equal to that pressure multiplied

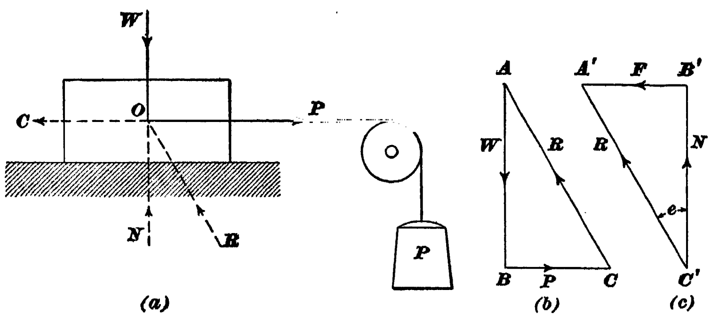


FIG. 39

by a certain number; this number is called the **coefficient of friction**, or what is the same thing the coefficient of friction is the ratio of the friction F to the perpendicular or normal pressure N. This coefficient is denoted by f and has different values for different materials and under different conditions. Thus we have

$$f = \frac{F}{N}, \text{ or } F = f \times N$$

The experimental data as to the coefficients of friction are meager and somewhat unsatisfactory. At best the table can serve only as a guide.

47. Angle of Sliding Friction.—Fig. 39 (a) is Fig. 38 repeated. Suppose that the weight P is such that it is just

equal to the friction, which acts in the direction OC, and that any increase in P will cause the body to move. The body is then in equilibrium, and is acted on by three forces—the weight W, the force P, and a reaction R whose value and direction must be determined. Draw AB vertical, Fig. 39 (b), and make it equal in value to W, to any convenient scale; draw BC horizontal (parallel to P) and make it equal to P to the same scale; then CA must be the reaction sought, and its direction must be from C to A, to produce equilibrium. Resolving CA into its horizontal and vertical components, as shown in Fig. 39 (c), $B'A'$ is the horizontal component, $C'B'$ is the vertical component, and their directions are indicated by the arrowheads. Since the triangles ABC and $A'B'C'$ are similar and equal, $A'B' = BC$, in magnitude, = the friction, and $C'B' = AB$ = the reaction caused by the weight of the body $= N = W$, in magnitude. In other words, R may be regarded as the resultant of the friction F and the reaction N, which is equal to the perpendicular pressure against the plane. The angle e between R and N is called the **angle of friction.**

The direction in which the friction F acts is always at right angles to the reaction N of the load, and $\tan e = \frac{F}{N}$. But $\frac{F}{N}$ is the coefficient of friction f; hence, $\tan e = f$; i. e., the coefficient of friction is equal to the tangent of the angle of friction.

48. Angle of Repose.—Suppose that a body of a weight, W, Fig. 40, rests on an inclined plane ab. Draw AB vertical to represent the weight of the body, and draw AC and BC perpendicular and parallel, respectively, to ab. Then AC represents the perpendicular pressure against the plane and BC represents that component of AB that tends to move the body down the plane. Since BAC and bac are equal angles, $BC = P \times \tan\ BAC = P \times \tan\ bac = P \tan a$. If the inclination of ab to ac is such that the body is just on the point of moving, BC equals the friction

caused by the perpendicular pressure AC. The angle a is then called the **angle of repose**, and when its value is such that BC equals the friction, angle a equals angle e of the last article, i. e., $P \tan a = N \tan e = F$. If a is less than or just equal to e, the body will remain stationary; but as soon as a exceeds e, no matter how slightly, the body will begin to move down the plane.

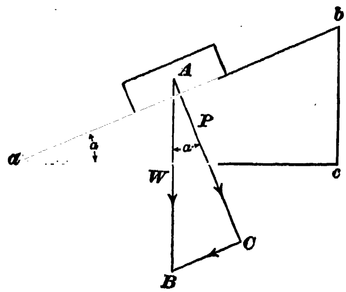


FIG. 40

49. Let ML, Fig. 41, be an inclined plane on which rests a body, as shown, which is kept from sliding down by a cord P, parallel to the plane. Suppose that it is desired to find the tension of (pull on) the cord; (a) when the cord and friction just keep the body from sliding down and (b) when the pull on the cord is just sufficient to start the body up the plane. If there were no friction, AC would evidently represent the tension of the cord. Friction, however, lessens the pull, or tension, in the first case and increases it in the second case. Hence, if OD and OB be drawn from O, so that $\tan DOC = \tan BOC = \tan e = f$, the coefficient of friction $DC = CB$ = the friction. In the first case, the tension in the cord will be represented by AD, the friction acting from D toward C, and in the second case by AB, the friction acting from B toward C, the reaction CO, perpendicular to the plane, acting in both cases from C toward O.

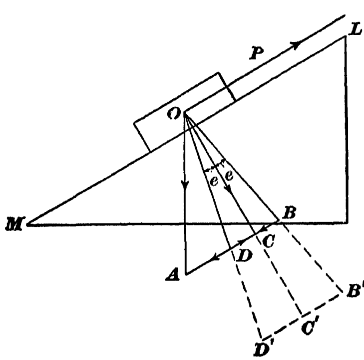


FIG. 41

50. To show the application of the foregoing principles, several examples will now be given.

EXAMPLE 1.—A body whose weight is 200 pounds, Fig. 41, rests on an inclined plane. The angle of the plane is 30° and f = .20. (*a*) What force P will just prevent the body from sliding down? (*b*) What is the force P required to draw the body up the plane, if P acts parallel to the plane?

SOLUTION.—The graphical solution would be the same as described in Art. **49**. Make OA = 200 lb. to some convenient scale. Draw AC parallel and OC perpendicular to ML. To lay off the angles e, the best method is to prolong OC to C', OC' being any convenient length, say 5 in.; draw $D'C'B'$ at right angles to OC'; then lay off on either side of C', $C'D' = C'B' = 5 \times .20 = 1$ in., and draw OD' and OB', which intersect AB in D and B, respectively. This construction is evidently correct, since $\tan e = \frac{C'D'}{OC'} = \frac{C'B'}{OC'} = \frac{1}{5} = .20$, the coefficient of friction. The angle M = 30° of course.

The analytical solution would be as follows: It was shown in Art. **18** that, neglecting friction, the force AC, Fig. 41, that tends to pull the body down the plane is $W \sin M = 200 \times \sin 30° = 200 \times .5 = 100$ lb. and the perpendicular pressure OC against the plane is $W \cos M = 200 \times .86603 = 173.2$ lb. The friction is $173.2 \times .20 = 34.64$ lb. Hence, (*a*) the force that will just prevent the body from sliding down is $100 - 34.64 = 65.36$ lb., and (*b*) the force required to start the body up the plane is $100 + 34.64 = 134.64$ lb. Ans.

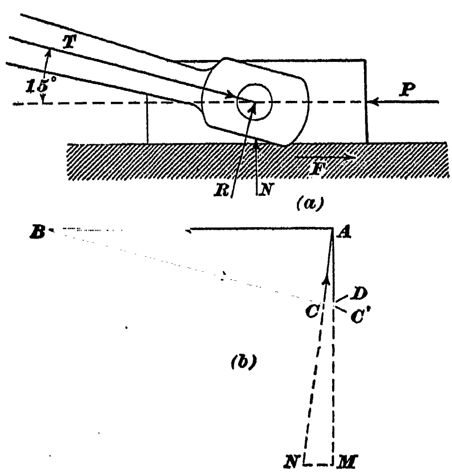


FIG. 42

EXAMPLE 2.—In Fig. 42 (*a*), suppose that the pressure P acting on the crosshead is 15,000 pounds; find the force T transmitted through the connecting-rod when the center line of the rod makes an angle of 15° with the horizontal, the coefficient of friction between the crosshead and guide being .12 and the friction of the pin being neglected.

SOLUTION.—There are three forces acting on the crosshead—the horizontal force P, the thrust T in the connecting-rod, which makes an angle of 15° with the horizontal, and the reaction R, which is the resultant of the perpendicular pressure N of the guides against the

block and the friction F. Draw AB, Fig. 42 (b), horizontal and make it equal to 15,000 lb. to some scale; draw an indefinite line BC, making an angle of 15° with BA; draw AC, making an angle with AD such that its tangent will be .12, intersecting BC in C. This last is done in the same manner as in the preceding example, by making AM some convenient length, as 5 in., drawing MN at right angles to AM, making $MN = .12 \times 5 = .6$ in., and joining N and A by the line AN. BC will be the force T and CA the force R. Drawing the horizontal line CD, CD represents the amount of friction, acting from C toward D, and DA represents the perpendicular pressure against the crosshead. Had friction been neglected, the values of T and N would have been represented respectively by BC' and $C'A$.

The analytical solution is as follows: In the triangle ABC, angle $B = 15°$, angle $A = 90°$ − the angle whose tangent is .12 $= 90° - 6° \; 50' \; 34'' = 83° \; 9' \; 26''$, angle $C = 180° - (15° + 83° \; 9' \; 26'') = 81° \; 50' \; 34''$, and the side $AB = 15{,}000$. Solving this triangle, by trigonometry, $BC = 15{,}045$ lb. Ans.

EXAMPLE 3.—In Fig. 43 (a), suppose that the weight of the block B and its load is 600 pounds; that the angle of the wedge W

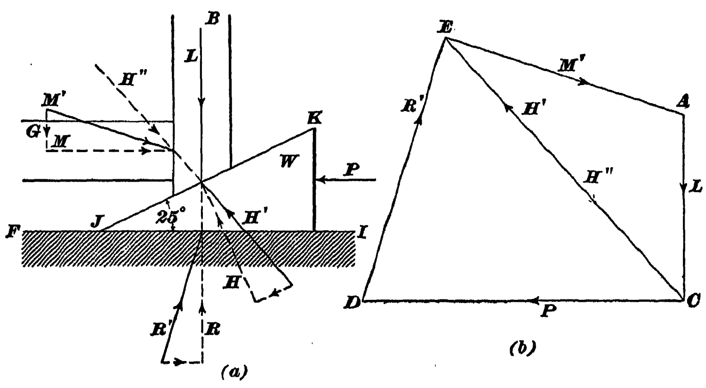


FIG. 43

is 25°; and that the coefficient of friction between the guide G and block B, between block B and wedge W, and between wedge W and surface FI is in each case .321. Find what the value of the horizontal force P must be in order to just start B to moving upwards.

SOLUTION.—Consider, first, the forces acting on the block B. These are three in number: the load L, the reaction M', and the reaction H'. To determine M' and H', consider each separately.

If there were no friction the reaction M would be horizontal; but since there is friction, a line M' must be drawn making an angle with M whose tangent is equal to the coefficient of friction .321 or

$17° 48'$, very nearly. To determine on which side of M the line M' must lie, it will be noticed that if the wedge W moves under the action of P that the block B will move upwards, hence the friction acts downwards, and M' must lie above M, and must make an angle of $17° 48'$ with M.

If there were no friction, the reaction H' would have the direction of the dotted line H, perpendicular to side JK of the wedge, which side is parallel to the bottom of the block. On account of friction, H' must make an angle with H, in this case $17° 48'$. If the wedge were to move under the action of P, the motion of the block would be *up* the plane; hence, the friction must act *down* the plane and H' must be drawn to the right of H, as shown.

Now draw AC, Fig. 43 (*b*), vertical, i. e., parallel to L, and make it equal to 600 lb. to some convenient scale; draw AE parallel to M' and CE parallel to H'; they intersect at E, and EA and CE represent the magnitudes of M' and H', respectively.

Next, consider the wedge W, Fig. 43 (*a*); this is acted on by three forces: P, a reaction R', and a reaction H'', which must be equal and opposite to H'. If there were no friction, R' would have the direction R, equal and opposite to L; but by reason of the friction between the wedge and the surface FI, R' must make an angle of $17° 48'$ with R. If the wedge moves under the action of P, the direction of the movement will be from right to left; hence, the friction must act in the opposite direction, as shown, and R' must be drawn to the left of R.

In the triangle EAC, Fig. 43 (*b*), EC represents the reaction H'', the direction being from E to C, as indicated by the dotted arrowhead. Draw CD horizontal, i. e., parallel to P, and ED parallel to R'; they intersect in D, and CD and DE represent the magnitudes of P and R', respectively.

If it be desired to find the value of P analytically, proceed as follows: Angle $A = 90° + 17° 48' = 107° 48'$. To determine angle ECA, notice that the angle that H makes with the upper side of FI, on the left, is $90° - 25° = 65°$, and that the angle that it makes with the vertical L is $90° - 65° = 25°$; hence, the angle that H' makes with the vertical is $25° + 17° 48' = 42° 48' = ECA$. Angle $CEA = 180° - (107° 48' + 42° 48') = 29° 24'$. Since $AC = 600$, $CE = 1{,}163.7$ lb., by trigonometry.

In triangle CDE, angle $ECD = 90° - 42° 48' = 47° 12'$; angle $D = 90° - 17° 48' = 72° 12'$; angle $CED = 180° - (47° 12' + 72° 12') = 60° 36'$; and $EC = 1{,}163.7$. Solving by trigonometry, $P = 1{,}064.8$ lb. Ans.

51. It will be interesting to ascertain what the effect of the friction was in the last example in increasing P. If there were no friction, the quadrilateral $ACDE$, Fig. 43,

would be a rectangle, in which $M' = M = P$. The angle E would be 25°; angle A, 90°; and angle CEA, 65°. Hence, EA would equal $CD = P = AC \tan ECA = 600 \times \tan 25° = 600 \times .46631 = 279.786$ lb. (See Arts. **20** and **25**.)

52. Journal Friction.—**Journal friction,** as was stated in Art. **43,** is a special kind of sliding friction and plays a very important part in connection with machinery. In Fig. 44 is shown a journal resting in its bearing.

As in sliding friction, the friction is equal to the perpendicular pressure between the surfaces, multiplied by the coefficient of journal friction, which will be denoted by f', whose value in particular cases is to be determined by experiment. It is not necessarily the same as f even for the same materials in contact. It may vary from .05 to .10, depending on the lubricants used.

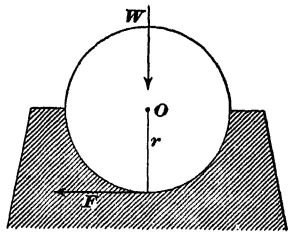


FIG. 44

If r, Fig. 44, denotes the radius of the journal, $F \times r$ is the moment of journal friction about the center O and is, therefore, the moment of the external couple that must be applied to turn the journal. Hence, the greater the diameter of the journal with the same load W the greater will be the value of the couple required to turn it.

EXAMPLE.—The load on a journal is 4,000 pounds, the diameter of the journal is $3\frac{1}{2}$ inches, and $f' = .075$; what is the friction and the moment of the friction?

SOLUTION.—Friction is $f'W = .075 \times 4{,}000 = 300$ lb. Ans. The moment is $300 \times \frac{3\frac{1}{2}}{2} = 525$ in.-lb. Ans.

53. Bearing Pressure.—If d and l denote, respectively, the diameter and length of a journal, the product dl is called the **projected area** of the journal. Let the load W be divided by this projected area; the quotient is the pressure per square inch of projected area or more simply the *bearing pressure.* Call this p; then $p = \frac{W}{dl}$.

The value of p must not exceed certain limits or the lubricant will be squeezed out and the journal will heat. For metals other than steel, p should not exceed 800 pounds per square inch; while for steel it is sometimes as high as 1,200 pounds per square inch.

The pressure p should be smaller the higher the speed of a point on the circumference of the journal; thus with the speed of 200 feet per minute, Professor Thurston found that p should not exceed 30 to 75 pounds per square inch of projected area, with ordinary lubricants.

ROLLING FRICTION

54. Rolling friction is much less than sliding friction, from which fact sliding surfaces are, wherever permissible, supplanted by rolling surfaces in machine practice. The most familiar examples are the various vehicles on wheels. The same general law obtains as in sliding friction; namely, that *the rolling friction is proportional to the pressure between the surfaces in contact*, so that we have:

$$\text{Rolling friction} = W \times f''$$

in which f'' is the **coefficient of rolling friction.**

The rolling friction is, however, not to be considered as a force, as sliding friction is, but as a *moment*, of which W is the force and f'' the arm, as will be understood from the following. Assume an impediment, owing to the inevitable roughness of all surfaces, to lie in the path of the rolling body, then this impediment will for the time being form the fulcrum about which motion must take place. For equilibrium, we then have, referring to Fig. 45 (a),

$$P \times AB = W \times OB$$

AB is very nearly equal to r, as it must be kept in mind that the impediment cannot but be very small. Thus we have, if we furthermore designate the distance OB by f'',

$$P \times r = W \times f'' = \text{rolling friction}$$

The force necessary to overcome the rolling friction is, from the above equation,

$$P = \frac{Wf''}{r} \qquad (1)$$

when the force P is applied at the center of the rolling body as shown in Fig. 45 (*a*).

When P is applied at the periphery of the rolling body and is horizontal, as shown in Fig. 45 (*b*), the moment of the force is $P \times 2r$, which must again be equal to the rolling friction

$$P \times 2r = W \times f''$$

from which

$$P = \frac{Wf''}{2r} \qquad (2)$$

Again, if a load is moved on a roller between two planes, as shown in Fig. 45 (*c*), two frictional moments, one being the friction between the lower plane and the roller and the

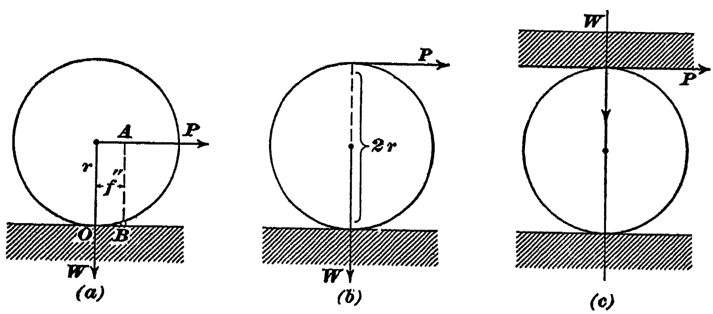


FIG. 45

other being the friction between the upper plane and the roller, must be overcome by the moment of the applied force and we have

$$P \times 2r = 2 \times W \times f''$$

which again gives formula **1**.

55. Coefficients of Rolling Friction.—Experiments on rolling friction are very incomplete. The following are a few values for f'', when W is given in pounds and r in *feet*.

Wood on wood		$f'' = .06$
Metal on metal		$f'' = .005$
Railroads	ordinary	$f'' = .003$
	well laid	$f'' = .002$
	best possible	$f'' = .001$

TABLE I

COEFFICIENTS OF SLIDING FRICTION, *f*, FOR SEVERAL METALS, HIGH PRESSURES PER SQUARE INCH, AND SCANTILY LUBRICATED SURFACES, AFTER RENNIE

NOTE.—The surfaces were lubricated and then wiped off, so that the lubricant could not prevent the close contact of the surfaces.

Pressure in Pounds per Square Inch of Contact Surface	Wrought Iron on Wrought Iron	Cast Iron on Wrought Iron	Steel on Cast Iron	Brass on Cast Iron
125	.140	.174	.166	.157
186	.250	.275	.300	.225
226	.271	.292	.166	.219
260	.285	.321	.300	.214
298	.297	.329	.333	.211
336	.312	.333	.340	.215
370	.350	.351	.344	.206
385	.376	.363	.347	.205
448	.395	.365	.351	.208
485	.403	.366	.353	.221
523	.409	.366	.354	.223
560	Surfaces abraded	.367	.356	.233
600		.367	.357	.234
630		.367	.358	.235
672		.376	.359	.233
710		.434	.367	.234
785		Surfaces abraded	.403	.232
820			Surfaces abraded	.273

TABLE III

SOME SPECIAL COEFFICIENTS OF FRICTION

1. **Mechanisms,** total friction, in the average . $f = .05$

2. **Iron tires on dry iron rails.**

Velocity, in miles per hour	10	16	20	32	45	49
Coefficient of friction f . . .	.209	.206	.171	.145	.136	.112

3. **Steel tires on steel rails.**

Velocity, in miles per hour	low	7	34	60
Coefficient of friction, f . .	.242	.088	.065	.027

4. **Cast-iron brake shoes on steel tires.**

Velocity, in miles per hour. .	6	12	19	25	31	37	44	50	56
Coefficient of friction average f .	.201	.164	.142	.128	.117	.109	.103	.098	.093

5. **Pumping machinery.**

(*a*) *Bronze or lignum vitæ slides on bronze.* f is constant with slow reciprocating motion and a pressure of from 30 to 1,500 pounds per square inch.

Slides constantly greased . . . . . . $f = .06$

Slides wetted by water by means of numerous grooves . . . . $f = .10$

Slides running dry and groaning . . . f up to .30

(*b*) *Stuffingboxes* packed with hemp, cotton, or leather cups. f is constant with pressures of from 15 to 750 pounds per square inch, and independent of the height of the packing. The friction, in pounds, is equal to the water pressure in pounds per square inch $\times f \times$ circumference of packed surface.

1. Cotton or hemp, loose or braided, soaked in hot tallow, rod smooth, box not firmly

TABLE III—*Continued*

pressed down, so that packing is still elastic, ordinary dimensions; even after several months . . . . f = .06 to .11

2. Cotton or hemp, difficult packing (heavy boxes, difficult of access, etc.) . . . . f up to .25

3. Leather-cup packing
- soft leather, good workmanship . . . . f = .03 to .07
- hard leather, well tanned . . . . f = .10 to .13
- unfavorable conditions (rough piston, dirty water, etc.) . . . . f = up to .20

6. **Grindstones.**

Coarse-grained sandstone and . . . .
- cast iron . . . f = .21 to .24
- steel . . . . . . f = .29
- wrought iron . f .41 = to .46

according to whether the stone has been newly sharpened or has become dull.

Fine-grained sandstone, wet and . (average)
- cast iron . . f = .72
- steel . . . . f = .94
- wrought iron f = 1.00

7. **Coefficients of total friction for street vehicles.**

Smooth granite road	.006
Street-car rails, average	.006 to .008
Good asphalt paving	.010
Excellent stone paving	.015
Poor stone paving	.020
Macadamized road: in excellent condition	.016
in good condition	.023
covered with dust, etc.	.028
covered with mud and rutted	.035
in very bad condition	.050
Mud roads: in excellent condition	.045
in good to bad condition	.080 to .160
Loose sand	.15 to .30

TABLE II

COEFFICIENT OF SLIDING FRICTION FOR LOW PRESSURES (4 – 20 POUNDS) PER SQUARE INCH, AFTER MORIN

Description of Surfaces in Contact	Disposition of Fibers*	State of the Surfaces	Coefficient of Friction	
			of Rest	of Motion
Cast iron on cast iron		slightly unctuous	.16	.15
		with water		.31
Wrought iron on cast iron or bronze		dry	.19	.18
Wrought iron on wrought iron		dry		.44
		slightly unctuous	.13	
Bronze on cast iron		dry		.21
Bronze in wrought iron		slightly unctuous		.16
Bronze on bronze		dry		.20
Cast iron on oak	=	dry		.49
	=	with water	.65	.22
	=	slightly unctuous		.19
Wrought iron on oak	=	with water	.65	.26
	=	with tallow	.11	.08
Bronze on oak	=	dry	.62	
Oak on oak	=	dry	.62	.48
	=	with dry soap	.44	.16
	‡	dry	.54	.34
	‡	with water	.71	.25
	†	dry	.43	.19
Wood (medium hard) on oak	=	dry	.55	.38
Leather on oak	leather, flat	dry	.61	
	leather on edge	dry	.43	.33
		with water	.79	.29
Leather belt on oak drum	=	dry	.47	.2[illegible]
Hemp rope on oak	=	dry	.50	
Leather belt on cast iron	flat	dry	.28	.56
	flat	with water	.38	.36
Leather packing	flat	without lubrication		.56
	flat	with water	.62	.36
	flat	with oil, soap	.12	.15
	flat	greasy and damp		.23
Steel on ice		dry	.027	.014

* = means that the motion takes place in the direction of the fibers of both bodies; ‡, that the motion takes place against the fibers of the moving body; †, that cross-grain of one body moves along the fibers of the other body.

TABLE III—*Continued*

8. **Coefficients of friction for sledges.**

Unarmed wooden runners on smooth wood or stone road . . .	unlubricated	.38
	lubricated with dry soap	.15
	lubricated with tallow	.07
Unarmed wooden runners on snow and ice . .		.035
Armed wooden runners on snow and ice . . .		.02

ELEMENTARY MECHANICS
(PART 4)

KINEMATICS

VELOCITY

1. In *Elementary Mechanics*, Parts 1, 2, and 3, forces have been considered as producing, or tending to produce, equilibrium in the body or system of bodies on which the forces acted. In the present Section, forces will be considered as being the cause of motion.

UNIFORM VELOCITY

2. **Velocity** is the rate of motion; it is the rate at which a body changes its position between two points. To determine the velocity of a body, two units are necessary—the unit of length and the unit of time. When these have been decided on, velocity is the number of units of length passed over in one unit of time. If a body moves 20 feet in 1 second and the unit of length is 1 foot and the unit of time is 1 second, the velocity is 20 feet per second. If the body moves 72 feet in 4 seconds with an unchanging speed, it will move $72 \div 4 = 18$ feet in 1 second, and the velocity will be 18 feet per second.

3. When in passing over a distance a body moves over each unit of length of that distance in the same time, the velocity is **uniform**; in other words, when a body has uniform velocity it passes over *equal spaces in equal times*. For

example, if a railroad train travels 4,200 feet in 1 minute 10 seconds, without any change in its speed during that time, it has a uniform velocity of $4{,}200 \div 70 = 60$ feet per second, or of $4{,}200 \div 1\frac{1}{6} = 3{,}600$ feet per minute, or of $3{,}600 \times 60 = 216{,}000$ feet per hour, or of $\frac{216000}{5280} = 40\frac{10}{11}$ miles per hour.

From the foregoing, it is evident that to find the uniform velocity all that is necessary is to divide the distance passed over by the time consumed.

Let s = distance or space traversed;
t = time consumed;
v = uniform velocity;

then,
$$v = \frac{s}{t} \qquad (1)$$

Before applying this formula or the two following formulas, s, t, and v should always be expressed in the required units.

From formula **1**,
$$s = vt \qquad (2)$$
and
$$t = \frac{s}{v} \qquad (3)$$

Example 1.—If a body moves 2.5 miles in 3 minutes 20 seconds, what is its velocity, in feet per second?

Solution.—In this case, s, in formula **1**, must be expressed in feet and t in seconds. Hence,

$$v = \frac{2.5 \times 5{,}280}{3\frac{20}{60} \times 60} = \frac{13{,}200}{200} = 66 \text{ ft. per sec.} \quad \text{Ans.}$$

Example 2.—If the uniform velocity of a railroad train for several miles is 45 miles per hour, how many seconds will it take to traverse 400 feet?

Solution.—In formula **3**, t must be in seconds, s in feet, and v in feet per second. The problem may be worked in two ways: (1) by substituting in formula **1** and finding v and then substituting in formula **3**, or (2) by substituting in formula **3** direct.

First Solution.—

$$v = \frac{s}{t} = \frac{45 \times 5{,}280}{60 \times 60} = 66 \text{ ft. per sec.}$$

$$t = \frac{s}{v} = \frac{400}{66} = 6\frac{2}{33} \text{ sec.} \quad \text{Ans.}$$

Second Solution.—

$$t = \frac{s}{v} = \frac{400}{\dfrac{45 \times 5{,}280}{60 \times 60}} = \frac{400 \times 60 \times 60}{45 \times 5{,}280} = 6\tfrac{2}{33} \text{ sec. Ans.}$$

EXAMPLE 3.—Suppose that a wheel 6 feet in diameter turns around 1,425 times in 3 minutes. (*a*) What is the velocity, in feet per second, of a point on the outside? (*b*) What is the velocity, in miles per minute? (*c*) How many yards will the point travel in 1 hour?

SOLUTION.—(*a*) $s = 6 \times 3.1416 \times 1{,}425$ ft.; $t = 3 \times 60$ sec.

$$v = \frac{s}{t} = \frac{6 \times 3.1416 \times 1{,}425}{3 \times 60} = 149.226 \text{ ft. per sec. Ans.}$$

(*b*)

$$v = \frac{s}{t} = \frac{\dfrac{6 \times 3.1416 \times 1{,}425}{5{,}280}}{3} = \frac{6 \times 3.1416 \times 1{,}425}{5{,}280 \times 3} = 1.69575 \text{ mi. per min.}$$

Ans.

(*c*) $$s = vt = \frac{1.69575 \times 5{,}280}{3} \times 60 = 179{,}071.2 \text{ yd. Ans.}$$

4. Conditions for Uniform Motion.—Uniform motion, and consequently uniform velocity, cannot be conceived as the immediate result of the action of a force, since it takes time for a body to acquire a certain velocity under the action of a force. When a body is at rest and is constantly acted on by a force moving it, the velocity gradually increases. If after a certain velocity has been reached the force ceases to act, uniform motion sets in according to Newton's first law; this is strictly true only when friction is not considered. Considering friction, a constant force must still be acting on a uniformly moving body just balancing the constant frictional resistance.

5. Variable Motion.—**Variable motion** is a motion in which, in equal intervals of time, unequal paths are traversed; that is, the velocity is variable. When a body moves with a variable velocity formula **1** of Art. **3** gives the **average** or **mean velocity** for the path in question. For example, if a railway train passes over 240 miles of track in 6 hours, the mean velocity is 40 miles per hour, though the actual velocity may be at times much greater and at other times less.

Consider the motion of a train starting from a station. At the instant of starting, the velocity is zero and from this value it increases gradually until the greatest value is reached. Evidently the velocity at any instant has a definite value greater than that an instant before and less than that an instant later. The *velocity at any instant* is measured as follows: Suppose that at the instant considered the velocity were to become constant; then the distance the point will move in one unit of time at this constant velocity is the measure of the velocity at the given instant.

6. Graphical Representation of Motion.—On the horizontal line OX, Fig. 1, let each space represent an interval of time, say 1 second; then the number of seconds, from the instant we start to consider the motion, is given by the number of spaces to the right of O. At each division point of the horizontal line, let perpendiculars be erected representing the velocity the moving point has at that particular time. Join the end points of the perpendicular lines by a continuous line, *the velocity curve*. Then the area of the closed figure thus obtained represents, by its area, the

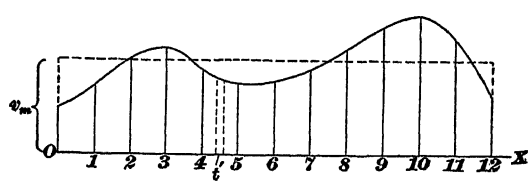


FIG. 1

path traversed by the point in the time OX. To prove this, consider a narrow strip of this area of the very small width t'. During the small space of time represented by t', the motion may be consided to have been uniform, that is, the velocity to have been constant. The path traversed during this time t' is then $t' \times v = s$, which is, however, the area of the narrow strip, considered to be a rectangle. The whole figure may be thought to be composed of such narrow strips, and thus the whole path traversed is represented by the area of the figure. The mean velocity is obtained from this

figure by drawing a rectangle over the line OX having a length and area equal to that of the irregular figure. The height of this rectangle v_m is the mean velocity. For uniform motion, the diagram is at once a rectangle.

EXAMPLES FOR PRACTICE

1. The piston speed of a steam engine is 10 feet per second; how many miles will the piston travel in 1 hour? Ans. $6\frac{9}{11}$ mi.

2. If a railroad train travels 70 miles in 1 hour, what is its speed in feet per second? Ans. $102\frac{2}{3}$ ft. per sec.

3. A man runs 100 yards in 12 seconds; how long will it take him to run a mile at the same rate? Ans. 3 min. 31.2 sec.

4. The outside diameter of an engine flywheel is 13 feet 9 inches; if a point on the rim travels 45,000 feet in 5 minutes, what is the speed in feet per second? Ans. 150 ft. per sec.

5. The Empire State express has made the trip from New York to Buffalo, 440 miles, in 412 minutes; find the mean speed in miles per hour and feet per second. Ans. $\begin{cases} 64.08- \text{ mi. per hr.} \\ 93.98 \text{ ft. per sec.} \end{cases}$

COMPOSITION AND RESOLUTION OF VELOCITIES

7. Composition of Velocities.—Suppose that a point A, Fig. 2, has a uniform motion relative to a plane $KLMN$ and that it moves, in the unit of time, a distance AB equal to its constant velocity v_1. Suppose that the plane $KLMN$ has, at the same time, a uniform motion relative to the earth and

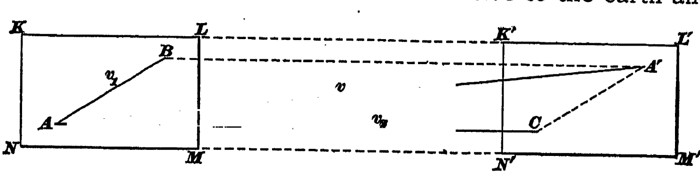


FIG. 2

that it moves, in the unit of time, a distance AC equal to its constant velocity v_2. Thus, after the unit of time the plane will occupy the position $K'L'M'N'$ and the point A will have the position A' relative to the earth; that is, it will be at the end of the diagonal AA' of the parallelogram $ABA'C$; in

other words, it will have moved from A to A' with a constant velocity equal to $v = A\,A'$, the diagonal of the parallelogram formed by the original given velocities. A **parallelogram of velocities** may thus be constructed in precisely the same manner as a parallelogram of forces.

EXAMPLE.—A man rows across a river at the rate of 3 miles per hour and at the same time the current carries the boat down stream at the rate of 5 miles per hour. (*a*) What is the velocity of the boat, relative to the river bed? (*b*) What is the direction of the velocity?

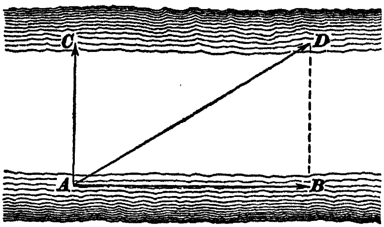


FIG. 3

SOLUTION.—(*a*) Lay off $A\,C = 3$ to any convenient scale (see Fig. 3), and $A\,B = 5$ to the same scale. Complete the parallelogram $A\,B\,D\,C$, and draw the diagonal $A\,D$. The length $A\,D$ may be measured, but since $A\,B\,D$ is a right triangle it can easily be computed; thus, $\overline{AD}^2 = \overline{AB}^2 + \overline{BD}^2 = 5^2 + 3^2 = 34$, and $A\,D = \sqrt{34} = 5.831$. The resultant velocity is therefore 5.831 mi. per hr. Ans.

(*b*) The direction is that of the line $A\,D$. To obtain the angle $D\,A\,B$ that this direction makes with the direction of the current,

$$\tan D\,A\,B = \frac{DB}{AB} = \frac{3}{5} = .6, \text{ and } D\,A\,B = 31°, \text{ nearly. Ans.}$$

8. Resolution of Velocities.—Since two velocities can be combined into a single resultant velocity, it follows that a given velocity may be considered as equivalent to two velocities in any two directions. The magnitudes of these component velocities can be determined by making the given velocity, the diagonal of a parallelogram whose sides are parallel to the direction it is desired that the components should have. In Fig. 4, let $A\,D$ represent the given velocity. On $A\,D$ as a diagonal, any number of parallelograms can be constructed and the adjacent sides of each give a pair of component velocities

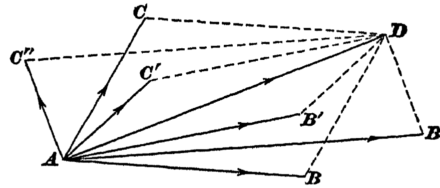


FIG. 4

equivalent to the velocity represented by *A D*; thus *A D* may be replaced by the component velocities *A B* and *A C*, *A B′* and *A C′*, *A B″* and *A C″*, etc.

EXAMPLES FOR PRACTICE

1. A train is running toward the southeast on a straight track that makes an angle of 30° with an east and west line; if the velocity of the train is 45 miles per hour, (*a*) what velocity has it toward the east? (*b*) toward the south?

Ans. $\begin{cases}(a) \text{ 38.97 mi. per hr. east} \\ (b) \text{ } 22\frac{1}{2} \text{ mi. per hr. south}\end{cases}$

2. A baseball is thrown due north, its velocity on leaving the hand being 100 feet per second; a wind is blowing in a direction 35° degrees east of north, with a velocity of 15 miles per hour. What will be the general direction of the ball just before striking the ground, leaving the resistance of the air out of consideration, but assuming that the ball has imparted to it the additional velocity of the wind before striking?

Ans. 6° 6′ east of north, nearly

VARIABLE VELOCITY

9. Acceleration.—The rate at which the velocity of a moving body changes, that is, the increase or decrease per unit of time, is called **acceleration.** If the velocity changes by equal amounts in equal time intervals, the acceleration is said to be *constant* or *uniform*, otherwise it is *variable.* Suppose that at a given instant the velocity of a moving point is 10 feet per second, and that after 1, 2, 3, 4, etc. seconds the velocities are, respectively, 13, 16, 19, 22, etc. feet per second. The increase of the velocity in each second is 3 feet per second and the acceleration, therefore, is constant.

The measure of a constant acceleration is the change of velocity in a unit of time and is obtained by dividing the total change of velocity by the time in which the change takes place.

Let v_1 = velocity at beginning of interval;
v_2 = velocity at end of interval;
t = time interval;
a = acceleration;

then,

$$a = \frac{v_2 - v_1}{t}$$

EXAMPLE.—A body moving in a straight line has a velocity of 20 feet per second; this velocity is constantly accelerated and at the end of 12 seconds reaches 116 feet per second. What is the acceleration?

SOLUTION.—Here $v_1 = 20$, $v_2 = 116$, and $t = 12$; substituting in the formula,

$$a = \frac{v_2 - v_1}{t} = \frac{116 - 20}{12} = \frac{96}{12} = 8 \text{ units of acceleration}$$

The **unit of acceleration** is a change of velocity of 1 ft. per second accomplished in 1 sec. When we say the acceleration is 8 units, we mean that in each second the change of velocity is 8 ft. per sec. When the unit of time is 1 sec. and the unit of space or distance is 1 ft., the unit of acceleration is 1 ft. per sec. per sec., or, as some writers express it, 1 ft. per sec^2.

The unit of time is universally taken as 1 second, but the unit of space or distance varies, being generally taken as 1 foot or 1 meter.

10. Uniformly Accelerated Motion Starting From Rest.—If the point starts from rest so that $v_1 = 0$, then denoting the velocity at the end of a time interval by v, we have, inserting these values in the formula of Art. **9**,

$$a = \frac{0 + v}{t} = \frac{v}{t} \qquad (1)$$

$$v = a\,t \qquad (2)$$

$$t = \frac{v}{a} \qquad (3)$$

For $t = 1$, the first of these three equations becomes $a = v$; hence, when a body starts from rest and moves with uniformly accelerated velocity, the acceleration is numerically equal to the velocity at the end of the first second.

The second equation shows that the velocity attained at the end of any time interval, starting from rest, is the product of the time and acceleration.

EXAMPLE.—A train starting from a station attains a velocity of 30 miles per hour in 22 seconds with uniform acceleration. (*a*) What is the acceleration? (*b*) What is the velocity at the end of 10 seconds?

SOLUTION.—In 1 mi. there are 5,280 ft.; hence, 30 mi. per hr. $= \frac{30 \times 5,280}{60 \times 60} = 44$ ft. per sec.

(*a*) $$a = \frac{v}{t} = \frac{44}{22} = 2 \text{ units. Ans.}$$

(*b*) The velocity at the end of 10 sec. is $v = at = 2 \times 10 = 20$ ft. per sec. Ans.

11. Space Traversed.—Fig. 5 shows the velocity curve for the motion of a point starting from rest and having uniform acceleration. As in Fig. 1, the spaces along OX represent times and the verticals represent velocities. Starting from rest, the velocity at the end of the first second is given by A_1A; that at the end of the second second, by B_1B; at the end of the third second, by C_1C; at the end of t seconds, by XY, and so on. The change of velocity between the first and

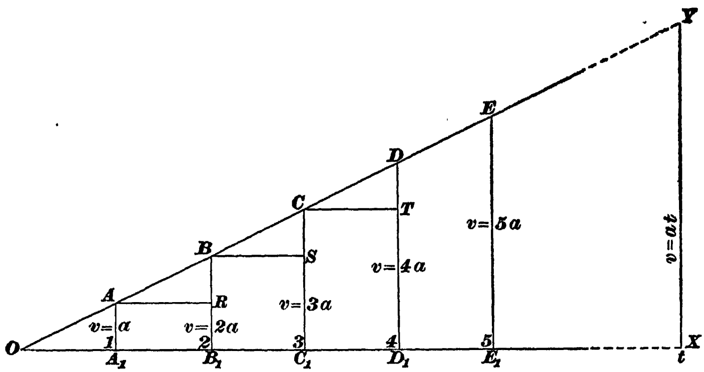


FIG. 5

second seconds is represented by $B_1B - A_1A = BR$, the change in the next second, by CS; that in the next, by DT; and so on. These changes are all equal because of the uniform acceleration, and this can only be the case when $OABCD$ is a straight line. Since, now, the space traversed is equal to the area of the figure, one side of which is the velocity curve, in this case the triangle OXY, the space traversed is $s = \frac{XY \times OX}{2}$; hence, since $XY = v = at$, and $OX = t$,

$$s = \frac{vt}{2} \qquad (1)$$

$$s = \frac{at \times t}{2}$$

$$s = \tfrac{1}{2}at^2 \qquad (2)$$

The same result is arrived at by means of the mean velocity. If v denotes the velocity at the end of a given

time t, since the velocity at the start was zero, the average velocity is $\frac{0+v}{2}=\frac{v}{2}$. The distance moved through is the product of this mean velocity and the time; or, using symbols, $s=\frac{v}{2}\times t=\frac{v\,t}{2}=\frac{1}{2}a\,t^2$.

EXAMPLE.—In the example of Art. **10**: (*a*) how far had the train moved at the end of the 22 seconds? (*b*) at the end of 10 seconds?

SOLUTION.—(*a*) The value of a was found to be 2 units. By formula **2** of this article,

$$s=\tfrac{1}{2}a\,t^2=\tfrac{1}{2}\times 2\times 22^2=484 \text{ ft. Ans.}$$

(*b*) At the end of 10 sec.,

$$s=\tfrac{1}{2}\times 2\times 10^2=100 \text{ ft. Ans.}$$

12. Other important formulas are obtained as follows: First, by transforming formula **2**, Art. **11**, to find t:

$$t=\sqrt{\frac{2\,s}{a}} \qquad (1)$$

Second, from formula **3**, Art. **10**, $t=\frac{v}{a}$; and this value substituted for t in formula **1**, Art. **11**, gives

$$s=\frac{v}{2}\times\frac{v}{a}=\frac{v^2}{2\,a} \qquad (2)$$

from which $v^2=2\,a\,s$, or $v=\sqrt{2\,a\,s}$ (3)

EXAMPLE 1.—What velocity has a uniformly accelerated body after traveling 625 feet, starting from rest, the acceleration being 8 units?

SOLUTION.— $v=\sqrt{2\,a\,s}=\sqrt{2\times 8\times 625}=100$ ft. per sec. Ans.

EXAMPLE 2.—Suppose that a body starts from rest and after traveling 900 feet has a velocity of 150 feet per second; the acceleration being uniform throughout, what was it?

SOLUTION.—Solving formula **3** for a, and substituting 900 for s and 150 for v,

$$a=\frac{v^2}{2\,s}=\frac{150^2}{2\times 900}=12\tfrac{1}{2} \text{ units, or } 12\tfrac{1}{2} \text{ ft. per sec. per sec. Ans.}$$

13. Falling Bodies.—The most important case of uniformly accelerated motion in a straight line is that of bodies falling toward the earth's surface. It has been found by

experiment that *all* bodies falling freely (i. e., in a space free from air and meeting with no resistances) have an acceleration of about 32.16 units. In other words, in 1 second the velocity increases 32.16 feet per second. This acceleration is always denoted by the letter g. The distance fallen through is usually denoted by the letter h. Using g and h in place of the general symbols a and s, the formulas for falling bodies are obtained.

TABLE I

Sought	Given	Formulas	
a, (g)	v, t	$a = \frac{v}{t}$	(1) Art. 10
v	a, t (g, t)	$v = a\,t$; $(v = g\,t)$	(2) Art. 10
	s, a (h, g)	$v = \sqrt{2\,a\,s}$; $(v = \sqrt{2\,g\,h})$	(3) Art. 12
t	v, a (v, g)	$t = \frac{v}{a}$; $\left(t = \frac{v}{g}\right)$	(3) Art. 10
	s, a (h, g)	$t = \sqrt{\frac{2\,s}{a}}$; $\left(t = \sqrt{\frac{2\,h}{g}}\right)$	(1) Art. 12
s (h)	v, t	$s = \frac{v\,t}{2}$	(1) Art. 11
	a, t (g, t)	$s = \frac{1}{2}\,a\,t^2$; $(h = \frac{1}{2}\,g\,t^2)$	(2) Art. 11
	v, a (v, g)	$s = \frac{v^2}{2\,a}$; $\left(h = \frac{v^2}{2\,g}\right)$	(2) Art. 12

By means of the various formulas evolved, any one of the values a, v, t, and s, or g, v, t, and h, respectively, can be found when two of the others are given. For greater convenience the Table I is prepared, in which the parentheses enclose the formulas for falling bodies. These formulas are so important that it would be well to memorize all of them.

EXAMPLE 1.—From what height must a stone be dropped to acquire a velocity of 24,000 feet per minute, neglecting the resistance of the air?

SOLUTION.— 24,000 divided by 60 = 400 ft. per sec.; using formula **2**, Art. **12**, v and g being given and h sought,

$$h = \frac{v^2}{2g} = \frac{400^2}{2 \times 32.16} = \frac{160,000}{64.32} = 2,487.6 \text{ ft. Ans.}$$

EXAMPLE 2.—A body falls from a height of 400 feet; what will be its velocity at the end of its fall?

SOLUTION.—Using formula **3**, Art. **12**, h and g given and v sought,

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 400} = 160.4 \text{ ft. per sec. Ans.}$$

EXAMPLE 3.—How far will a body fall in 10 seconds?

SOLUTION.—Using formula **2**, Art. **11**, t and g given and h sought,

$$h = \tfrac{1}{2} g t^2 = \tfrac{1}{2} \times 32.16 \times 10^2 = 1,608 \text{ ft. Ans.}$$

EXAMPLE 4.—How long will it take a body to fall 4,116.48 feet?

SOLUTION.—Using formula **1**, Art. **12**, h and g given and t sought,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 4116.48}{32.16}} = 16 \text{ sec. Ans.}$$

14. Uniformly Accelerated Motion With Initial Velocity.—If the velocity at the beginning of the motion considered is not zero but equal to v_1, the velocity after the first second is $v_1 + a$; after the second, $v_1 + 2a$, etc.; after t seconds, the velocity is

$$v = v_1 + at \qquad (1)$$

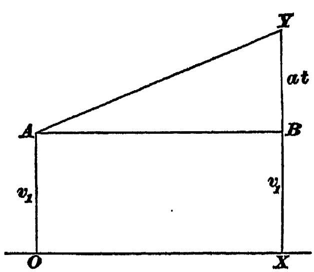


FIG. 6

The area of the figure representing the space traversed is then a trapezoid, as shown in Fig. 6, and thus the space traversed,

$$s = \frac{v + v_1}{2} \times t \qquad (a)$$

or also considering the trapezoid composed of the rectangle $OABX$ and the triangle AYB,

$$s = v_1 t + \tfrac{1}{2} a t^2 \qquad (2)$$

From formula **1**, by transposition,

$$v - v_1 = at \qquad (3)$$

and from equation (a)

$$v + v_1 = \frac{2s}{t} \qquad (4)$$

Multiplying formulas **3** and **4** together, member by mei

$$(v - v_1) \times (v + v_1) = a\,t \times \frac{2\,s}{t}$$

$$v^2 - v_1^2 = 2\,a\,s$$

$$v^2 = v_1^2 + 2\,a\,s$$

$$v = \sqrt{v_1^2 + 2\,a\,s} \qquad (5)$$

For $v_1 = 0$, these formulas, as will be readily perceived, become the original formulas of Arts. **10** to **12.**

EXAMPLE.—A motor car traveling at the speed of 12 miles an hour reaches a slope; the power is then shut off and the car allowed to run down hill with an acceleration of 2 feet per second per second. What will be the velocity after it has run 200 feet farther?

SOLUTION.—A velocity of 12 mi. per hr. is $\frac{12 \times 5{,}280}{60 \times 60} = 17.6$ ft. per sec. Substituting, in formula **5**, $v = \sqrt{17.6^2 + 2 \times 2 \times 200} = 33.313$ ft. per sec.; $\frac{33.313 \times 60 \times 60}{5{,}280} = 22.713$ mi. per hr. Ans.

15. Uniformly Retarded Motion, or Negative Acceleration.—It is the custom to use the word acceleration in case of a change of velocity whether that change is an increase or a decrease. When the motion of a body is retarded and its velocity is decreasing, the acceleration is said to be **negative.** Negative acceleration, therefore, means the same thing as **retardation.**

The same formulas already derived for positive acceleration serve for motion with negative acceleration, its value being inserted with the negative sign. There can, of course, be no retarded motion without an initial velocity. Thus, for formulas **1**, **2**, and **5**, Art. **14**,

$$v = v_1 - a\,t \qquad (1)$$

$$s = v_1\,t - \tfrac{1}{2}\,a\,t^2 \qquad (2)$$

$$v = \sqrt{v_1^2 - 2\,a\,s} \qquad (3)$$

For $v = 0$, that is, if the condition obtains that the body comes to rest, the above formulas correspond with formula **1**, Art. **10**, formula **2**, Art. **11**, and formula **3**, Art. **12**, respectively. Thus, formula **1** then becomes $0 = v_1 - a\,t$; or

$$v_1 = a\,t \qquad (4)$$

Substituting this value of v_1 in formula **2**,

$$s = a\,t \times t - \tfrac{1}{2}\,a\,t^2 = \tfrac{1}{2}\,a\,t^2 \qquad \textbf{(5)}$$

Making $v = 0$ in formula **3**,

$$0 = \sqrt{v_1^2 - 2\,a\,s}$$

or $$v_1^2 = 2\,a\,s$$

and $$v_1 = \sqrt{2\,a\,s} \qquad \textbf{(6)}$$

In these equations, however, a is the value of the negative acceleration and v_1 the initial velocity.

EXAMPLE 1.—A ball is thrown vertically upwards with a velocity of 60 feet per second. (*a*) How high will it rise? (*b*) How long before it comes to rest and begins to fall again? (*c*) What velocity will it have when it has risen 30 feet? (*d*) How high will it have risen after 1 second?

SOLUTION.—(*a*) From formula **6**, substituting g for a and h for s,

$$h = \frac{v_1^2}{2\,g} = \frac{60^2}{2 \times 32.16} = 55.97 \text{ ft. Ans.}$$

(*b*) From formula **4**, substituting g for a,

$$t = \frac{v_1}{g} = \frac{60}{32.16} = 1.866 \text{ sec. Ans.}$$

(*c*) Applying formula **3**, substituting g for a and h for s,

$$v = \sqrt{v_1^2 - 2\,g\,h} = \sqrt{60^2 - 2 \times 32.16 \times 30} = 40.871 \text{ ft. per sec. Ans.}$$

(*d*) Applying formula **2**, substituting g for a and h for s,

$$h = 60 \times 1 - \tfrac{1}{2} \times 32.16 \times 1^2 = 43.92 \text{ ft. Ans.}$$

EXAMPLE 2.—A train has a velocity of 45 miles per hour. If the brakes are applied in such a manner that the decrease in speed each second is 3 feet per second: (*a*) in how many seconds will the train stop? (*b*) how far will it travel in stopping?

SOLUTION.—(*a*) The retardation, or negative acceleration, is 3 ft. per sec., that is, $a = 3$, the initial speed is $v_1 = 45$ mi. per hr. $= 66$ ft. per sec. The final speed is to be zero, thus $v = 0$. From formula **4**,

$$t = \frac{v_1}{a} = \frac{66}{3} = 22 \text{ sec. Ans.}$$

(*b*) Applying formula **5**,

$$s = \tfrac{1}{2}\,a\,t^2 = \tfrac{1}{2} \times 3 \times 22^2 = 726 \text{ ft. Ans.}$$

or from formula **6**,

$$s = \frac{v_1^2}{2\,a} = \frac{66^2}{2 \times 3} = 726 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. A point starting from rest has its velocity accelerated at the rate of 10 feet per second each second; make a table showing the velocity and space passed over at the end of each second from 1 to 12 seconds.

2. A train starting from a station attains a speed of 45 miles per hour in 24 seconds with uniform acceleration. (*a*) What is the acceleration? (*b*) What is the velocity at the end of 15 seconds? (*c*) What distance has the train traveled at the end of 15 seconds? (*d*) At the end of 24 seconds?

$$\text{Ans.}\begin{cases}(a)\ 2\tfrac{3}{4}\text{ ft. per sec. per sec.}\\(b)\ 41\tfrac{1}{4}\text{ ft. per sec.}\\(c)\ 309\tfrac{3}{8}\text{ ft.}\\(d)\ 792\text{ ft.}\end{cases}$$

3. In 8 seconds, the speed of a point changes from 90 feet per second to 54 feet per second. (*a*) What is the acceleration? (*b*) Is it positive or negative? Ans. (*a*) $4\frac{1}{2}$ ft. per sec. in each sec.

4. A train running 60 miles per hour is stopped in traveling 968 feet; what is the negative acceleration? Ans. 4 ft. per sec. per sec.

5. A body starts from a state of rest and falls freely; how far will it fall in 9 seconds? Ans. 1,302.48 ft.

6. What initial velocity must a body have in order to carry it upwards 500 feet vertically? Ans. 179.33 ft. per sec.

7. A baseball is thrown vertically upwards to a height of 200 feet; how long a time must elapse before it strikes the ground? Ans. 7.05 sec.

8. What will be the velocity of a freely falling body at the end of 6 seconds? Ans. 192.96 ft. per sec.

9. A bullet falls from a tower 100 feet high; with what velocity will it strike the ground? Ans. 80.2 ft. per sec.

10. A bullet is dropped from a high tower; if it takes $4\frac{1}{4}$ seconds to reach the ground, how high is the tower? Ans. 290.445 ft.

11. A body falling freely has a velocity of 400 feet per second; how long has it been falling? Ans. 12.438 sec.

KINETICS

FORCE, MASS, AND GRAVITATION

FORCE AND MASS

16. According to Newton's first law of motion, a body cannot be put in motion nor can its motion be changed without the application of force. According to his second law, the change in motion (or from rest to motion) is proportioned to the value of the force producing it. Although nothing is stated about the size (weight) of the body, it is evident that a greater force will be required to move a large (heavy) body a certain distance in a given time than will be necessary to move a small (light) body an equal distance in the same time. For example, a man can throw a baseball, say, 80 feet with very little effort, but try as he may he cannot throw a 16-pound iron ball that distance. Other examples, similar to this, will at once suggest themselves to the reader. It is therefore obvious that there is some relation between the value of the force and the amount of matter contained in the body.

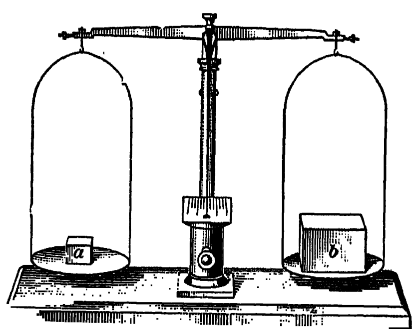


FIG. 7

17. In place of the cumbersome expression—the amount of matter contained in a body—the word **mass** is used. Two bodies have equal masses when they exactly balance each other on being placed in the scale pans of a beam scale, as

shown in Fig. 7. The sizes of the bodies are immaterial; if one balances the other when the beam scale is placed in an air-tight vessel from which all air has been removed (a vacuum), then the mass of one is equal to the mass of the other. For example, the body *a* may be of iron and the body *b* of cork, in which case *b* will be very much larger than *a*; but if they balance in a vacuum, their masses are equal.

18. If two bodies have equal masses, they also have equal weights, as will be explained more fully later; if one body has two, three, ten, or fifty times the mass of another, its weight is two, three, ten, or fifty times that of the other. Consider two bodies *a* and *b* and suppose the mass of *b* to be ten times that of *a*. If a certain force f_1 act on *a* and produce a certain effect in altering the velocity of *a*, it will take just ten times as great a force to produce the same effect on *b*. Referring to Fig. 7, if the mass of *b* is ten times that of *a*, the pan containing *a* will rise with a certain acceleration; in order to make *b* rise with the same acceleration, it will be necessary to replace *a* with a body having ten times the mass of *b*. In other words, *forces, in order to produce the same acceleration, must be directly proportional to the masses of the bodies on which they act.*

GRAVITATION

19. Gravitation is the force with which masses attract one another. All bodies attract one another with a force, the magnitude of which depends on their distance apart and on their masses. In the case of ordinary bodies, this force is too small to be measured; but in the case of an immense mass like the earth, the result is very noticeable, and produces the effect called **weight.** The weight of a body, then, is the attraction exerted on it by the earth. Of course the body also attracts the earth, but the ratio of the mass of the body to the mass of the earth is so small that the effect of this latter attraction is negligible. When a body falls to the earth, the earth rises to meet it; but this movement of the earth is so exceedingly small that it is always neglected.

Gravitation is a universal force and is a term applied to the attraction between masses wherever they may be situated in the universe; when, however, the attraction between the earth and bodies on or near its surface is referred to, the word **gravity** is used instead of gravitation.

20. Universal Law of Gravitation.—The **law of gravitation** was discovered by Sir Isaac Newton and is as follows: *Every particle attracts every other particle in the* **direct** *ratio of its mass, and in the* **inverse** *ratio of the square of its distance.* For example, a particle A attracts a particle B with a certain force; if the mass of A be doubled, the ratio of their masses will be doubled and the force will also be doubled. If, however, the masses remain the same, but the distance between the particles A and B is doubled, the force will be only $\frac{1}{4}$, since $(\frac{1}{2})^2 = \frac{1}{4}$; that is, representing the original force by f_1 and the force after the distance a is increased, by f_2,

$$f_1 : f_2 = (2a)^2 : a^2, \text{ or } f_2 = \frac{a^2 f_1}{(2a)^2} = \frac{1}{4} f_1$$

21. Gravity.—As before stated, gravity is the force with which the earth attracts bodies on or near its surface. The effect on the earth, of the attraction of the earth by bodies, is not considered, for the following reason: The weight of a heavy express train, including the locomotive, is say 500 tons; 2,000 such trains would weigh 1,000,000 tons. The weight of the earth, in millions of tons, is about 6,500,000,000,000,000. The respective masses are in proportion to the weights; hence, if the trains were above the earth, the distance the 2,000 trains would move toward the earth is 6,500,000,000,000,000 times the distance the earth would move toward the trains. If the trains were 1,000,000 miles above the earth and fell, the earth would rise less than $\frac{1}{100000}$ inch.

The mass of the earth remains constant; hence, gravity may be considered as a constant force for any particular locality, but it varies inversely as the square of the distance

of the locality from the center of the earth. The total force gravity exerts on any body is directly proportional to the mass of the body; that is, if a body A has ten times the mass of a body B, the force exerted by gravity on the body A will be ten times that exerted on the body B.

22. The force exerted by gravity is called **weight;** hence, the weight of a body is in direct proportion to its mass. The weight of a body, however, is not the same in all places. There are three reasons for the variation in weight.

1. The general shape of the earth is not that of a sphere. The earth is flattened at the poles; hence, the polar diameter, or diameter measured along the axis, is shorter than the equatorial diameter, the difference being about 26 miles. As a consequence, a body at the poles is 13 miles nearer the center than when situated on the equator; it therefore weighs least at the equator and most at the poles, and between these two extremes its weight depends on the latitude.

2. By reason of the rotation of the earth on its axis, a body on the equator revolves about the earth's axis, the velocity being about 1,000 miles an hour; this creates centrifugal force (to be explained later), which acts in opposition to gravity. This also depends on the latitude, and at the poles is zero. At the equator, it diminishes the weight about $\frac{1}{289}$th.

3. Taking sea level as a basis of measurement, a body situated above sea level weighs less and below sea level weighs more. This follows directly from the law of gravitation, Art. **20.** For example, taking mean radius of the earth as 3,956 miles, the weight of a body on top of a mountain 4 miles high, that weighed 1,000 pounds at sea level would be, $3{,}956^2 : 3{,}960^2 = x : 1{,}000$, or $x = 998$ pounds, nearly. It will be noticed from this calculation that, for small variations from sea level (several hundred feet or so), the change in weight due to this increase in the distance from the center may be neglected.

23. Gravity may be regarded as a constant force equal to the weight of the body, which acts on its center of mass and

tends to push the body against its support; in the case of a freely falling body, gravity constantly pushes it to the earth, with a uniform acceleration equal to the value of g at that place. The value of g varies in the same manner as weight varies; at the equator its value is 32.0902, at the poles 32.2549, for the latitude of London it is 32.2 very nearly, and for the latitude of Scranton, Pennsylvania (about 41° 25′ north), its value is almost exactly 32.16. This latter value will be used in all cases hereafter.

24. Unit of Force.—In English-speaking countries, the unit of force most generally used in engineering and in commerce is the standard pound, which is a piece of platinum kept in the Exchequer office in London. If this be placed on the scale pan of a *spring balance* in the latitude of London it will deflect the spring and move the indicator to a certain point, and is said to exert a force of 1 pound on the spring. Any other body that moves the indicator to the same position will also exert a force of 1 pound on the spring. If the spring balance be located on the equator, it will take a greater mass to move the indicator to the desired point, but the force will still be 1 pound. When it is stated that the weight of a body varies, as in Art. 22, it is always understood that the weight is measured with a standard spring balance.

25. Unit of Mass.—The mass of a body never changes on account of the position of the body. The weight varies and so does g, and they both vary in the same proportion; therefore their ratio $\frac{W}{g}$ must be constant. For suppose that the weight of a body at a place where $g = 32.16$ is 100 pounds; the ratio is $\frac{100}{32.16}$. If, now, the body be removed to some place where it weighs 1 pound more (has increased 1 per cent. in weight), g has also increased 1 per cent., its value then being 32.4816, and the ratio $\frac{W}{g} = \frac{101}{32.4816} = \frac{100\ (1 + .01)}{32.16\ (1 + .01)} = \frac{100}{32.16}$, the same as before. Since this ratio is always the same,

and the mass of the body having the weight W is likewise always the same, this ratio is taken to represent the mass. Representing the mass of a body by M,

$$M = \frac{W}{g} \qquad (1)$$

from which

$$W = Mg \qquad (2)$$

Making $M = 1$, $W = 1 \times g = g$. Hence, the unit of mass is g times the weight of a standard pound, the weighing being done with a beam scale, similar to that shown in Fig. 7, and g having the value corresponding to the latitude of London. For ordinary practical purposes, the weighing may be done with any scale; the value of g may be taken as 32.16 and the mass of a body may be determined by dividing its weight by 32.16.

26. Relation of Force to Acceleration.—A freely falling body is acted on by a constant force equal to the weight of the body, which produces a constant acceleration (see Art. 23) having the value of g at that place. Assuming that there are no resistances, all bodies will fall the same distance in the same time, as can be shown by direct experiment. The only reason that a feather does not drop to the ground in the same time as a bullet is because of the greater resistance of the air; but if both were placed in a long tube from which all air had been exhausted, on inverting the tube both feather and bullet would strike the bottom at the same instant.

Since the weight of a body and g vary in the same proportion, it follows that accelerations are directly proportional to the forces producing them. For instance, a body falling freely and weighing 1 pound is acted on by a constant force of 1 pound and the resulting acceleration is 32.16, or g, feet per second per second. Had the force been 2 pounds, acting on the same body, the acceleration would have been $2 \times 32.16 = 64.32$, or $2g$, feet per second per second, etc. Hence, no matter whether a body falls freely or whether it moves on a frictionless horizontal plane, the acceleration is proportional to the constant force acting.

Let W = weight of a body in pounds;
M = its mass;
F = a constant force acting on it;
a = acceleration produced by F;

then, $$F : W = a : g$$

from which, $$F = \frac{W a}{g} = M a$$

since $\frac{W}{g} = M$. This is a fundamental formula and is one of the most important in kinetics.

The following examples are given to show some applications of this important formula:

EXAMPLE 1.—What constant force is required to produce an acceleration of 5 units (feet per second in a second) in a body weighing 80 pounds?

SOLUTION.— $F = M a = \frac{W}{g} a = \frac{80}{32.16} \times 5 = 12.44$ lb. Ans.

EXAMPLE 2.—A train weighing 400 tons attains a speed of 30 miles per hour in 2 minutes after starting; assuming the acceleration to have been uniform, what must have been the constant pull exerted by the locomotive in producing the acceleration?

SOLUTION.— 30 mi. per hr. $= \frac{30 \times 5{,}280}{60 \times 60} = 44$ ft. per sec., and 2 min. $= 2 \times 60 = 120$ sec. By formula **1**, Art. **10**, the acceleration is $a = \frac{v}{t} = \frac{44}{120}$. The mass of the train is $M = \frac{W}{g} = \frac{400 \times 2{,}000}{32.16}$. Using these values of M and a,

$$F = M a = \frac{400 \times 2{,}000}{32.16} \times \frac{44}{120} = 9{,}121.1 \text{ lb.} = 4.5605 \text{ T.} \quad \text{Ans.}$$

EXAMPLE 3.—A constant force of 60 pounds is applied to a body weighing 800 pounds. (a) What acceleration would be produced if there were no resistances to the motion? (b) How far would the body move in 4 seconds?

SOLUTION.—(a) From the formula, $F = M a$, $a = \frac{F}{M}$. In this case, $F = 60$ lb. and $M = \frac{800}{32.16}$; hence, $a = 60$ divided by $\frac{800}{32.16} = 2.412$ units. Ans.

(b) By formula **2**, Art. **11**, $s = \frac{1}{2} a t^2 = \frac{1}{2} \times 2.412 \times 4^2 = 19.296$ ft. Ans.

EXAMPLE 4.—A locomotive in starting a train exerts a net pull of 6 tons and gets the train up to a speed of 45 miles per hour in

80 seconds; assuming the acceleration to be uniform and neglecting all resistances, what is the weight of the train?

SOLUTION.—The force F is 6 T. = 12,000 lb. 45 mi. per hr. = 66 ft. per sec. and the acceleration is $a = \frac{v}{t} = \frac{66}{80}$. From $F = Ma$; $M = \frac{F}{a} = 12{,}000 \div \frac{66}{80} = 14{,}545$. The weight is therefore $W = Mg = 14{,}545 \times 32.16 = 467{,}767$ lb. = 234 T., nearly. Ans.

CENTRIFUGAL FORCE

27. Suppose that a body is forced to move in a circle, being held perhaps by a string to a central point or guided by a circular ring. If it were not for this constraint, the motion starting from any point A, Fig. 8, would be in the straight line AX; the string or guide deflects the motion. Such a deflection can, however, be due only to a force, which is the tension in the string or the pressure of the ring on the body. This force is called the **centripetal force** and acts *toward* the center of motion. Like any other force it produces an acceleration a.

Since there is always action and reaction, there is a force equal and opposite to the centripetal force, which is called the **centrifugal force,** which acts *away* from the center of motion.

There is a common impression that centrifugal force is a force pulling the body away from the center about which it moves, and if it were not for the cord or other restraining agent the body would fly out radially from the center; this notion is wholly incorrect. The body, if left to itself, would, in obedience to Newton's first law, move in a straight line, and it is the deflection or centripetal force that pulls it out of the straight line. Let the restraint be removed, that is, cut the string in Fig. 8, and the body will move in a tangent to the circle, the line AX in the figure. So far as the body is concerned, there is no centrifugal force that might cause it

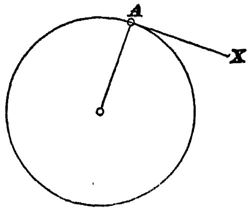


FIG. 8

to move radially outwards. The centrifugal force acts on the restraining agent, and not on the body.

28. Assuming the velocity of the body around the circle to be uniform, the acceleration toward the center is also uniform and is just sufficient to keep the body moving in a circle. Representing the velocity of the body along its circular path by v and the radius of the circle by r, it can be proved by the principles of limits or by the calculus that the acceleration a of the body toward the center is equal to $\frac{v^2}{r}$, or

$$a = \frac{v^2}{r}$$

Since force = mass × acceleration, let F_c equal the centrifugal (= centripetal) force, m the mass, and W the weight of the revolving body; then,

$$F_c = m\frac{v^2}{r} = \frac{Wv^2}{gr} \qquad (1)$$

This formula may be expressed more conveniently as follows:

Let F_c = centrifugal force, in pounds;
W = total weight of body, in pounds;
r = radius, usually taken as distance between center of motion and center of gravity of body, in feet;
N = number of revolutions of body per minute.

In formula **1**, v is in feet per second; since the circumference of a circle having a radius r is $2\pi r$ the distance traveled by the body in 1 minute is $2\pi r N$, and in 1 second $\frac{2\pi r N}{60}$, which is the velocity v. Substituting this value of v, and 32.16 for g in formula **1**,

$$F_c = \frac{\left(\frac{2r\pi N}{60}\right)^2}{r} \times \frac{W}{32.16}$$

or

$$F_c = .00034\, W r N^2 \qquad (2)$$

EXAMPLE.—A segment of a flywheel, Fig. 9, weighs 1,200 pounds and its center of gravity G is at a distance of $5\frac{1}{4}$ feet from the shaft

center O; the wheel makes 120 revolutions per minute. Calculate the pull on the arm exerted by the segment.

SOLUTION.—By formula 2,

$$F_c = .00034 \times 1{,}200 \times 5.25 \times 120^2$$
$$= 30{,}844.8 \text{ lb. Ans.}$$

NOTE.—In this example, the arm is the restraining agent and the pull exerted by the segment on the arm is the centrifugal force. The equal and opposite pull of the arm on the segment is the centripetal force.

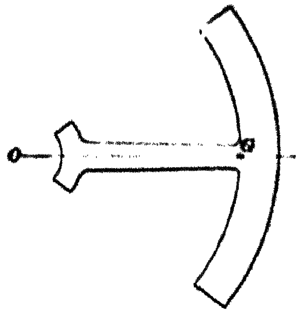


FIG. 9

29. In calculating the bursting effect due to centrifugal force acting on a flywheel, it is customary to neglect the effect of the arms and to consider the weight of the rim as concentrated in a circle passing through the center of gravity of the cross-section of the rim. The centrifugal force tends to separate one-half of the rim from the other half, and the magnitude of this force is equal to the centrifugal force acting on one-half of the wheel. According to *Elementary Mechanics*, Part 2, the radius of the center of gravity of a semicircular arc is $\frac{2}{\pi} r$, in which r is the distance of the center of the cross-section from the center of rotation. Let F_c' represent the bursting effect due to centrifugal force, r' the distance from the center of the shaft to the center of the rim cross-section in feet, and W' the weight of one-half of the rim. Then, by substituting in formula 2, Art. **28**, the value $\frac{2}{\pi} r'$ for r, $F_c' = .00034\, W' \frac{2}{\pi} r' N^2 = .0002165\, W' r' N^2$. This force is resisted by the metal at the two ends of the half rim, and the force resisted at either end or at any section of the rim is one-half as great, or $.0001082\, W' r' N^2$.

In practical flywheel calculations the radius r' is generally taken as the distance from the center of the shaft to a point midway between the inside and outside of the rim.

EXAMPLE. **What is the effect of the centrifugal force at any section of the rim of a cast-iron flywheel whose outside diameter is**

10 feet, width of face 20 inches, and thickness of rim 6 inches, turning at the rate of 80 revolutions per minute?

SOLUTION.—First calculate the weight of one-half the rim. The diameter of the rim is $10 \times 12 = 120$ in.; the diameter of the circle midway between the inside and outside diameters of the rim is $120 - 6 = 114$ in. The number of cubic inches in the whole rim is $114 \times 3.1416 \times 20 \times 6$. Since 1 cu. in. of cast iron weighs .261 lb., the weight of one-half the rim is $W' = 114 \times 3.1416 \times 20 \times 6 \times .261 \times \frac{1}{2}$. The value of r' is $10 \div 2 - \frac{1}{4} = 4\frac{3}{4}$ ft. Substituting in the formula,

$$F_c' = .0001082 \times 114 \times 3.1416 \times 20 \times 6 \times .261 \times \tfrac{1}{2} \times 4.75 \times 80^2$$
$$= 18{,}448 \text{ lb., nearly. Ans.}$$

PROJECTILES

30. Any body thrown into the air is a **projectile**, and is acted on by three forces—the original or initial force, the force of gravity, and the resistance of the air. Only those projectiles that are thrown horizontally will be considered here.

The **range** is the horizontal distance between the starting point and the point where the body strikes the ground. In Fig. 10, suppose that A represents the starting point of the projectile, and that it is shot horizontally outwards in the direction of the arrow with a velocity of 70 feet per second. If the resistance of the air be neglected, the velocity in the horizontal direction will be uniform and the projectile will pass over equal spaces in equal times. Let $A\,1$ represent 70 feet, or the space passed over in 1 second. At the end of 5 seconds, if gravity had not acted on the projectile, it would have been at 5; but as gravity has acted, it *falls* 16.08 feet the first second; at the end of the second second it has fallen 64.32 feet, etc.

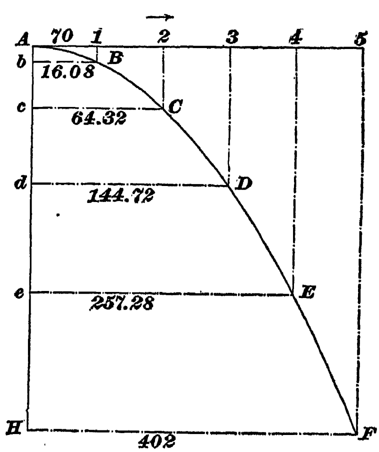


FIG. 10

Let Ab represent the fall in 1 second—that is, 16.08 feet, drawn to the same scale as $A\,1$, which represents 70 feet. Now, complete the parallelogram $A\,1\,B\,b$, and B will be the point that the projectile has reached at the end of 1 second. If Ac represents 64.32 feet, and the parallelogram $A\,2\,C\,c$ is completed, the projectile will be at C at the end of the second second. Proceeding in this manner, find the points D, E, and F, the positions of the projectile at the end of 3, 4, and 5 seconds, respectively. Drawing the curve $ABCDEF$ through the points thus found, it represents the path of the projectile; this curve is called a **parabola.**

The distance HF is the *range*, and, as is easily seen, *equals the time*, *in seconds*, *multiplied by the original velocity*, *in feet per second.*

If the height AH and the initial velocity are given, and it is desired to find the range HF, *calculate the time that it will take to fall through a height equal to the given height*, *and multiply the time thus found by the initial velocity.*

EXAMPLE 1.—A cannon ball is fired in a horizontal direction with an initial velocity of 1,500 feet per second; if the mouth of the cannon is 25 feet above the ground, what is its range?

SOLUTION.—Applying formula **1**, Art. **12**,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 25}{32.16}} = 1.247 \text{ sec., nearly.}$$

$$\text{Range} = vt = 1{,}500 \times 1.247 = 1{,}870.5 \text{ ft. Ans.}$$

EXAMPLE 2.—A projectile has an initial velocity of 90 feet per second; if it is desired to strike an object 15 feet away, how far below the horizontal line of direction must the object be located?

SOLUTION.—The object must be located as far below as the distance that the body would fall, through the action of gravity, during the time it would take in passing over a distance of 15 feet at a velocity of 90 feet per second. Hence, $15 \div 90 = \frac{1}{6}$ sec. Applying formula **2**, Art. **11**,

$$h = \tfrac{1}{2}gt^2 = \tfrac{1}{2} \times 32.16 \times (\tfrac{1}{6})^2 = .447 \text{ ft., nearly, } = 5.36 \text{ in. Ans.}$$

EXAMPLES FOR PRACTICE

1. (*a*) What acceleration will a constant force of 50 pounds produce when acting on a body weighing 100 pounds? (*b*) How far would the 100-pound body move in 10 seconds?

Ans. $\begin{cases} (a) & 16.08 \text{ ft. per sec. per sec.} \\ (b) & 804 \text{ ft.} \end{cases}$

2. If the acceleration of a railroad train, starting from rest, is 1 foot per second, how long will it take to get the speed of the train up to 60 miles an hour? Ans. 88 sec.

3. If the train in example 2 weighs 500 tons, what force is required to produce the acceleration, neglecting all resistances?

Ans. 15.547 T.

4. A segment of a flywheel weighs 2,500 pounds and its center of gravity is $7\frac{1}{2}$ feet from the axis of the shaft. Calculate the pull on the arm exerted by the segment, when the wheel makes 90 revolutions per minute. Ans. $51{,}637\frac{1}{2}$ lb.

5. The rim of a flywheel weighs 16,000 pounds and the diameter at the middle of the rim is 18 feet; at how many revolutions per minute will it be running when the effect of the centrifugal force at any section is 77,904 pounds? Ans. 100 rev. per min.

6. A man on top of a building throws a ball horizontally, at a height of 50 feet above the ground, so that it strikes the ground 200 feet from the building; with what velocity does the ball leave the man's hand? Ans. 113.42 ft. per sec.

WORK, ENERGY, AND POWER

WORK

31. Definition of Work.—When a constant force F acts on a body and the point of application of F is displaced a distance s *in the direction of* F, the force is said to do work, and the product Fs is the measure of the work done. If the direction of the displacement is opposite to that of the force, we say that work is done *against the force.*

For example, let a body rest on an inclined plane AB, Fig. 11, let the angle $BAC = a$, and let a force P be applied sufficient to pull the body up the plane. Suppose that the body moved up the plane from the position O to the position O', and let $OO' = s$. The point of application of

P has moved the distance s *in the direction of P;* hence, the work of P is Ps. The point of application O of the weight W has moved to O', *in the direction of W*, that is, vertically upwards the distance $MO' = s \sin a$. The upward displacement MO' is opposite to the direction of W (downwards), hence the work $W \times MO' = Ws \sin a$ is done *against* the force W. Since there has been no displacement in the direction of the reaction N, that is, the distance of O from the plane is the same for all positions, the work of N is zero. The friction F acts down the plane; hence, the displacement s is opposite to F and the work Fs is done against the force F.

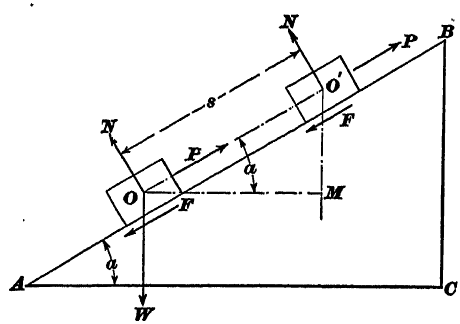


FIG. 11

The work done *by* a force may be considered *positive* (+) that done *against* a force, negative (−); thus, in Fig. 11, Ps is positive while $Ws \sin a$ and Fs are negative.

32. The Principle of Work.—*If the forces acting on a body are in equilibrium, the sum of the works of the separate forces for any displacement of the body is equal to zero.* To prove this, consider the case of the body on the inclined plane, Fig. 11. The four forces are P, N, W, and F and are supposed to be in equilibrium. Resolve the forces into components in the direction of the plane AB and perpendicular to the plane. Since the algebraic sum of the components parallel to AB must be zero for equilibrium,

$$P + 0 - W \sin a - F = 0$$

Multiplying each term by s, we get,

$$Ps - Ws \sin a - Fs = 0$$

But Ps, $-Ws \sin a$, and $-Fs$ are the works of the forces P, W, and F, when the body is displaced a distance s along the plane, and thus the sum of these works is zero.

Another statement of the above principle is as follows: Transposing the last equation,

$$Ps = Ws \sin a + Fs$$

Now P is the acting force or *effort*, as it is sometimes called; while W and F resist the upward motion of the body and are, therefore, *resistances;* hence, *the work of the effort is equal to the sum of the works of the resistances.*

In computing the work in any case, it may be considered as done either *by* the effort or *against* the resistances; thus in Fig. 11 we may find the effort required to move the body up the plane and multiply by the distance moved; or we may, without calculating P, find the works done against the resistances F and W and their sum must be equal to the work of P.

The principle above evolved for the special case of the inclined plane can be shown to hold good in all cases. It has been shown that in every one of the simple machines previously treated "the power multiplied by the distance it moves through is equal to the load multiplied by the distance it moves through." But this statement is nothing else but that "the work of the power (effort) is equal to the work of the resistances (load)."

33. The Unit of Work.—Taking the unit of force as the pound and the unit of distance as the foot, the unit of work is taken as the work done by a force of 1 pound acting through a distance of 1 foot; this unit is called the **foot-pound.**

EXAMPLE 1.—What work is done in lifting a casting weighing 3,500 pounds a height of 8 feet?

SOLUTION.—Work = Fs = 3,500 × 8 = 28,000 ft.-lb. Ans.

EXAMPLE 2.—The average pull of a certain locomotive on the draw-bar is $7\frac{3}{4}$ tons; what work is done in hauling the train 1 mile?

SOLUTION.— $7\frac{3}{4}$ T. = 15,500 lb. 1 mi. = 5,280 ft. Work = Fs = 15,500 × 5,280 = 81,840,000 ft.-lb. Ans.

EXAMPLE 3.—Suppose that the weight of the block in Fig. 11 is 1,200 pounds, that the angle a is 31°, and that the distance s is 500 feet; assuming the coefficient of friction to be .27, what work is required to move the block from O to O'?

SOLUTION.—The total work is the work required to raise the block through the distance $M O'$ plus the work required to overcome the friction.

$M O' = s \sin a = 500 \times \sin 31° = 500 \times .51504 = 257.52$ ft. The work done, neglecting friction, is $1,200 \times 257.52 = 309,024$ ft.-lb.

The perpendicular pressure against the plane is $W \cos a = 1,200 \times \cos 31° = 1,200 \times .85717 = 1,028.604$ lb. The force required to overcome friction is $1,028.604 \times .27 = 277.72+$ lb. The work done in overcoming friction is $277.72 \times 500 = 138,860$ ft.-lb.

The total work is $309,024 + 138,860 = 447,884$ ft.-lb. Ans.

NOTE.—The work in the last example might have been calculated by finding the value of P just sufficient to start the block and multiplying by the distance s through which it acted. Thus, $P = W \sin a + W \cos a \times .27$, and the work done $= W (\sin a + .27 \cos a) \times 500 = 447,884$ ft.-lb.

34. Graphic Representation of Work.—The work done by a force or against a resistance may be represented graphically by an area.

Case I. Let the effort P be constant in magnitude and direction. From some point A, Fig. 12, let $A S$ be laid off to some scale to represent the displacement s, and at right angles to $A S$ let a be laid off to represent to some scale the effort P (or resistance). Thus, in example 2, Art. **33**, if the scale of distances is 1 inch = 1,000 feet, $A S = 5,280 \div 1,000 = 5.28$ inches, and if the scale of force is 1 inch = 5,000 pounds, $A a = 15,500 \div 5,000 = 3.1$ inches. The work is the product of the force and displacement, that is, $A a \times A S$ = area of rectangle $A a s S$; hence, the area of the rectangle represents to some scale the work.

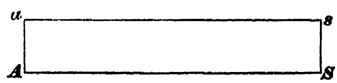


FIG. 12

The scale is obtained by *multiplying together the scale of distances and the scale of forces;* thus in the example just cited 1 inch = 1,000 feet and 1 inch = 5,000 pounds; hence, 1 square inch of the area $= 1,000 \times 5,000 = 5,000,000$ foot-pounds.

Case II. If the effort P (or resistance) varies, the work may still be represented by an area. Take the case of a locomotive pulling a train, the pull is greatest on the up grades and least on the down grades and, in practice, is continually varying. Let $A S$, Fig. 13, represent a distance moved by the train, say 1 mile. Divide $A S$ into a number of equal

parts as AB, BC, etc. and at each point A, B, C, etc. erect perpendiculars Aa, Bb, etc. representing to scale the pull of the locomotive at those points. A curve $abc \ldots s$ drawn through the ends of these lines gives an effort (or resistance), curve, and, as in the previous case, the area $AasS$ represents the work done by the variable pull of the locomotive, to some scale, the scale being determined as in Case I.

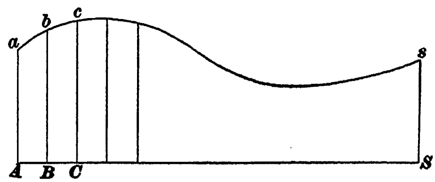


FIG. 13

Another example of work done by a variable effort is shown in the steam engine. The effort is the pressure of the steam on the piston. After the valve cuts off the supply of steam to the cylinder, the pressure falls rapidly. A mechanical device, called the indicator, draws a pressure curve for each stroke of the engine, and by measuring the areas under these curves, the work per stroke and the power of the engine are obtained.

35. Work of Stretching or Compressing a Spring. An important example of work done against a variable resistance is shown in the stretching or compressing of a helical spring, as in a spring scale. It is well known that the resistance to extension or compression is directly proportional to the amount of the extension or compression. Thus, if a force of 5 pounds stretches a spring 1 inch, 10 pounds will stretch it 2 inches, 20 pounds, 4 inches, and so on.

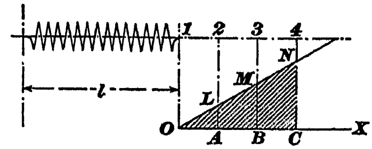


FIG. 14

Suppose that the original length of the unstretched spring is l, Fig. 14, and that keeping one end fixed the other is extended from the initial position *1*, to the successive positions *2*, *3*, *4*, etc. Take a point O below *1* and draw a horizontal line OX and let A, B, C, etc. be the projections of *2*, *3*, and *4* on OX. Let, now, AL be laid off on $A2$ to represent the resistance when

the spring is extended to *2*, and let a line be drawn through O and L; then the intercepts BM and CN represent to the same scale the resistances of the spring for the position *3* and *4*, respectively. For, from similar triangles, the intercepts AL, BM, and CN are proportional to the extensions OA, OB, OC, and these extensions are, in turn, proportional to the resistances for the three points. The straight line ON is, therefore, the resistance curve for the extension of the spring.

Let R_2, R_3, and R_4 denote the resistance for the positions *2*, *3*, and *4*; then $AL = R_2$, $BM = R_3$, and $CN = R_4$. During the extension from *1* to *2*, the average resistance is $\frac{0 + R_2}{2} = \frac{R_2}{2}$ and the displacement is *1–2* $= OA$. The work is, therefore, average resistance $\times$ distance $= \frac{1}{2}R_2 \times OA = \frac{1}{2}AL \times OA =$ area of triangle OAL. Similarly, the work of extending the spring from *1* to *3* is given by the area of the triangle OBM, and from *1* to *4* by the area of the triangle OCN. To extend the spring from *2* to *3* requires the work represented by the area of the trapezoid $ALMB$. For the average resistance is $\frac{R_2 + R_3}{2} = \frac{AL + MB}{2}$, and the displacement is AB; hence,

$$\text{work} = \frac{R_2 + R_3}{2} \times AB = \frac{AL + MB}{2} \times AB$$
$$= \text{area } ALMB$$

EXAMPLE.—A spring is extended 1 inch by a force of 30 pounds; what work is done in extending it an additional length of 2 inches?

SOLUTION.—An additional stretching of 2 in. means a total extension of 3 in. The resistance for 1 in. being 30 lb. that for 3 in. is $30 \times 3 = 90$ lb.; hence,

$$\text{work} = \text{mean resistance} \times \text{extension} = \frac{30 + 90}{2} \times 2$$
$$= 120 \text{ in.-lb.} = 120 \div 12 = 10 \text{ ft.-lb.} \quad \text{Ans.}$$

36. Work Done by Fluid Pressure or Against Fluid Resistance.—In many cases, the effort consists of the pressure of a fluid as in the case of the steam engine; or the resistance overcome may be a fluid pressure, as in the

case of pumping water or compressing air. By *fluid* is meant a liquid, gas, or vapor.

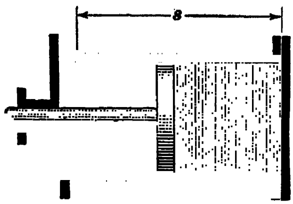


FIG. 15

In Fig. 15 is shown a cylinder containing a piston to the right of which is a fluid exerting a pressure on the piston. If the piston moves to the right, the fluid pressure is the resistance; while if it moves to the left the pressure is the effort. In either case,

Let P = pressure per square foot, in pounds;
p = pressure per square inch, in pounds;
A = area of piston, in square feet;
s = stroke of piston, in feet;
V = volume swept through by piston, in cubic feet;
W = work done per stroke, in foot-pounds.

The effort (or resistance) is PA and the distance is s; hence,

$$W = PAs$$

But $V = As$, since the volume swept through by the piston is that of a cylinder whose base has the area A and whose altitude is s. Substituting,

$$W = PV = 144\,p\,V^*$$

that is, *the work done is the product of the fluid pressure per unit area and of the volume of fluid displaced.* This formula has a wide application and should be thoroughly understood.

EXAMPLE.—In the water cylinder of a pump 4.2 cubic feet of water is displaced per stroke, and the pressure exerted by the water is 65 pounds per square inch. The pump makes 80 strokes per minute. Find the work done per minute.

SOLUTION.—Work per stroke $= 144\,p\,V = 144 \times 65 \times 4.2 = 39{,}312$ ft.-lb.

Work per minute $= 39{,}312 \times 80 = 3{,}144{,}960$ ft.-lb. per min. Ans.

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*Since p is the pressure, in pounds per square inch, and the area is in square feet, the pressure per square foot must be introduced, which is $144\,p$, since a square foot has 144 square inches.

POWER

37. Work, as defined in Art. **31,** has no reference to time. The work performed in raising a given weight through a given height is the same whether the time occupied is a second or an hour. However, when the capacities of different agents for doing work are to be compared, time must be considered; and to indicate the amount of work performed in a given time, or the rate of doing work, the term **power*** is used. If an engine does in an hour twice as much work as another engine, it is said to have twice the power of the second engine, or to be twice as powerful.

The unit of power is the power of an agent that does a unit of work per unit of time. The time unit is usually the minute; hence, power is generally expressed in foot-pounds per minute.

EXAMPLE.—In the second example of Art. **33,** what is the power of the locomotive if the train is moving 25 miles per hour?

SOLUTION.—To move a mile requires $\frac{60}{25} = 2.4$ min.; hence, the work per minute, or power, is $81{,}840{,}000 \div 2.4 = 34{,}100{,}000$ ft.-lb. per min. Ans.

38. Horsepower.—The foot-pound per minute is too small a unit for many purposes, and a larger unit is more generally used by engineers. This unit is called the **horsepower** and is equal to *33,000 foot-pounds per minute or 550 foot-pounds per second.* The abbreviation for horsepower is H. P.

Let P = effort (or resistance), in pounds;
s = distance, in feet, that point of application is moved;
t = time occupied, in minutes;
t_s = time occupied, in seconds;
H = horsepower;

*The term *power* as here used has a different meaning from that previously given to it, which simply meant force acting to produce motion in simple machines. The two should be carefully distinguished. There will be no difficulty in doing this, as the wording of the sentence in which it occurs will always show whether force or work per minute is meant.

then, $$H = \frac{Ps}{33{,}000\ t} = \frac{Ps}{550\ t_s}$$

EXAMPLE 1.—What horsepower is required to pull a train weighing 400 tons at a speed of a mile per minute if the resistance at this speed is 12 pounds per ton?

SOLUTION.—The force P to overcome the resistance is $400 \times 12 = 4{,}800$ lb. $s = 5{,}280$ ft. Substituting in the formula,

$$H = \frac{4{,}800 \times 5{,}280}{33{,}000 \times 1} = 768 \text{ H. P.} \quad \text{Ans.}$$

EXAMPLE 2.—A crane hoists a load of 5 tons a height of 22 feet in 20 seconds; what horsepower is developed?

SOLUTION.—The resistance P is 5 T. = 10,000 lb. 20 sec. = $\frac{1}{3}$ min. Substituting in formula,

$$H = \frac{10{,}000 \times 22}{33{,}000 \times \frac{1}{3}} = 20 \text{ H. P.} \quad \text{Ans.}$$

Or, $$H = \frac{10{,}000 \times 22}{550 \times 20} = 20 \text{ H. P.} \quad \text{Ans.}$$

39. Power Absorbed by Friction.—In connection with journal friction, *Elementary Mechanics*, Part 3, it was shown that the friction acting tangent to the circumference of a journal is $f' W$. If d = diameter of journal, in inches, and n = revolutions of shaft per minute, a point on the circumference moves per minute a distance $\frac{\pi d n}{12}$ feet. The work per minute done against friction is, therefore, $f' W \times \frac{\pi d n}{12}$ foot-pounds and the horsepower absorbed is

$$H = \frac{f' W \times \frac{\pi d n}{12}}{33{,}000} = \frac{\pi f' W d n}{396{,}000}$$

EXAMPLE.—A journal 5 inches in diameter is loaded with 32,000 pounds and runs at 80 revolutions per minute; if $f' = .05$, what is the horsepower absorbed in friction?

SOLUTION.— $H = \frac{\pi f' W d n}{396{,}000} = \frac{3.1416 \times .05 \times 32{,}000 \times 5 \times 80}{396{,}000}$ = 5.077 H. P. Ans.

ENERGY

40. Kinetic Energy.—If the forces acting on a body are not in equilibrium, they will, in general, have a single resultant, and this unbalanced force acting alone on the body

will produce a change of velocity, that is, an acceleration. For example, take the case of a train starting from a station. The locomotive at the start exerts a pull P on the drawbar that exceeds the frictional resistance R of the train; hence, there is an unbalanced force $P - R$ urging the train forwards and accelerating its motion. As the speed increases, the resistance R also increases and finally becomes equal to the effort P. Now the forces are in equilibrium, the acceleration ceases, and according to the principle of work, the work of the effort is just equal to that of the resistance. So long as $R = P$ the speed remains constant. To bring the train to rest, brakes are applied, thus suddenly increasing R, while P is made zero by shutting off steam. The force R acting opposite to the train's motion produces a negative acceleration and the velocity decreases to zero.

Let P denote the effort and R the resistance, the unbalanced force is $P - R = F$. This force acts on a body of mass M and produces an acceleration denoted by a. Suppose that the body starts from rest and in moving a distance s attains a velocity of v feet per second.

From Art. **26**, $F = Ma$, whence $a = \frac{F}{M}$; and from formula **3**, Art. **12**, $v = \sqrt{2as}$; hence, $a = \frac{v^2}{2s}$. Placing the two values of a equal,

$$\frac{F}{M} = \frac{v^2}{2s}$$

or

$$Fs = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

since

$$M = \frac{W}{g}$$

The product Fs is the work done by the force F moving the body through the distance s; hence, to start a body of weight W from rest and give it a velocity v, $\frac{Wv^2}{2g}$ foot-pounds of work must be done on it. Conversely, if the body has the velocity v, it will, in coming to rest, give up $\frac{Wv^2}{2g}$ foot-pounds of work.

It appears, therefore, that the quantity $\frac{W v^2}{2 g}$ is the capacity the body has for doing work due merely to its velocity. The capacity of an agent for doing work is called the **energy** of the agent, and the energy that a body has in consequence of its velocity is called **kinetic energy**; hence, the kinetic energy E of a body weighing W pounds and moving with a velocity v is expressed by

$$E = \frac{W v^2}{2 g}$$

Since energy is capacity for doing work, it is measured in work units, that is, foot-pounds.

EXAMPLE 1.—What is the kinetic energy of a train weighing 200 tons and running at a speed of 30 miles per hour?

SOLUTION.— W = 200 T. = 400,000 lb., v = 30 mi. per hr. = 44 ft. per sec.

Kinetic energy $= \frac{W v^2}{2 g} = \frac{400{,}000 \times 44^2}{2 \times 32.16} = 12{,}039{,}800$ ft.-lb. Ans.

EXAMPLE 2.—If, when steam is shut off and the brake applied, the resistance is 15,000 pounds, in what distance will the train come to rest?

SOLUTION.—The kinetic energy of the train is all used up in doing work against the resistance of 15,000 pounds; if s denote the distance, the work is 15,000 s; hence, 15,000 s = 12,039,800, or s = 803 ft., nearly. Ans.

41. Equation of Energy.—Suppose that a body whose weight is W pounds is moving at a constant velocity of v_1 feet per second; its kinetic energy is $\frac{W v_1^2}{2g}$ foot-pounds. An unbalanced force F is applied to the body, and after a distance s is traversed, the velocity is increased to v_2, and the kinetic energy has increased therefore to $\frac{W v_2^2}{2g}$. The work Fs has thus been expended in increasing the kinetic energy, and the work must just be equal to that increase, or

$$Fs = \frac{W v_2^2}{2g} - \frac{W v_1^2}{2g} = \frac{W}{2g}(v_2^2 - v_1^2)$$

But, as in Art. 40, $F = P - R$, that is, the unbalanced force is the difference between the effort and the resistance. Substituting $P - R$ for F,

$$(P - R)s = \frac{W}{2g}(v_2^2 - v_1^2)$$

or
$$Ps = Rs + \frac{W}{2g}(v_2^2 - v_1^2)$$

This formula is called the *equation of energy* and may be stated thus:

The work done by the effort is equal to the work done against the resistance plus the increase of kinetic energy.

EXAMPLE.—In hoisting coal from a mine, the load to be hoisted including cage and car is 12,000 pounds. The load starts from rest and when it is 50 feet from the bottom it is moving with a velocity of 30 feet per second; what is the pull in the hoisting rope?

SOLUTION.—The resistance R is the weight, 12,000 lb. The effort P is the unknown pull in the rope. $v_1 = 0$, $v_2 = 30$ ft. per sec., and $s = 50$ ft. Substituting in the formula,

$$Ps = Rs + \frac{W}{2g}(v_2^2 - v_1^2) = 12{,}000 \times 50 + \frac{12{,}000}{2 \times 32.16}(30^2 - 0^2)$$
$$= 767{,}910 \text{ ft.-lb.}$$

$$P = \frac{767{,}910}{50} = 15{,}358.2 \text{ lb. Ans.}$$

NOTE.—In the solution of this example, it is assumed that when the load has reached a height of 50 feet it is still being accelerated at the average rate of acceleration necessary to give it a velocity of 30 feet per second in a space of 50 feet. When the velocity becomes uniform, the pull in the rope is that due to the load only, since there is then no increase in the kinetic energy.

42. Potential Energy.—As we have seen, a body may have a capacity for doing work by reason of its velocity. There are, however, other states or conditions than that of motion which give a body a capacity for work.

(*a*) A body is raised through a height h, and if W is the weight of the body, the work Wh is done against the force W. If now the body is permitted to descend through the same vertical distance h, the weight W becomes the acting force and the work Wh is done by the force W. The body in its highest position has the work Wh stored in it and thus possesses a stock of energy Wh because of its position.

(*b*) A spring is extended or compressed, as in Art **35**, and work is done against the resistance of the spring to change of length. If the spring is released, it will return to its original form, and in so doing can do precisely the amount of work that was expended on it. Thus, in the extended or

compressed state, there is an amount of work stored in the spring and it possesses energy because of its stretched or compressed condition. Similarly, compressed air possesses energy merely because it is compressed and can do work in returning to its original state.

The energy that a body possesses by reason of its *position*, *state*, or *condition* is called **potential energy.**

43. Conservation of Energy.—The law of the conservation of energy asserts that energy cannot be destroyed. When energy apparently disappears, it is found that an equal amount appears somewhere, though perhaps in another form. Frequently, potential energy changes to kinetic energy or vice versa. To illustrate, take the case of a steam hammer or a pile driver. The ram is raised to a height h above the pile and on being released strikes the head of the pile and comes to rest. In its highest position, the ram has the potential energy Wh. In falling, it attains a velocity v that, just at the instant of striking, has the magnitude $v = \sqrt{2gh}$; hence, at this instant, the ram has lost all its potential energy but has a kinetic energy $\frac{Wv^2}{2g} = Wh$. The loss of one kind is therefore just balanced by the gain of the other kind. After the blow, the ram comes to rest and has neither potential nor kinetic energy. Apparently there is a loss of energy, but really the energy has for the most part been expended in doing the work of driving the pile a certain distance into the earth. A small part has been expended in heating the ram and head of the pile, and another part in producing sound.

Heat is a form of energy due to the rapid vibration of the molecules of bodies. In many cases where energy apparently disappears, it reappears in the form of heat; thus the work done against frictional resistances appears as heat, which is usually dissipated into the air as fast as produced.

Conversely, there are examples of heat energy transformed into other forms. A pound of coal has potential energy, which is liberated when the coal is burned and is changed into heat energy. The heat applied to water produces steam

and the steam has potential energy. Finally, the steam in giving up its energy does work in a steam engine, and this work is expended in overcoming the friction of shafts, belts, and machine parts and ultimately reappears in the form of heat. However, in all these transformations, the total amount of energy, the sum of the kinetic and potential energy, remains always the same. This last statement is termed **conservation of energy.**

EFFICIENCY

44. In any machine, there is an effort or driving force that produces motion in the machine and a resistance against which the machine does work. Take, for example, the raising of a load by a crane. The effort is exerted by the workman on the crank of the windlass, and the resistance is the weight of the load lifted. In the case of a machine tool for cutting metal, the effort is the pull of the belt on the driving pulley and the resistance is the resistance of the metal to the cutting tool. In a given time, say 1 minute, a quantity of work is done by the effort and another quantity of work is done against the resistance. Were it not for frictional resistances these two works would be just equal. In actual cases, however, only a part of the work of the effort is *usefully* expended in doing work against the resistance. The remainder is used in overcoming the friction between the various sliding surfaces, journals and bearings, etc. The work thus expended appears in the form of heat energy and serves no *useful* purpose.

Work done by effort = useful work done against resistance + lost work.

The ratio, $\frac{\text{useful work}}{\text{total work of effort}}$, is called the **efficiency** of the machine.

Representing the efficiency by η (Greek letter *eta*), the useful work by W_u, and the total work (= energy supplied) by W_t,

$$\eta = \frac{W_u}{W_t}$$

Consider the last example of Art. **33.** It was found that in pulling the body up the inclined plane, the work of the

effort was 447,884 foot-pounds. Of this amount, 309,024 foot-pounds was the useful work of raising the body through the vertical height of 257.52 feet, while the remaining 138,860 foot-pounds was the work done against friction, and therefore expended to no useful purpose. Hence, the efficiency was

$$\eta = \frac{W_u}{W_t} = \frac{309{,}024}{447{,}884} = .69\text{, nearly, or 69 per cent.}$$

45. The work W_u and the work W_t are products obtained by multiplying two different forces by the same distance, since for the same machine or operation the distance through which the force acts is the same regardless of whether friction is considered or not. Hence, W_u may be replaced by the product of the force F_u and s and W_t by the product of the force F_t and s, and

$$\eta = \frac{W_u}{W_t} = \frac{F_u s}{F_t s} = \frac{F_u}{F_t} \qquad (1)$$

in which F_u is the force theoretically required to do the work and F_t is the force actually required.

Thus, if there were no friction, a machine whose velocity ratio was 5 would, by an application of 100 pounds of force, raise a weight of 500 pounds.

Now, suppose that the friction in the machine is equivalent to 10 pounds of force, then it would take 110 pounds of force to raise 500 pounds.

If, in the above illustration, friction were neglected, 110 pounds $\times$ 5 = 550 pounds, or the weight that 110 pounds would raise; but, owing to the frictional resistance, it only raised 500 pounds; therefore, the ratio between the two is $\frac{500}{550}$ = .91, or 91 per cent.

In the case of simple machines,

Let F = force applied to machine;
V = velocity ratio of machine;
W = weight actually lifted, or equivalent resistance overcome;
η = efficiency of machine.

Then,
$$\eta = \frac{W}{FV} \qquad (2)$$

EXAMPLE 1.—In a machine having a combination of pulleys and gears, the velocity ratio of the whole is 9.75; a force of 250 pounds just lifts a weight of 1,626 pounds; what is the efficiency of the machine?

SOLUTION.— $\eta = \frac{W}{FV} = \frac{1,626}{250 \times 9.75} = .6671$, or 66.71 per cent. Ans.

Since the total amount of friction varies with the load, it follows that the efficiency will also vary for different loads.

EXAMPLE 2.—A pulley block with a velocity ratio of 7 has an efficiency of 38 per cent for a load of 3,000 pounds; what power is required to start the load?

SOLUTION.— $\eta = \frac{W}{FV}$, or $.38 = \frac{3,000}{7F}$. Hence,

$$F = \frac{3,000}{.38 \times 7} = 1,128 \text{ lb., nearly.} \quad \text{Ans.}$$

46. If a machine is made up of several mechanisms, and the efficiency of each of these mechanisms is known, the efficiency of the combination is equal to the product of the efficiencies of all the separate mechanisms. For example, consider a lathe driven by an electric motor; suppose that the efficiency of the motor is 91 per cent. and of the lathe 73 per cent., then the efficiency of the combination is $.91 \times .73 = .6643 = 66.43$ per cent.

MOMENTUM AND IMPACT

47. Fundamental Equation.—The fundamental relation between the acceleration of a body and the force producing it is expressed by formula

$$F = Ma = \frac{W}{g}a$$

If the force is constant, the acceleration a is constant, and according to the formula of Art. **9** the value of a is $\frac{v_2 - v_1}{t}$, where v_2 and v_1 are the final and initial velocities, respectively, and t is the time required to effect the change of velocity. Substituting this value of a in the formula of Art. **9**,

$$F = M\left(\frac{v_2 - v_1}{t}\right)$$

or

$$Ft = Mv_2 - Mv_1$$

This formula is merely the preceding formula in another form, which is more convenient in the solution of some problems. Many problems may be solved by either of these formulas or by the application of the equation of energy, Art. **41**. The example of Art. **41** is of this character.

EXAMPLE 1.—A body whose weight is 40 pounds is drawn along a horizontal plane by a force of 9 pounds, and the coefficient of friction is .10; if the body starts from rest, what will be its velocity at the end of 6 seconds?

SOLUTION.—The force of friction is $.10 \times 40 = 4$ lb.; and as this opposes the pulling force, 9 lb., the net force is $F = 9 - 4 = 5$ lb. The mass is $\frac{W}{g} = \frac{40}{32.16}$; $v_1 = 0$ and v_2 is unknown. Substituting in the preceding formula, $5 \times 6 = \frac{40}{32.16} v_2$, whence,

$$v_2 = \frac{5 \times 6 \times 32.16}{40} = 24.12 \text{ ft. per sec.} \quad \text{Ans.}$$

EXAMPLE 2.—Solve the example of Art. **41** by the two preceding formulas.

SOLUTION.—Assuming the acceleration constant, we have from formula **3**, Art. **12**, $a = \frac{v^2}{2s} = \frac{30^2}{2 \times 50} = 9$ ft. per sec. per sec. and the time is, from formula **3**, Art. **10**, $t = \frac{v}{a} = \frac{30}{9} = 3\frac{1}{3}$ sec.

The force F producing the acceleration is, from the formula of Art. **26**,

$$F = Ma = \frac{W}{g} a = \frac{12{,}000}{32.16} \times 9 = 3{,}358.2 \text{ lb.}$$

or, from the preceding formula,

$$F = \frac{M(v_2 - v_1)}{t} = \frac{12{,}000}{32.16} \times \frac{(30 - 0)}{3\frac{1}{3}} = 3{,}358.2 \text{ lb.}$$

Now the force F producing the upward acceleration is the difference between the tension T in the rope and the downward weight W; that is, $F = T - W$, whence

$$T = F + W = 3{,}358.2 \text{ lb.} + 12{,}000 \text{ lb.} = 15{,}358.2 \text{ lb.} \quad \text{Ans.}$$

48. Momentum.—*The momentum of a moving body is the product of its mass and velocity.*

Denoting by M and v the mass and velocity, respectively,

$$\text{momentum} = Mv$$

In the formula of Art. **47**, Mv_2 is the final momentum, Mv_1 the initial momentum, and the second member of the equation $Mv_2 - Mv_1$ is therefore the change of momentum.

Hence, the change of momentum is the product of the force and time, and the force is rate of change of momentum, or the change of momentum per unit of time.

The product Ft of a force and the time during which it acts is called the **impulse** of the force during that time. The formula of Art. 47 may therefore be thus expressed: *The impulse of a force is equal to the change of momentum.*

49. Impulsive Forces.—In the formula of Art. 47, suppose that the change of momentum, as given by the second member, has some definite value. To obtain this change, a force F must act through such a time t that the product Ft shall be equal to the given change. However, the factors F and t may be varied at pleasure, provided that the product is the same. If t is made large, the necessary force F is small; if t is decreased, F must be increased.

If t is made very small, F must be very large. For example, to produce a change of momentum of 20 units in $\frac{1}{100}$ second requires the force F to be $20 \div \frac{1}{100} = 2{,}000$ pounds.

There are many examples of large forces acting during short intervals of time. Thus, the explosion of powder in a gun, the impact of a hammer on an anvil or the head of a nail, the impact of two billiard balls, are examples of such cases. The time t is the very short interval during which the bodies are in contact, and the force F is the pressure between the two bodies.

Forces of this kind are called **impulsive, or instantaneous, forces.**

It is not usually possible to determine the short time t during which an impulsive force acts, and the magnitude of the force F cannot therefore be determined. However, it is the impulse Ft that is of importance and this is determined by the change of momentum.

50. Impact.—Let two spheres, Fig. 16, have masses M and M', respectively, and let their centers o and o' move in the same straight line and in the same direction, with velocities u and u', respectively. Assuming that u is greater than u',

the spheres will ultimately collide and the velocities u and u' will be changed. Let v and v' denote the velocities after impact; the problem is to determine these velocities.

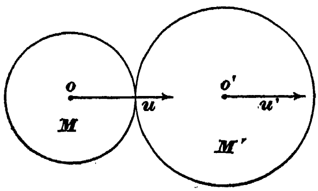

FIG. 16

In connection with impact two general principles have been established:

1. The total momentum after impact is equal to that before impact.

2. The relative velocity after impact bears a constant ratio to the relative velocity before impact, and this ratio depends on the elasticity of the bodies.

These principles give the following equations:

$$Mv + M'v' = Mu + M'u' \qquad (1)$$

$$\frac{v' - v}{u - u'} = e \qquad (2)$$

in which e is some constant quantity not greater than 1. In the first equation, the first member is the momentum after impact and the second member is that before impact. In the second equation, $v' - v$ is the relative velocity after and $u - u'$ the same before impact.

For bodies with little elasticity, as clay, lead, etc., e is very small, and may be made zero; hence, in this case,

$$v' - v = 0; \text{ or } v' = v$$

and from (1), $\quad (M + M')v = Mu + M'u'$

whence $$v = \frac{Mu + M'u'}{M + M'} \qquad (3)$$

or since $M = \dfrac{W}{g}$, $M' = \dfrac{W'}{g}$, the g's canceling out,

$$v = \frac{Wu + W'u'}{W + W'} \qquad (4)$$

For perfectly elastic bodies, $e = 1$; hence, from (2), $\dfrac{v' - v}{u - u'} = 1$, or $v' - v = u - u'$, from which

$$v' + u' = v + u \qquad (5)$$

In the case of non-elastic bodies, therefore, the two bodies impact move with the same velocity, while with perfectly elu bodies the sum of the initial and final velocities of one body is equal to the same sum for the other body.

EXAMPLE.—A ball weighing 8 pounds and moving at a speed of 30 feet per second overtakes a second ball weighing 20 pounds and moving in the same line with a speed of 15 feet per second. (*a*) If the balls are inelastic, what is the common velocity after impact? (*b*) If the balls are perfectly elastic, what is the velocity of each after impact?

SOLUTION.—(*a*) From formula **4**,

$$v = \frac{8 \times 30 + 20 \times 15}{8 + 20} = 19\tfrac{2}{7} \text{ ft. per sec. Ans.}$$

(*b*) Substituting known values in formula **1**, and canceling the factor g,

$$8\,v + 20\,v' = 8 \times 30 + 20 \times 15 = 540 \qquad (1)$$

With $e = 1,\ v' - v = u - u' = 30 - 15 = 15 \qquad (2)$

Solving the simultaneous equations for v and v', we obtain $v' = 15 + v$, which, inserted in (1), gives $8\,v + 20\,(15 + v) = 540$; $8\,v + 300 + 20\,v = 540$; $28\,v = 540 - 300$;

$$v = \tfrac{240}{28} = 8\tfrac{4}{7} \text{ ft. per sec.}$$

and finally from (2),

$$v' = 15 + 8\tfrac{4}{7} = 23\tfrac{4}{7} \text{ ft. per sec.}$$

Ans.

If the balls are moving toward each other, their velocities are opposite. To take account of this case, consider the direction of the greater initial velocity u as positive; then if u' is opposite to u, give it the negative sign.

51. Impact of a Hammer.—Consider the case of a nail driven into a wooden plank by a hammer. The mass of the hammer is M that of the nail M'; the resistance of the wood to penetration is denoted by F and the distance the nail is driven by s. The velocity with which the hammer strikes is u, and the initial velocity of the nail is, of course, 0. Assuming that the bodies are inelastic, formula **3**, Art. **50**, gives

$$v = \frac{M}{M + M'}u$$

as the velocity of nail and hammer after the stroke. The kinetic energy of the nail and hammer is (see Art. **40**)

$$(M + M')\frac{v^2}{2}$$

Substituting the value of $v = \frac{M}{M+M'}u$; this gives

$$\frac{1}{2}(M+M')\frac{M^2u^2}{(M+M')^2} = \frac{M}{M+M'} \times \frac{1}{2}Mu^2$$

Since the original kinetic energy of the hammer just as it struck the nail was $\frac{1}{2}Mu^2$, we see that only the fraction $\frac{M}{M+M'}$ of this energy remains in the hammer and nail together. The remainder is

$$\frac{Mu^2}{2} - \frac{M}{M+M'} \times \frac{Mu^2}{2} = \left(1 - \frac{M}{M+M'}\right)\frac{Mu^2}{2}$$

$$= \left(\frac{M+M'-M}{M+M'}\right)\frac{Mu^2}{2} = \frac{M'}{M+M'} \times \frac{Mu^2}{2} = \frac{1}{\frac{M}{M'}+1} \times \frac{Mu^2}{2}$$

This remainder has been spent in battering or distorting the nail head and in heat, sound, etc. The nail and hammer in coming to rest give up their kinetic energy and this is utilized in doing the work Fs of driving the nail; hence,

$$Fs = \frac{M}{M+M'} \times \frac{Mu^2}{2} = \frac{\frac{M}{M'}}{\frac{M}{M'}+1} \times \frac{Mu^2}{2}$$

It is easily seen from the second form of the term on the right side of this equation that the greater the ratio $\frac{M}{M'}$, that is, the heavier the hammer, the larger is the fraction $\frac{\frac{M}{M'}}{\frac{M}{M'}+1} = \frac{M}{M+M'}$ of the original energy $\frac{Mu^2}{2}$ that is expended in driving, and the smaller is the fraction $\frac{1}{\frac{M}{M'}+1}$ $= \frac{M'}{M+M'}$ that is spent in distorting the nail. In some cases however, as in riveting and forging, the distortion of the body struck is the object of the blow, and the energy $\frac{M'}{M+M'} \times \frac{Mu^2}{2}$ becomes the useful instead of the lost

energy. In such cases, it is evidently advantageous to have M', the mass of the body struck, large in comparison with M, the mass of the striking body (the hammer).

In the expressions $\frac{M}{M+M'}$ and $\frac{M'}{M+M'}$, W and W' may be substituted for M and M', since $M = \frac{W}{g}$ and $M' = \frac{W'}{g}$, the g's canceling out.

EXAMPLE.—A hammer weighs 2 pounds and a nail .04 pound. The velocity of the hammer as it strikes the nail is 30 feet per second and the penetration is $\frac{1}{4}$ inch. Find: (*a*) the resistance offered by the wood; (*b*) the percentage of energy lost.

SOLUTION.—The energy of the hammer just at striking is $\frac{Wu^2}{2g} = \frac{2 \times 30^2}{2 \times 32.16} = 28$ ft.-lb., nearly. Of this $28 \times \frac{2}{2 + .04} = 27.45$ ft.-lb. is expended in driving the nail and the remainder $28 - 27.45 = .55$ ft.-lb. is lost.

(*a*) $s = \frac{1}{4}$ in. $= \frac{1}{48}$ ft.

$Fs = 27.45$, or $F = 27.45 \div \frac{1}{48} = 1{,}317.6$ lb. Ans.

(*b*) The fraction of the energy lost is

$$\frac{W'}{W + W'} = \frac{.04}{2 + .04} = .02, \text{ or 2 per cent., nearly.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. How much work is required to move a block of ice weighing 600 pounds up a slide 200 feet long, inclined at 30° to the horizontal, if the coefficient of friction is .035. Ans. 63,637 ft.-lb.

2. If a force of 1,800 pounds extends a spring 16 inches, how much work is the spring capable of exerting? Ans. 1,200 ft.-lb.

3. The pressure in the water cylinder of a pump is 70 pounds per square inch and 3.6 cubic feet of water is displaced per stroke; if the pump makes 60 strokes per minute, what horsepower is required? Ans. 65.98 H. P.

4. A journal 8 inches in diameter is loaded with 96,000 pounds and runs at 100 revolutions per minute; if the coefficient of journal friction is .05, what is the horsepower absorbed in friction? Ans. 30.464 H. P.

5. What is the kinetic energy of a train weighing 500 tons and running at a speed of 60 miles per hour? Ans. 120,398,000 ft.-lb.

6. A sledge weighs 8 pounds and a spike .1 pound. The velocity of the sledge as it strikes the spike is 20 feet per second and the penetration is $\frac{1}{8}$ inch. Find: (*a*) the resistance offered by the wood; (*b*) the percentage of lost energy.

Ans. $\begin{cases} (a) \ 4{,}717 \text{ lb.} \\ (b) \ 1.235 \text{ per cent., nearly} \end{cases}$

DENSITY AND SPECIFIC GRAVITY

52. The **density of a body** is its mass divided by its volume in cubic feet.*

Let D be the density; then, the density of a body is,

$$D = \frac{M}{V}. \quad \text{Since } M = \frac{W}{g}, D = \frac{W}{gV}$$

EXAMPLE 1.—Three cubic feet of cast iron weighs 1,350 pounds; what is the density of cast iron?

SOLUTION.— $D = \dfrac{W}{gV} = \dfrac{1{,}350}{32.16 \times 3} = 13.992$. Ans.

EXAMPLE 2.—A cubic foot of water weighs 62.42 pounds: (*a*) what is its density? (*b*) What is the ratio between the density of cast iron and the density of water?

SOLUTION.—(*a*) $D = \dfrac{W}{gV} = \dfrac{62.425}{32.16 \times 1} = 1.9411 = \text{density}$. Ans.

(*b*) $\dfrac{13.992}{1.941} = 7.2083 = \text{ratio}$. Ans.

53. The **specific gravity of a body** *is the ratio between its weight and the weight of a like volume of water.*

Since gases are so much lighter than water, it is usual to take the specific gravity of a gas as the ratio between the weight of a certain volume of the gas and the weight of the same volume of air.

EXAMPLE.—A cubic foot of cast iron weighs 450 pounds; what is its specific gravity, a cubic foot of water weighing 62.425 pounds?

SOLUTION.—According to the definition,

$$\frac{450}{62.425} = 7.2086. \quad \text{Ans.}$$

NOTE.—Notice that this answer is the same as that of the preceding example: hence, the specific gravity of a body is also the *ratio of the density of a body to the density of water.*

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*Some writers define density as the weight of a unit of volume of the material. When English measures are used, the density of any material, according to this definition, is the weight of a cubic foot of the material, in pounds

54. The specific gravities of different bodies are given in printed tables; hence, if it is desired to know the weight of a body that cannot be conveniently weighed, *calculate its cubical contents, and multiply the specific gravity of the body by the weight of a like volume of water, remembering that a cubic foot of water weighs 62.425 pounds.*

EXAMPLE 1.—How much will 3,214 cubic inches of cast iron weigh? Take its specific gravity as 7.21.

SOLUTION.—Since 1 cu. ft. of water weighs 62.425 lb., 3,214 cu. in. weigh $\frac{3{,}214}{1{,}728} \times 62.425 = 116.108$ lb.

Then, $116.108 \times 7.21 = 837.139$ lb. Ans.

EXAMPLE 2.—What is the weight of a cubic inch of cast iron?

SOLUTION.— $\frac{62.425}{1{,}728} \times 7.21 = .26047$ lb. Ans.

One cubic foot of pure distilled water at a temperature of 39° Fahrenheit weighs 62.425 pounds, but *the value usually taken in making calculations is 62.5 pounds.*

EXAMPLE 3.—What is the weight, in pounds, of 7 cubic feet of oxygen?

SOLUTION.—One cubic foot of air weighs .08073 lb., and the specific gravity of oxygen is 1.1054, compared with air; hence,

$.08073 \times 1.1054 \times 7 = .62467$ lb., nearly. Ans.

The table of specific gravities gives the specific gravities of a variety of substances likely to be met with in ordinary practice. The weights per cubic foot are calculated on a basis of 62.425 pounds of water per cubic foot. It will be noted that the weight of a cubic foot of wrought iron is given as 485.67 pounds; this is the weight when chemically pure. The wrought iron of commerce weighs less than this, and it is customary in engineering calculations to take 480 pounds as the weight of a cubic foot.

TABLE II

APPROXIMATE SPECIFIC GRAVITIES OF VARIOUS SUBSTANCES

Substance	Specific Gravity	Weight per Cubic Foot Pounds
METALS		
Aluminum	2.60	162.31
Antimony	6.715	419.18
Brass {80 copper + 20 zinc	8.60	536.86
Brass {60 copper + 40 zinc	8.20	511.89
Bronze (80 copper + 20 tin)	8.70	543.10
Copper	8.95	558.70
Gold	19.33	1,206.68
Iron, cast	7.21	450.00
Iron, wrought	7.78	485.67
Lead	11.37	709.77
Magnesium	1.76	109.87
Manganese	8.00	499.40
Mercury, at 32° F.	13.59	848.36
Nickel	8.90	555.58
Osmium	22.48	1,403.31
Platinum	21.50	1,342.14
Silver	10.50	655.46
Steel	7.85	490.00
Tin	7.29	455.08
Zinc	7.00	436.98
MISCELLANEOUS		
Brick	1.90	118.61
Cement, Portland (loose)		84.00
Cement, Portland (solid)	3.12	194.77
Chalk	2.78	173.54
Clay	1.93	120.48
Coal, anthracite (solid)	1.50	93.64
Coal, anthracite (loose)		56.00
Coal, bituminous (solid)	1.27	79.28

TABLE II—*Continued*

Substance	Specific Gravity	Weight per Cubic Foot Pounds
MISCELLANEOUS—*Continued*		
Coal, bituminous (loose)		49.94
Concrete	2.20	137.34
Earth, loose dry		75.00
Earth, rammed wet	1.80	112.37
Emery	4.00	249.70
Glass, average	2.80	174.79
Granite	2.65	165.43
Ice	.92	57.43
Lime	.80	49.94
Limestone	3.16	197.26
Marble	2.70	168.55
Masonry, dressed	2.68	167.30
Masonry, rough rubble	2.40	149.82
Masonry, brick	2.00	124.85
Mortar, hardened	1.64	102.38
Plaster Paris	2.00	124.85
Salt, common (solid)	2.13	132.97
Sand, rammed		120.00
Sand, loose		100.00
Stone, common	2.52	157.31
WOODS		
Ash	.845	52.75
Beech	.852	53.19
Cedar	.561	35.02
Cork	.240	14.98
Ebony (American)	1.331	83.09
Elm	.640	39.95
Hemlock	.386	24.10
Lignum-vitæ	1.333	83.21
Mahogany	1.063	66.36
Maple	.750	46.82

TABLE II—*Continued*

Substance	Specific Gravity	Weight per Cubic Foot Pounds
WOODS—*Continued*		
Oak (old heart)	1.170	73.04
Oak, white	.860	53.69
Pine, yellow	.660	41.20
Pine, white	.554	34.58
Poplar, white	.529	33.02
Spruce	.500	31.21
Walnut	.671	41.89
LIQUIDS		
Acetic acid	1.063	66.36
Alcohol	.800	49.94
Ammonia (liquid 27.9 per cent.)	.891	55.62
Gasoline	.660	41.20
Hydrochloric acid	1.212	75.66
Milk	1.032	64.42
Nitric acid (single, 46 per cent. pure)	1.290	80.53
Nitric acid (double, 67 per cent. pure)	1.420	88.64
Petroleum	.878	54.81
Sulphuric acid	1.837	114.67
Turpentine	.870	54.31
Water (pure, at maximum density, 39.1° F.)	1.000	62.425
Water, ordinary sea-water	1.026	64.05
GASES		
(At 32° F., and under a pressure of 1 atmosphere.)		
Air, atmospheric	1.0000	.08073
Ammonia	.6362	.05136
Carbon dioxide	1.5290	.12344
Carbon monoxide	.9674	.07810
Chlorine	2.4500	.19779
Hydrogen	.0693	.00559
Nitrogen	.9713	.07841

TABLE II—*Continued*

Substance	Specific Gravity	Weight per Cubic Foot Pounds
GASES—*Continued*		
(At 32° F., and under a pressure of 1 atmosphere.)		
Oxygen	1.1054	.08924
Smoke (bituminous coal)	.1020	.00823
Smoke (wood)	.0900	.00727
*Steam at 212° F.	.4700	.03794

*The specific gravity of steam at any temperature and pressure compared with air at the same temperature and pressure is .622.

HYDROSTATICS

PROPERTIES OF LIQUIDS

1. Perfect and Viscous Liquids.—**Hydrostatics** treats of liquids at rest under the action of forces. A **liquid body** is one whose molecules change their relative positions easily. In some liquids, as water, alcohol, ether, etc., a change of form takes place almost instantly when the liquid is acted on by a force; thus, any one of these liquids will almost instantly assume a level surface under the action of gravity. Other liquids change form gradually, as, for example, tar, pitch, or molasses.

A **perfect liquid** is one whose particles can move on each other with the greatest freedom and without friction; that is, a liquid that instantly changes form under the action of a disturbing force. There is no such thing as an absolutely perfect liquid, but for practical purposes water, mercury, alcohol, and ether, at ordinary temperatures, may be treated as perfect liquids.

Liquids that do not change their form readily are prevented from doing so by the property called *viscosity*. All liquids are more or less viscous, and the degree of viscosity varies greatly. The liquids with great viscosity, as tar and pitch, are usually called **viscous liquids.**

In that which follows, the liquid that will be dealt with chiefly is water. The principles stated will, however, apply to other perfect liquids, and approximately to viscous liquids.

2. Compressibility of Liquids.—Liquids are only slightly compressible. A pressure of 15 pounds per square inch will compress water only about $\frac{1}{20000}$ of its volume.

For this reason, in all practical problems, water is generally regarded as incompressible.

3. **Heaviness of Liquids.**—The weight of a cubic unit of a liquid is called its **heaviness.** The weight, in pounds per cubic foot, of a substance is taken as the standard for heaviness in the United States and England. For example, the heaviness of distilled water at the temperature of maximum density, 39.1° F., is 62.425 pounds, and of ordinary seawater, 64.05 pounds. In engineering calculations, the heaviness of fresh water is generally taken as 62.5 pounds.

FLUID PRESSURE

TRANSMISSION OF PRESSURE

4. **Pascal's Law.**—Fig. 1 shows two cylindrical vessels of the same size. Vessel *a* is fitted with a wooden block of the same size as the cylinder; the vessel *b* is filled with water, whose depth is the same as the length of the wooden block in *a*. Both vessels are fitted with air-tight pistons *p*, whose areas are each 10 square inches.

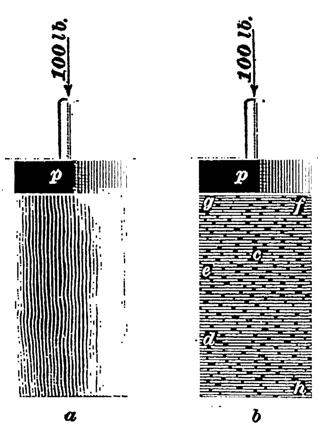


FIG. 1

Suppose, for convenience, that the weights of the cylinders, pistons, block, and water are neglected, and that a force of 100 pounds is applied to both pistons. The pressure per square inch is $100 \div 10 = 10$ pounds. In the vessel *a*, this pressure is transmitted without loss to the bottom of the vessel, and it is easy to see that there will be no pressure on the sides. In the vessel *b*, an entirely different result is obtained. The pressure on the bottom is the same as in the other case—that is,

10 pounds per square inch—but owing to the fact that the molecules of the water are perfectly free to move, this pressure of 10 pounds per square inch is transmitted in every direction with the same intensity; that is to say, the pressure at any point, *c*, *d*, *e*, *f*, *g*, *h*, etc. due to the force of 100 pounds, is exactly the same and is 10 pounds per square inch.

The truth of this statement may be proved, experimentally, by means of the apparatus shown in Fig. 2. Let the area of the piston *a* be 20 square inches; of *b*, 7 square inches; of *c*, 1 square inch; of *d*, 6 square inches; of *e*, 8 square inches; and of *f*, 4 square inches.

If the pressure due to the weight of the water is neglected and a force of 5 pounds is applied at *c*, whose area is 1 square inch, a pressure of 5 pounds per square inch will be transmitted in all directions, and in order that there shall be no movement a force of $6 \times 5 = 30$ pounds must be applied at *d*, 40 pounds at *e*, 20 pounds at *f*, 100 pounds at *a*, and 35 pounds at *b*.

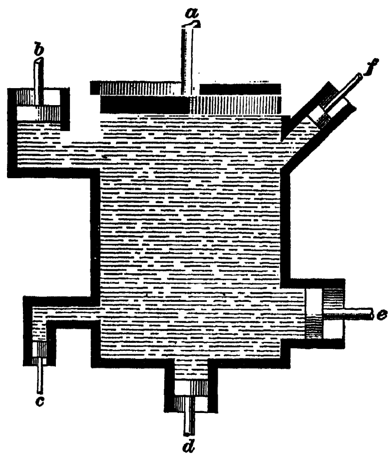


Fig. 2

If a force of 99 pounds is applied at *a*, instead of 100 pounds, the piston *a* will rise, and the other pistons *b*, *c*, *d*, *e*, and *f* will move inwards; but if the force applied at *a* is 100 pounds, they will all be in equilibrium. Should 101 pounds be applied at *a*, the pressure per square inch will be $101 \div 20 = 5.05$ pounds, which will be transmitted in all directions; and, since the pressure due to *c* is only 5 pounds per square inch, it is evident that the piston *a* will move downwards, and the pistons *b*, *c*, *d*, *e*, and *f* will be forced outwards.

The whole may be summed up in the following law:

The pressure per unit of area exerted anywhere on a mass of liquid is transmitted undiminished in all directions, and acts

with the same intensity on all surfaces in a direction at right angles to those surfaces.

This law was first discovered by Pascal, and is the most important in hydrostatics. Its meaning should be thoroughly understood.

Example.—If the area of the piston e in Fig. 2 is 8.25 square inches, and a force of 150 pounds is applied to it, what forces must be applied to the other pistons to keep the water in equilibrium, assuming that their areas are the same as those just given?

Solution.—The intensity of pressure is $150 \div 8.25 = 18.182$ lb. per sq. in., nearly. Then,

$$\left.\begin{array}{rll} 20 \times 18.182 = & 363.64 \text{ lb.} = & \text{force required to balance } a \\ 7 \times 18.182 = & 127.274 \text{ lb.} = & \text{force required to balance } b \\ 1 \times 18.182 = & 18.182 \text{ lb.} = & \text{force required to balance } c \\ 6 \times 18.182 = & 109.092 \text{ lb.} = & \text{force required to balance } d \\ 4 \times 18.182 = & 72.728 \text{ lb.} = & \text{force required to balance } f \end{array}\right\} \text{Ans.}$$

5. Application of Pascal's Law.—Let the area of the piston a, Fig. 3, be 1 square inch and that of b 40 square inches. According to Pascal's law, 1 pound placed on a will balance 40 pounds placed on b.

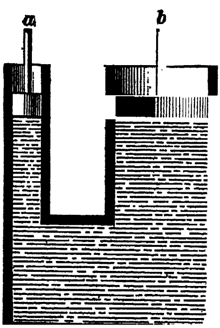


Fig. 3

Suppose that a moves downwards 10 inches; 10 cubic inches of water will then be forced into the tube b. This will be distributed over the entire area of the tube b in the form of a cylinder, whose cubic contents must be 10 cubic inches, whose base has an area of 40 square inches, and whose height must therefore be $\frac{10}{40} = \frac{1}{4}$ inch; that is, a movement of the piston a of 10 inches causes a movement of the piston b of $\frac{1}{4}$ inch.

This is an example of the familiar principle of work: *The applied force multiplied by the distance through which it moves is equal to the resistance multiplied by the distance through which it moves.*

6. The Hydraulic Press.—The foregoing principle is made use of in the hydraulic press shown in Fig. 4. As the lever a is depressed, the plunger b is forced down on the

water in the cylinder *c*. The water is forced through the bent tube *d* into the cylinder in which the large plunger *e* works, and causes the plunger to rise, thus lifting the platform *f*, and compressing the bales that lie on it.

Let the area of the plunger *b* be 1 square inch and that of *e* 100 square inches. Also, assume the length of the lever between the hand and fulcrum to be 10 times the length between the fulcrum and plunger *b*. If the end of the lever is

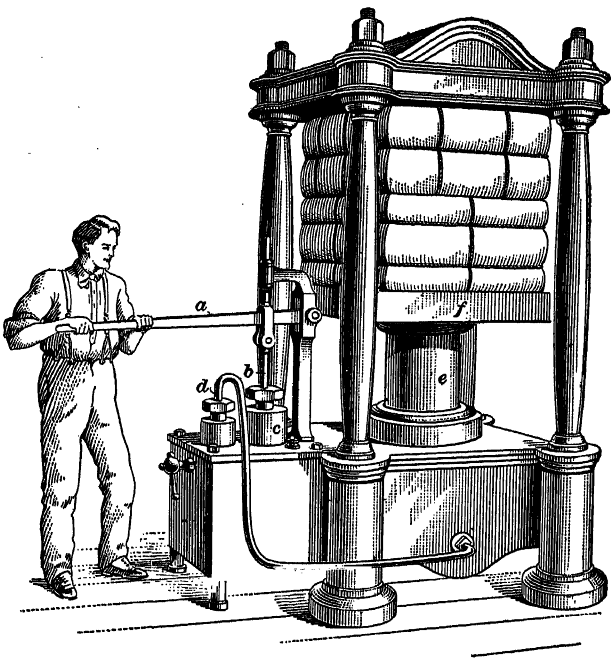


FIG. 4

depressed 10 inches, the plunger *b* is depressed one-tenth of 10 inches = 1 inch, and since 1 cubic inch of water is displaced and the area of plunger *e* is 100 square inches, it is raised $\frac{1}{100}$ inch. If p represents the force applied to the lever *a*, and q represents the pressure on the platform *f*, then $p \times 10$ inches $= q \times \frac{1}{100}$ inch. From this, $q = 1{,}000\,p$ or $p = \frac{1}{1000}\,q$. A force of 40 pounds applied by the hand will therefore cause a pressure of $40 \times 1{,}000 = 40{,}000$ pounds

to be exerted by the piston e. But if the average movement of the hand per stroke is 10 inches, and since a 10-inch movement of the hand lever moves the plunger e, $\frac{1}{100}$ inch, it will require $1 \div \frac{1}{100} = 100$ strokes to raise the platform 1 inch. It is here seen that what is gained in pressure is lost in speed.

Applications of Pascal's law are seen also in hydraulic machines used for forcing locomotive drivers on their axles, in punching plates, in bending rails, and in testing the strength of boiler shells.

EXAMPLE.—In a hydraulic punch, the plunger that carries the punch is $4\frac{1}{2}$ inches in diameter and the forcing plunger is $\frac{3}{8}$ inch in diameter. The latter plunger is joined to a lever 3 inches from the fulcrum, and the hand pressure is applied 36 inches from the fulcrum. The resistance to the punch is 40,000 pounds. What pressure must be exerted on the end of the lever?

SOLUTION.—When the end of the lever is depressed 12 in., the forcing plunger moves $12 \times \frac{3}{36} = 1$ in. The ratio of the areas of the two plungers is $(4\frac{1}{2})^2 : (\frac{3}{8})^2 = \frac{81}{4} \div \frac{9}{64} = 144$; hence, when the smaller plunger moves 1 in., the larger moves $\frac{1}{144}$ in. The hand pressure p exerted through 12 in. causes the resistance of 40,000 lb. to move $\frac{1}{144}$ in. Hence, $p \times 12 = 40{,}000 \times \frac{1}{144}$, or

$$p = \frac{40{,}000}{12 \times 144} = 23.15 \text{ lb. Ans.}$$

PRESSURES ON SURFACES

7. Pressure on the Bottom of a Vessel.—*The pressure on the flat horizontal bottom of a vessel due to the weight of the contained liquid is independent of the shape of the vessel and is equal to the weight of a prism of the liquid whose base has the same area as the bottom of the vessel and whose height is the distance between the bottom and the upper surface of the liquid.*

The truth of this principle may be shown by the following example: In Fig. 5, the pressure on the bottom of the vessel a is, of course, equal to the weight of the water it contains. If the area of the bottom of the vessel b and the depth of the liquid contained in it are the same as in the vessel a, the pressure on the bottom of b will be the same as on the bottom of a. Suppose that the bottoms of the vessels a and b are 6 inches square, and the part $c\,d$ in the vessel b is 2 inches square, and that both vessels are filled with

water. Then, since the weight of 1 cubic inch of water is $62.5 \div 1{,}728 = .03617$ pound and the volume of vessel *a* 6 in. $\times$ 6 in. $\times$ 24 in. = 864 cubic inches, the weight of the water in *a* is $864 \times .03617 = 31.25$ pounds. Hence, the total pressure on the bottom of the vessel *a* is 31.25 pounds. The weight of water contained in the part *ec* of vessel *b* is $6 \times 6 \times 10 \times .03617 = 13.02$ pounds; hence, the pressure on the bottom due to this weight is 13.02 pounds. The weight of the part contained in *cd* is $2 \times 2 \times 14 \times .03617 = 2.0255$ pounds.

Imagine a thin partition to be placed horizontally at the bottom of the narrow part *cd*. The pressure on this partition would be 2.0255 pounds, the weight of the water above it; and, the area being 4 square inches, the pressure per square inch would be $2.0255 \div 4 = .5064$ pound. It is evident, therefore, that the upper horizontal layer of water in *ec*, just like the imaginary partition, is subjected to a downward pressure of .5064 pound per square inch.

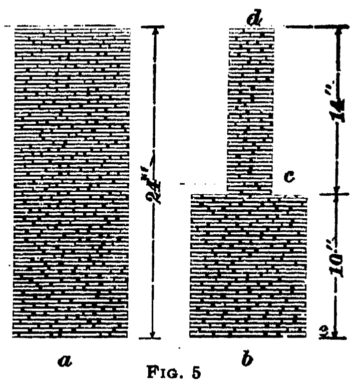


FIG. 5

According to Pascal's law, this pressure is transmitted equally in all directions; therefore, every square inch of the large part of the vessel *b* is subjected to a pressure of .5064 pound, owing to the body of water above *c*. The horizontal area of the part *ec* is $6 \times 6 = 36$ square inches, and the total pressure due to the weight of the water in the small part is $.5064 \times 36 = 18.23$ pounds. Hence, the total pressure on the bottom of *b* is $13.02 + 18.23 = 31.25$ pounds, the same result as in the case of the vessel *a*. In both cases, the pressure is uniformly distributed over the bottom, and its intensity, or the pressure per square inch, is $31.25 \div 36 = .868$ pound.

If an additional pressure of 10 pounds per square inch were applied to the upper surface of both vessels, the total

pressure on their bottoms would be $31.25 + (6 \times 6 \times 10) = 31.25 + 360 = 391.25$ pounds. In case this pressure were obtained by means of a weight placed on a piston, as shown in Figs. 1 and 2, the weight for the vessel a would be $6 \times 6 \times 10 = 360$ pounds, and for the vessel b it would be $2 \times 2 \times 10 = 40$ pounds.

8. Pressure Due to a Given Head.—The pressure of water at any given level in a tank or reservoir is due to the weight of the column of water above that level. This column of water is known as the **head** and is measured vertically from the level of the surface of water to the given level in the tank or reservoir. For example, suppose that a vessel, as shown in Fig. 6, is filled with liquid, and imagine a horizontal partition or thin piston placed at $m n$. The water above produces a downward pressure on the partition, just as if it were the bottom of a vessel, and, as shown in Art. **7**, this pressure depends only on the area of the partition, and its depth below the surface of the liquid. The pressure per square inch on this partition is the weight of a prism of the liquid whose base is 1 square inch and whose height is the distance from $m n$ to the surface.

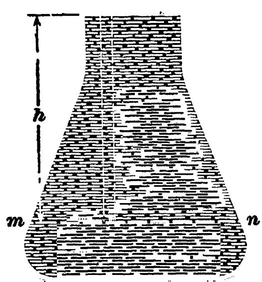


FIG. 6

The horizontal layer of the liquid at $m n$ may now be considered as the partition or piston, and the downward pressure per square inch on this layer will be the same as with the thin piston first assumed.

The distance from any horizontal layer of a body of liquid to the surface of the liquid is termed the head for that layer.

Let h = head, in feet, for any horizontal layer;

p = pressure per square inch on the layer, in pounds;

w = weight of a column of liquid 1 foot long and 1 square inch in cross-section.

Then, $$p = w h \qquad \textbf{(1)}$$

A column 1 foot long and 1 square inch in cross-section contains 12 cubic inches. Since water weighs .03617 pound per cubic inch, for water $w = .03617 \times 12 = .434$ pound.

Hence, when the liquid is water,

$$p = .434\,h \qquad (2)$$

EXAMPLE.—The depth of water in a stand pipe is 80 feet. (*a*) What is the pressure, per square inch, on the bottom? (*b*) What is the pressure, per square inch, on a layer 65 feet from the surface?

SOLUTION.—

(*a*) $p = .434\,h = .434 \times 80 = 34.72$ lb. per sq. in. Ans.

(*b*) $p = .434 \times 65 = 28.21$ lb. per sq. in. Ans.

9. Head Required for Given Pressure.—Using the symbols of Art. 8, and solving formula 1 for h,

$$h = \frac{p}{w} \qquad (1)$$

When the liquid is water,

$$h = \frac{p}{.434} = 2.304\,p \qquad (2)$$

EXAMPLE.—What must be the height of water in a stand pipe to give a pressure of 80 pounds per square inch on the bottom?

SOLUTION.—The required head is

$h = 2.304 \times 80 = 184.32$ ft. Ans.

10. Upward and Lateral Pressure.—So far, only downward pressure has been discussed. Upward pressure and lateral pressure will now be considered. Let the vessel shown in Fig. 7 be filled with liquid to the level a. The part of the liquid in $a\,b$ acts on the layer at b and produces over that surface an intensity of pressure of $w\,h_1$ pounds per square inch, where h_1 is the head on the layer at b. According to Pascal's law, this pressure per unit

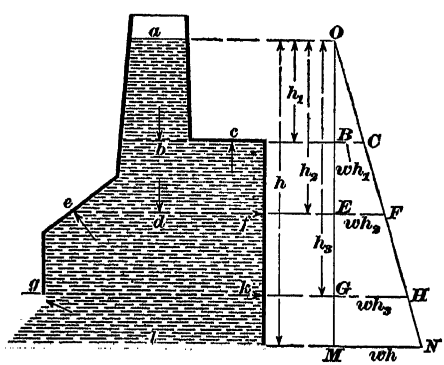


FIG. 7

area acts on all the bounding surfaces below the level *b*; hence, the liquid *a b* exerts a pressure of $w h_1$ pounds per square inch at *c*, *e*, *f*, and *k*, at right angles to the surfaces. At *e* and *f*, however, there is additional pressure due to the weight of liquid between the level *e f* and the level *b*; but at *c*, the liquid below the level *b* can exert no additional pressure; hence, the total upward pressure per unit of area at *c* is $w h_1$ pounds, the same as the downward pressure on the layer at *b*.

Consider now the layer of liquid *e f* at a distance h_2 below the surface *a*. The pressure on this layer due to the weight of the liquid above is $w h_2$ pounds per square inch; and by Pascal's law, this pressure is transmitted to all parts of the bounding surface below the level *e f*, just as though the layer *e f* were a solid piston. The liquid below *e f* can exert no pressure at the points *e* and *f*; hence, at these points the pressure per unit area is the same as the downward pressure on the layer *e f*; namely, $w h_2$ pounds per square inch. The same reasoning shows that the lateral pressure per unit area at the points *g* and *k* is $w h_3$, where h_3 is the head for the layer *g k*. The following important law, therefore, is a direct consequence of Pascal's law:

The pressure per unit area at any point of a surface, whether downwards, upwards, lateral, or oblique, depends only on the depth of the point below the surface of the liquid. The magnitude of this pressure is given by formula **1** of Art. **8**, $p = w h$.

EXAMPLE.—In Fig. 7, suppose the depth of the various layers below the level *a* to be as follows: *b*, 10 feet; *e f*, 17 feet; *g k*, 25 feet; and *l*, 30 feet. The liquid is water. What are the pressures per square inch at points *c*, *e*, *f*, *g*, *k*, and *l*?

SOLUTION.—Using formula **2** of Art. **8**, in each case,

$$\left.\begin{array}{l} \text{Pressure at } c \text{ is } .434 \times 10 = 4.34 \text{ lb. per sq. in.} \\ \text{Pressure at } e \text{ is } .434 \times 17 = 7.378 \text{ lb. per sq. in.} \\ \text{Pressure at } f \text{ is } .434 \times 17 = 7.378 \text{ lb. per sq. in.} \\ \text{Pressure at } g \text{ is } .434 \times 25 = 10.85 \text{ lb. per sq. in.} \\ \text{Pressure at } k \text{ is } .434 \times 25 = 10.85 \text{ lb. per sq. in.} \\ \text{Pressure at } l \text{ is } .434 \times 30 = 13.02 \text{ lb. per sq. in.} \end{array}\right\} \text{Ans.}$$

The varying pressure per unit area for different points below the surface of a liquid may be determined, graphically,

as follows: In Fig. 7, the vertical line OM is drawn from the upper level a to the lower level l, and from M a length MN is laid off horizontally to represent, to some scale, the pressure per square inch at l. Points O and N are joined by a straight line and horizontal lines are drawn through c, f, and k to cut OM and ON. Then, the horizontal intercepts BC, EF, and GH represent the pressures per unit area at their respective depths to the same scale that MN represents the pressure per unit area on the surface l.

11. Pressure Due to External Load.—If the surface of the liquid is subjected to a pressure, this pressure, according to Pascal's law, is transmitted undiminished to all points of the enclosing vessel and must be added to the pressure due to the weight of the liquid. For example, suppose the surface a, Fig. 7, to be subjected to a pressure of 30 pounds per square inch. In the example of Art. **10**, the pressure at c due to the head of water is 4.34 pounds per square inch. To this is added the 30 pounds per square inch, giving as the total pressure $4.34 + 30 = 34.34$ pounds per square inch.

The pressure at e and f is $7.378 + 30 = 37.378$ pounds per square inch.

The pressure at g and k is $10.85 + 30 = 40.85$ pounds per square inch.

The pressure at l is $13.02 + 30 = 43.02$ pounds per square inch.

Let G = total load on surface of liquid;

A = area of surface loaded in square inches;

$p_0 = \dfrac{G}{A}$ = pressure per square inch on surface;

p = pressure per square inch at a point h feet below surface of liquid.

Then,
$$p = wh + p_0 = wh + \frac{G}{A} \qquad (1)$$

When the liquid is water,
$$p = .434\,h + p_0 = .434\,h + \frac{G}{A} \qquad (2)$$

EXAMPLE 1.—A vessel filled with ordinary sea-water has a circular bottom 13 inches in diameter. A column of ordinary sea-water 12 inches

high by 1 square inch in cross-section weighs .445 pound. The top of the vessel is fitted with a piston 3 inches in diameter, on which is laid a weight of 75 pounds; what is the pressure per square inch on the bottom, if the depth of the water is 18 inches?

SOLUTION.—The head is 18 in. = 1.5 ft. Using formula **1**,

$$p = wh + \frac{G}{A} = .445 \times 1.5 + \frac{75}{3 \times 3 \times .7854} = 11.28 \text{ lb. per sq. in. Ans.}$$

EXAMPLE 2.—In a vertical boiler, the water level is 5 feet above the top of the firebox and the steam pressure is 65 pounds per square inch; what is the pressure per square inch on the top of the firebox?

SOLUTION.—Here p_0 = 65 lb. per sq. in. By formula **2**,

$$p = .434\,h + p_0 = .434 \times 5 + 65 = 67.17 \text{ lb. per sq. in. Ans.}$$

EXAMPLE 3.—A suspended vertical cylinder is tested for the tightness of its heads by filling it with water. A pipe whose inside diameter is $\frac{1}{4}$ inch and whose length is 20 feet is screwed into a hole in the upper head and then filled with water; what is the pressure per square inch on each head if the cylinder is 5 feet long?

SOLUTION.—For the upper cylinder head, h = 20 ft.; and for the lower, h = 25 ft. From formula **2** of Art. **8**,

$$p = .434\,h = .434 \times 20 = 8.68 \text{ lb. per sq. in., on the upper head}$$

$$p = .434 \times 25 = 10.85 \text{ lb. per sq. in., on the lower head. Ans.}$$

EXAMPLE 4.—In example 3, if the pipe is fitted with a piston weighing $\frac{1}{4}$ pound, and a 5-pound weight is laid on it, what is the pressure per square inch on the upper head?

SOLUTION.—The area of the surface subjected to the load of $5\frac{1}{4}$ lb. is $(\frac{1}{4})^2 \times .7854 = .0491$ sq. in. The head h is 20 ft. Using formula **2**,

$$p = .434 \times 20 + \frac{5\frac{1}{4}}{.0491} = 115.6 \text{ lb. per sq. in. Ans.}$$

12. Surface Level of Liquids.—Since the pressure on the bottom of a vessel due to the weight of the liquid is dependent only on the height of the liquid, and not on the shape of the vessel, it follows that if a vessel has a number of radiating tubes, as shown in Fig. 8, the water in all tubes will be at the same level, no matter what may be the shape of the tubes. For, if the water were higher in one tube than in the others, the downward pressure at the bottom due to the height of the water in this tube would be greater than that due to the height of the water in the other tubes. Consequently, the upward pressure in the other tubes would also be greater; the equilibrium would be destroyed, and water

would flow from this tube into the vessel, and rise in the other tubes until it stood at the same level in all, when equilibrium would be restored. This principle is expressed in the familiar saying, "water seeks its level."

This explains why city water reservoirs are located on high elevations, and why water leaving the hose nozzle spouts

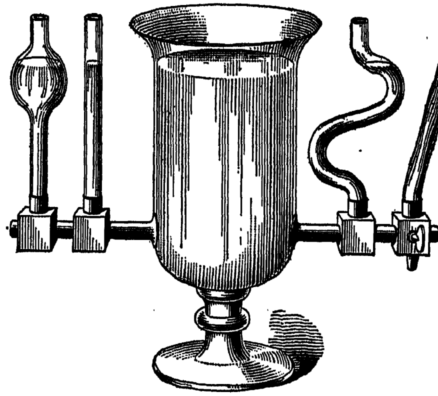


FIG. 8

so high. If there were no frictional and air resistance, the water would spout to a height equal to the level of the water in the reservoir. If a vertical pipe with a length equal to the vertical distance between the nozzle and the level of the water in the reservoir were attached to the nozzle, the water would just reach the end of the pipe; and, if the top of the pipe were lowered slightly, the water would flow over. Fountains, canal locks, and artesian wells are examples of the working of this principle.

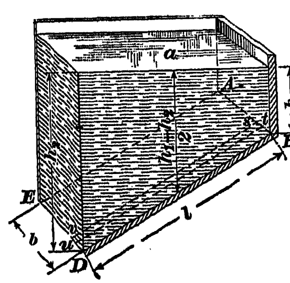


FIG. 9

13. Total Pressure on a Flat Surface.—Suppose $ABDE$, Fig. 9, to be a rectangular plane surface sustaining liquid pressure. In practice, such surfaces occur in dams, sluice gates, tanks with sloping sides, etc. Let the edges AB and DE be horizontal, and let the surface be

inclined at any angle with the horizontal. The depth of the edge AB below the liquid level a is h_1; hence, the pressure per square inch at this edge is $w h_1$. Likewise, the depth of the edge DE being h_2, the pressure at this edge is $w h_2$. Consider a strip of the surface $s t u v$, 1 inch wide and parallel to the edge BD. The pressure at the end $s t$ being $w h_1$ pounds per square inch, and that at the other end, $u v$, being $w h_2$ pounds per square inch, the average pressure per square inch is $\frac{w h_1 + w h_2}{2}$. Let the length of the edge BD be l inches; then the area of the strip is l square inches; and the total pressure on the strip is $l \frac{w h_1 + w h_2}{2}$ pounds. If b denotes the width DE of the surface, in inches, there will be b strips like $s t u v$, each 1 inch wide, and the total pressure P on the surface is

$$P = b l \frac{w h_1 + w h_2}{2} = b l w \frac{h_1 + h_2}{2} \qquad (1)$$

Now, $b \times l$ is the area of the surface and $\frac{h_1 + h_2}{2}$ is the depth of the center of gravity G of the surface below the liquid level. Hence, *the total liquid pressure on a rectangular plane surface is the product of the area of the surface and the pressure per unit area due to the head of the liquid above the center of gravity.* Sometimes the following statement is used: *The total liquid pressure is equal to the weight of a prism of liquid whose base is the area of the surface and whose height is the distance of the center of gravity of the surface below the liquid level.*

It can be proved that this law holds good for all flat surfaces whatever their outline. For a flat plate, only part of whose surface sustains liquid pressure, the law holds good for the part below the liquid level.

Let A = area in square inches, of a plane surface sustaining liquid pressure;

h_o = head, in feet, of liquid above center of gravity;

w = weight of a column of liquid 1 foot long and 1 square inch in cross-section;

P = total pressure on surface, in pounds.

Then,
$$P = A w h_o \qquad (2)$$

EXAMPLE.—A vertical sluice gate, Fig. 10, is $3\frac{1}{2}$ feet wide, and 5 feet of it is below the water level; what is the total pressure on the gate?

SOLUTION.—The area sustaining liquid pressure is $3\frac{1}{2} \times 5 \times 144 = 2{,}520$ sq. in. The center of gravity of the submerged part is $5 \div 2 = 2\frac{1}{2}$ ft. below the water surface; hence, by formula **2**, $P = A w h_0$ $= 2{,}520 \times .434 \times 2\frac{1}{2} = 2{,}734.2$ lb. Ans.

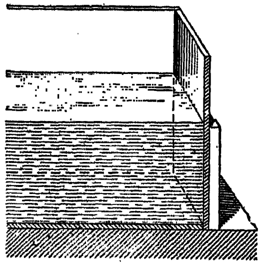


FIG. 10

14. Resolved Pressure in a Given Direction.—Let $BCDE$, Fig. 11, represent a rectangular plane area and p the normal pressure on it per unit of area. The force p may then be resolved into the rectangular components f_1 and f_2. Let m denote the angle between p and its component f_1; then $f_1 = p \cos m$. Suppose, now, that a plane is passed through the edge BC perpendicular to the force f_1, and let D and E be projected on this plane, giving the points D' and E'. The rectangular area $BCD'E'$ is the projection of the area $BCDE$ on the plane perpendicular to f_1. Evidently, angle $EBE' = m$, since p is perpendicular to BE and f_1 is perpendicular to BE'; then $BE' = BE \cos m$, and area $BCD'E'$ $=$ area $BCDE \times \cos m$.

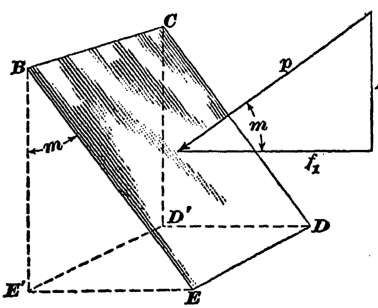


FIG. 11

Now, f_1 is the pressure per unit of area in its direction; hence, if A denotes the area of $BCDE$, the total pressure on this area in the direction of f_1 is

$$f_1 A = p \cos m \times A = p A \cos m$$

But, $A \cos m$ is the area of $BCD'E'$, the projection of $BCDE$ on the plane perpendicular to f_1. Hence, *the pressure exerted by a fluid in any direction on a plane surface, is equal to the weight of a prism of the fluid whose base is the area of the projection of the surface on a plane at right angles to the direction considered, and whose height is the*

depth of the center of gravity of the surface below the level of the liquid.

EXAMPLE.—The earthwork dam, Fig. 12, sustains a water pressure along the face $A\,B$, inclined at 60° to the horizontal; the distance $A\,B$ in contact with water is 18 feet, and the length of the dam is 60 feet. What is the total pressure on the dam in a horizontal direction and in a vertical direction?

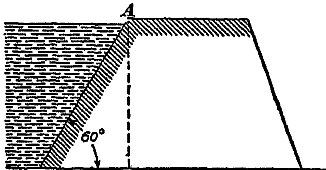


FIG. 12

SOLUTION.—The projection of $A\,B$ on a vertical plane is $A\,C$, and the length of $A\,C$ is $A\,B \times \sin 60° = 18 \times .86603 = 15.589$ ft. The length being 60 ft., the projection on the vertical plane of the oblique surface sustaining pressure is $15.589 \times 60 = 935.34$ sq. ft. The center of gravity of $A\,B$, and also of $A\,C$, lies at a distance of $15.589 \div 2 = 7.794$ ft. below the water level. The prism of water with a base of 935.34 sq. ft. and a height of 7.794 ft. weighs $935.34 \times 7.794 \times 62.5 = 455{,}627$ pounds, which is the total pressure in a horizontal direction. Ans.

The projection $B\,C$ has a length $A\,B \times \cos 60° = 18 \times .5 = 9$ ft., and the area of the projection of the oblique surface on the horizontal plane is, therefore, $9 \times 60 = 540$ sq. ft. A prism of water with this base and with a height of 7.794 ft. weighs $540 \times 7.794 \times 62.5 = 263{,}050$ pounds, which is the total downward pressure of the water on the dam.

15. Pressures on Curved and Irregular Surfaces. If the surface sustaining fluid pressures is not plane, but is curved or irregular, the total pressure on it in any direction is the same as the total pressure would be on the projection

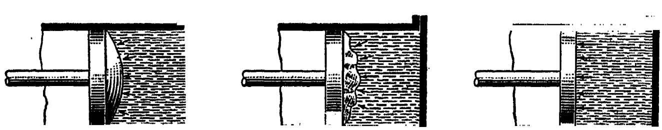

(a) (b) (c)

FIG. 13

of the surface on a plane at right angles to the given direction. To illustrate this statement, consider the three pistons shown in Fig. 13. In Fig. 13 (*a*) is shown a piston whose end has the form of a segment of a sphere; in Fig. 13 (*b*)

is shown one of irregular form; and in Fig. 13 (c) is shown one with a flat surface. In each case, the projection of the surface sustaining pressure, on a plane perpendicular to the piston rod, is the circular cross-section of the cylinder, and if the pressure per unit area is the same in the three cylinders, the thrust on the piston rod is the same. This law applies to all fluids, liquid or gaseous, and it assumes that the pressure per unit area is the same at all points of the surface.

In the case of a submerged curved or irregular surface, where the pressure per unit area varies with the depth below the surface, this law does not hold except for a projection on a vertical plane.

Suppose, for example, that the side wall of a vessel, Fig. 14, has a cylindrical part BFC. Then the total horizontal pressure on this part is equal to the total horizontal pressure on the vertical plane surface BC, supposing BC to be substituted for the curved surface. This pressure may be found by the use of formula **2**, Art. **13**, using for A the area of the projection, not that of the curved surface; that is, the pressure is equal to the product of the area of the projection and the pressure per square inch due to the head above the center of gravity of the curved surface. Let the diameter BC be 10 inches, the length of the cylinder 15 inches, and the depth of the center of gravity below the surface, 30 inches = $2\frac{1}{2}$ feet. The projected area is $10 \times 15 = 150$ square inches, and the total pressure is $P = AWh_0 = 150 \times .434 \times 2\frac{1}{2} = 162.75$ pounds.

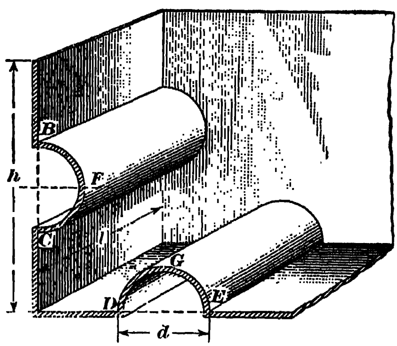


FIG. 14

For the cylindrical part DGE in the bottom of the vessel, this law is not true. To get the total vertical pressure, the projected area on the bottom, that is, the diameter DE multiplied by the length, must be multiplied by the pressure per

square inch due to a certain head. This head, however, is neither that above the center of gravity of the curved surface, nor that above the bottom. If d denotes the diameter DE of the cylinder, l its length, and h the head of water above the bottom, then the total downward pressure on the cylindrical part DGE is equal to the volume of the water resting on the curved surface, multiplied by the weight of water per unit volume. The total volume above the curved surface is equal to the volume $h\,d\,l$ above the flat side of the cylindrical part minus the volume $\frac{\pi d^2 l}{2 \times 4}$ of the cylindrical part. The volume, per unit of length, resting on the curved surface, is, therefore, $\left(h\,d - \frac{\pi d^2}{8}\right)$. If h is in feet and d in inches, the total downward pressure, per inch of length of the cylindrical part, is $.434\left(h\,d - \frac{\pi d^2}{8}\right)$. If h_1 represents the average head, in feet, of the water above the curved surface, then $.434\,h_1 d$ is also the vertical pressure per inch of length of the cylindrical part, and therefore $.434\,h_1 d = .434\left(h\,d - \frac{\pi d^2}{8}\right)$. Hence,

$$h_1 = h - \frac{\pi d}{8}$$

That is, the head required to give the true total pressure is $h - \frac{\pi d}{8}$, which is quite different from that above the center of gravity.

The whole subject of the pressure in any direction on a curved or irregular surface may be summarized as follows:

1. If the external pressure is so great that the pressure due merely to the weight of the fluid may be neglected in comparison, the total pressure in any direction is equal to the product of the projection of the surface on a plane perpendicular to that direction, and the pressure per unit of area.

2. When the pressure is due wholly or in part to the weight of the fluid, as in the case of surfaces submerged in liquids, the total pressure in any direction, except the horizontal

direction, cannot in general be determined except for ı larly curved surfaces, such as those of spheres, cylin and cones, and for these the calculations are difficult and must be usually made by higher mathematics.

3. The total horizontal pressure on any surface, however, is easily found. It is precisely the same as the horizontal pressure on the projection of the given surface on a vertical plane.

16. Fluid Pressures in Cylinders.—Let a cylinder contain, under pressure, a fluid which may be a liquid, as in city water mains and stand pipes, or a gas, as in the case of pipes carrying steam, compressed air, etc. The pressure per square inch is assumed to be the same for all points; that is, the slight increase of pressure on the lower half of the pipe due to the weight of the fluid contained in it is neglected. Consider the upper half of a short section of a pipe or cylinder, as $AMBE$, Fig. 15. This is a curved surface whose under side is subjected to a uniform pressure of, say p pounds per square inch. Now, according to Art. **15**, the total pressure on this surface in a vertical direction is the same as if the pressure acted on the projection of the surface on a horizontal plane. If d denotes the inner diameter of the cylinder, and l the length BE of the given section, then $d\,l$ is evidently the area of this projection; hence, the total vertical pressure is $P = p\,d\,l$.

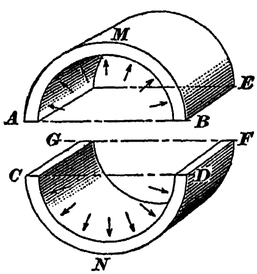


FIG. 15

The lower half $CNDFG$, likewise, is a curved surface subjected to a pressure of p pounds per square inch. It has the same projected area as the upper half; hence, the total vertical pressure on it, acting downwards, is likewise $P = p\,d\,l$.

In the figure, the two halves are shown separated; the same reasoning holds good, however, when they are united. The two forces, one of which tends to force the upper half upwards and the other to force the lower half downwards, are equal and opposite, and their combined tendency is to separate the two halves by tearing them apart along the

lines DF and CG. This tendency is resisted by the tenacity of the material comprising the cylinder. Evidently, the imaginary plane cutting the cylinder into halves may have any direction; and the force tending to separate the halves will be the same.

EXAMPLE 1.—The pressure in a water main is 60 pounds per square inch and the inner diameter of the main is 8 inches; what is the total force tending to separate one half from the other in a section of pipe 10 feet long?

SOLUTION.—Here $p = 60$, $d = 8$ in., and $l = 10$ ft. $= 120$ in.

$$P = p\,d\,l = 60 \times 8 \times 120 = 57{,}600 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—A section of stand pipe is 4 feet long and 10 feet in diameter, and its upper edge is 70 feet below the level of the water; what is the force tending to separate one half from the other?

SOLUTION.—The center of gravity of the section is 2 ft. below the upper edge, and therefore $70 + 2 = 72$ ft. below the water level. The pressure due to this head is $p = .434 \times 72$. The projected area of one half is $4 \times 10 = 40$ sq. ft. $= 5{,}760$ sq. in. The total pressure is therefore $.434 \times 72 \times 5{,}760 = 179{,}988.48$ lb. Ans.

17. Fluid Pressure in a Sphere.—For a hollow sphere filled with fluid under pressure, the same principle applies as in the case of the cylinder.

Let d = inner diameter of sphere in inches;
p = pressure per square inch;
P = total pressure in one direction tending to separate one half of sphere from the other.

The projection of one half of the sphere on the plane cutting it into two halves is a circle whose diameter is d and whose area is $\frac{1}{4}\pi d^2$; therefore,

$$P = \tfrac{1}{4}\pi d^2 p$$

EXAMPLE.—A spherical boiler 8 feet in diameter contains steam and water having a pressure of 45 pounds per square inch; what is the force tending to separate one half from the other?

SOLUTION.— $P = \frac{1}{4}\pi d^2 p = .7854 \times 96^2 \times 45 = 325{,}721$ lb. Ans.

EXAMPLES FOR PRACTICE

1. The diameter of the plunger of a hydraulic press used in an engineering establishment is 12 inches. Water is forced into the cylinder of the press by means of a small pump having a plunger

whose diameter is $\frac{3}{4}$ inch and stroke 4 inches. What pressure is exerted by the large plunger when the force acting on the small plunger is 125 pounds? Ans. 32,000 lb.

2. If the small plunger in example 1 makes 96 working strokes per minute: (*a*) how long will it take the large plunger to move 8 inches? (*b*) what is the velocity ratio? Ans. $\begin{cases} (a) & 5\frac{1}{3} \text{ min.} \\ (b) & 256:1 \end{cases}$

3. A vertical pipe, 88 feet high, is filled with water. (*a*) What is the pressure per square inch on the bottom? (*b*) If the diameter of the pipe is 8 inches, what is the total pressure tending to burst a section $2\frac{1}{2}$ inches high, whose center of gravity is 21 feet from the bottom?

Ans. $\begin{cases} (a) & 38.2 \text{ lb. per sq. in., nearly} \\ (b) & 581.56 \text{ lb.} \end{cases}$

4. A hollow sphere 8 inches in diameter is connected to a pipe in which a head of water 10 feet above the center of gravity of the sphere is maintained, and the upper surface of the water is subjected to a pressure of 8 pounds per square inch; what is the total pressure tending to rupture the sphere on a plane passing through its center? Ans. 620 lb., nearly

BUOYANT EFFECT OF LIQUIDS

IMMERSION AND FLOTATION

18. Buoyant Effort.—In a mass of liquid at rest, suppose a part of the liquid *m m*, Fig. 16, to become rigid without changing its form. Having the same density as before, the part will evidently remain at rest and will be held in equilibrium. Let the weight of the rigid part be denoted by B, the total downward pressure by P, and the total upward pressure by P_1. That the part may remain at rest, the upward force P_1 must balance the downward forces P and B; that is,

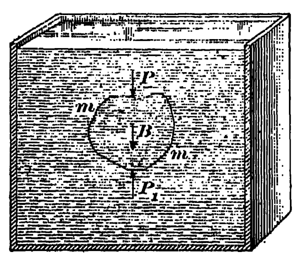


FIG. 16

$$P_1 = P + B; \text{ or, } P_1 - P = B$$

The difference between the total upward and downward pressures, $P_1 - P$, or the amount by which P_1 exceeds P, is called the **buoyant effort,** which acts upwards. Since

$P_1 - P = B$, it is evident that the buoyant effort is equal to B, the weight of the rigid mass $m\,m$ of the liquid.

Suppose that a solid body is immersed in the liquid, taking the place of the part $m\,m$, which is considered rigid, and let the weight of this body be denoted by G. Evidently, the body will be subjected to the same vertical pressures P and P_1; therefore, since the vertical forces acting on the body are G and P downwards and P_1 upwards, the net vertical force downwards is

$$P + G - P_1 = G - (P_1 - P) = G - B$$

B is equal to the buoyant effort and is the weight of the liquid displaced by the body. Hence, *when a solid body is immersed in a liquid, a buoyant effort equal to the weight of the liquid displaced acts upwards and opposes the action of gravity. The weight of the body, as shown by a scale, is decreased by an amount equal to the buoyant effort, that is, the weight of liquid displaced.* This is called the **principle of Archimedes,** because it was first stated by him.

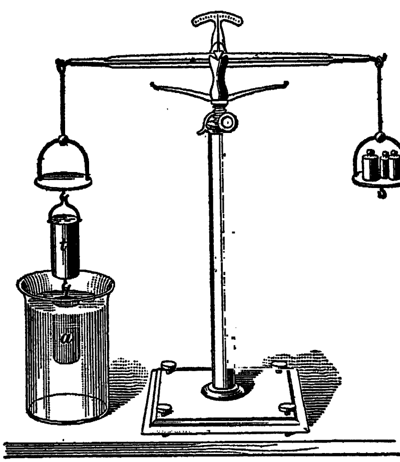


FIG. 17

Archimedes's principle may be experimentally demonstrated with the beam scales shown in Fig. 17. From one scale pan suspend a hollow cylinder of metal t, and below that a solid cylinder a of the same size as the hollow part of the upper cylinder. Put weights in the other scale pan until they exactly balance the two cylinders. If a be immersed in water, the scale pan containing the weights will descend, showing that a has lost some of its weight. Now, fill t with water, and the volume of water that can be poured into t will equal that displaced by a. The scale pan that contains the weights will gradually rise until t is filled, when the scales balance again.

If a body immersed in a liquid has the same weight as the liquid it displaces, then $G = B$, the net vertical force $G - B$ is zero, and the body will remain at rest at any depth below the surface.

If the body is heavier than the liquid it displaces, then G is greater than B, the net vertical force $G - B$ is downwards, and the body will sink to the bottom.

If, on the other hand, the body is lighter than the liquid it displaces, B is greater than G, the net vertical force $B - G$ is upwards, and the body will rise to the surface.

An interesting experiment in confirmation of the above facts may be performed as follows: Drop an egg into a glass jar filled with fresh water. The mean density of the egg being a little greater than that of the water, the egg will fall to the bottom of the jar. Then, dissolve salt in the water, stirring it so as to mix the fresh and salt water. The salt water will presently become denser than the egg and the egg will rise. Now, if fresh water is poured in until the egg and the water have the same density, the egg will remain in any position in which it may be placed below the surface of the water when the latter is perfectly at rest.

19. Floating Bodies.—A body lighter than an equal volume of a liquid rises to the surface when immersed in the liquid, and floats. For equilibrium, the buoyant effort represented by B must be just equal to the weight G of the body. But since B is the weight of the liquid displaced, the following principle results: *The weight of the liquid displaced by a floating body is equal to the weight of the body.*

The depth at which a body floats in a liquid depends on the relative weights of equal volumes of the body and the liquid. If the body is nearly as heavy as the liquid, it will sink until it displaces nearly its own volume; if very light compared with the liquid, the larger part of the body will be above the liquid surface. For example, ice has a specific gravity of about .9; hence, about one-tenth of an iceberg appears above the surface of the water and nine-tenths is submerged. The specific gravity of pine is about

.5; hence, about one-half of a pine log is submerged and one-half is above water.

The following examples show applications of the principles given in the preceding paragraphs:

EXAMPLE 1.—Water-tight canvas air bags are used for raising sunken ships. They are sunk when collapsed, attached to the ship by divers, and then filled with air by means of pumps from above. (*a*) If the capacity of a bag is 200 cubic feet, what is its buoyant effort? (*b*) How many bags will be required to lift 600 tons?

SOLUTION.—The weight of the bag and enclosed air may be neglected. (*a*) The buoyant effort of one bag is the weight of water displaced by the full bag, that is, $200 \times 62.5 = 12{,}500$ lb. Ans.

(*b*) To raise 600 tons, therefore, $\frac{600 \times 2{,}000}{12{,}500} = 96$ bags are required. Ans.

EXAMPLE 2.—A cast-iron cylinder is 14 inches long and 8 inches in diameter on the outside; it is closed at the ends and the metal is $\frac{1}{2}$ inch thick throughout. Will the cylinder float or sink in water?

SOLUTION.—The volume of the entire cylinder is $.7854 \times 8^2 \times 14 = 703.72$ cu. in. The hollow part has a length of 13 in. and a diameter of 7 in.; its volume is therefore $.7854 \times 7^2 \times 13 = 500.3$ cu. in. The volume of metal is therefore $703.72 - 500.3 = 203.42$ cu. in. Taking the weight of cast iron as 450 lb. per cu. ft., the weight of the cylinder is $\frac{203.42}{1{,}728} \times 450 = 52.973$ lb. If immersed, the cylinder displaces 703.72 cu. in. of water, which weighs $\frac{703.72}{1{,}728} \times 62.5 = 25.453$ lb. The buoyant effort being less than the weight, the cylinder will sink.

SPECIFIC GRAVITY

20. Specific Gravity of Solids.—The specific gravity of a body has been defined as the ratio between the weight of the body and the weight of an equal volume of water.

Archimedes's principle affords an easy and accurate method of finding the specific gravity of solids not easily soluble in water. The body is weighed first in air, then in water, suspended by a string from a scale pan, thus taking the place of the two cylinders shown in Fig. 17. *The difference between the two weights will be the weight of an equal volume of water. The ratio of the weight in air to the difference thus found will*

be the specific gravity. The abbreviation for specific gravity is Sp. Gr.

Let G = weight of solid in air;

G' = weight in water;

$G - G'$ = weight of a volume of water equal to volume of solid.

Then,
$$\text{Sp. Gr.} = \frac{G}{G - G'} \qquad (1)$$

EXAMPLE 1.—A body in air weighs $36\frac{1}{4}$ ounces and in water 30 ounces; what is its specific gravity?

SOLUTION.—Substituting in formula **1**,
$$\text{Sp. Gr.} = \frac{36\frac{1}{4}}{36\frac{1}{4} - 30} = \frac{36\frac{1}{4}}{6\frac{1}{4}} = 5.8. \text{ Ans.}$$

If the body is lighter than water, a piece of iron or other substance sufficiently heavy to sink both must be attached to it. *Then, weigh both bodies in air and both in water. Weigh each separately in air, and weigh the heavier body in water. Subtract the weights of the bodies in air and in water, and the result will be the weight of a volume of the water equal to the volume of the two bodies. Find the difference of the weights of the heavy body in air and in water, and the result will be the weight of a volume of water equal to the volume of the heavy body. Subtract this last result from the former, and the result will be the weight of a volume of water equal to the volume of the light body. The weight of the light body in air divided by the weight of an equal volume of water is the specific gravity of the light body.*

Let G = weight of both bodies in air;

G' = weight of both bodies in water;

G_1 = weight of light body in air;

G_2 = weight of heavy body in air;

G_3 = weight of heavy body in water.

Then, the specific gravity of the light body is given by
$$\text{Sp. Gr.} = \frac{G_1}{(G - G') - (G_2 - G_3)} \qquad (2)$$

EXAMPLE 2.—A piece of cork weighs 4.8 ounces in air; a piece of cast iron weighs 36 ounces in air and 31 ounces in water; the weight of the iron and cork together in water is 15.8 ounces. (*a*) What is the specific gravity of the cork? (*b*) Of the cast iron?

SOLUTION.—(*a*) Substituting in formula **2** the values given,

$$\text{Sp. Gr.} = \frac{4.8}{(40.8 - 15.8) - (36 - 31)} = \frac{4.8}{20} = .24$$

the specific gravity of the cork. Ans.

(*b*) By formula **1**, Sp. Gr. $= \frac{G}{G - G'} = \frac{36}{36 - 31} = 7.2$, the specific gravity of the iron. Ans.

21. Specific Gravity of Liquids.—To find the specific gravity of a liquid:

Weigh an empty flask; fill it with water, then weigh it again and find the difference between the two results; this difference will equal the weight of the water. Then weigh the flask filled with the liquid, and subtract the weight of the flask; the result is the weight of a volume of the liquid equal to the volume of the water. The weight of the liquid divided by the weight of the water is the specific gravity of the liquid.

Let G = weight of the flask and liquid;
G_1 = weight of the flask and water;
G_2 = weight of the flask.

Then,

$$\text{Sp. Gr.} = \frac{G - G_2}{G_1 - G_2}$$

EXAMPLE.—If the weight of the flask is 8 ounces, the weight when filled with water is 33 ounces, and when filled with alcohol 28 ounces, what is the specific gravity of the alcohol?

SOLUTION.—Substituting in the formula,

$$\text{Sp. Gr.} = \frac{28 - 8}{33 - 8} = .8. \quad \text{Ans.}$$

22. Specific Gravity of Gases.—The specific gravity of a gas is found by dividing the weight of a given volume of the gas by the weight of an equal volume of air or hydrogen. Air is usually taken as the standard for gases, but hydrogen is sometimes used. Water is the standard for liquids and solids. The specific gravity of gases is usually taken at a temperature of 32° F. The determination of the weight of gases being a very difficult matter, the method of finding the specific gravity of gases will not be described.

23. Hydrometers.—Instruments called **hydrometers** are in general use for determining quickly and accurately the specific gravities of liquids and some forms of solids.

They are of two kinds: (1) hydrometers of constant weight, as **Beaume's**; (2) hydrometers of constant volume, as **Nicholson's**.

A hydrometer of constant weight is shown in Fig. 18. It consists of a glass tube near the bottom of which are two bulbs. The lower and smaller bulb is loaded with mercury or shot, so as to cause the instrument to remain in a vertical position when placed in the liquid. The upper bulb is filled with air, and its volume is such that the whole instrument is lighter than an equal volume of water.

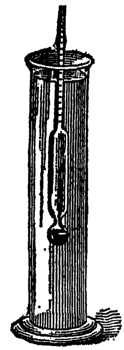


FIG. 18

The point to which the hydrometer sinks when placed in water is usually marked, the tube being graduated above and below the mark in such a manner that the specific gravity of the liquid can be read directly. It is customary to have two instruments, one with the zero point near the top of the stem for use with liquids heavier than water, and the other with the zero point near the bulb for use with liquids lighter than water.

These instruments are more commonly used for determining the degree of concentration or dilution of certain liquids,

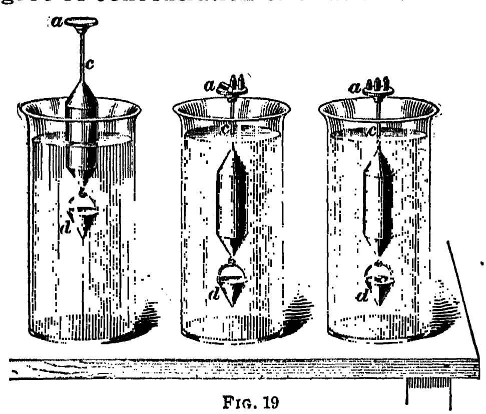


FIG. 19

as acids, alcohol, milk, solutions of sugar, etc., rather than their actual specific gravities. They are then known as *acidimeters*, *alcoholometers*, *lactometers*, *saccharimeters*, etc. according to the use to which they are put.

Nicholson's hydrometer is shown in Fig. 19. It consists of a hollow cylinder carrying at its lower end a basket d heavy enough to keep the apparatus upright when placed in water. At the top of the cylinder is a vertical rod, to which is attached a shallow pan a for holding weights, etc. The cylinder and its basket together must be so much lighter than water that a certain weight G must be placed in the pan in order to sink the apparatus to a given point c on the rod. The body whose specific gravity is to be found must weigh less than G. It is placed in the pan a, and enough weight G_1 is added to sink the point c to the water level. It is evident that the weight of the given body is $G - G_1$. The body is now removed from the pan a and placed in the basket d, an additional weight being added to sink the point c to the water level. Represent the weight now in the pan by G_2. The difference $G_2 - G_1$ is the weight of a volume of water equal to the volume of the body. Hence,

$$\text{Sp. Gr.} = \frac{G - G_1}{G_2 - G_1}$$

EXAMPLE.—The weight necessary to sink the hydrometer to the point c is 16 ounces; the weight necessary when the body is in the pan a is 7.3 ounces, and when the body is in the basket d, 10 ounces; what is the specific gravity of the body?

SOLUTION.—By the above formula, Sp. Gr. $= \frac{16 - 7.3}{10 - 7.3} = \frac{8.7}{2.7} = 3.222.$ Ans.

24. Volume of Irregularly Shaped Bodies.—Archimedes's principle gives a very easy and accurate method of finding the volume of an irregularly shaped body. Thus, subtract its weight in water from its weight in air, and the difference will be the weight of an equal volume of water. Divide this weight by .03617, and the result will be the volume in cubic inches; or divide by 62.5, and the result will be the volume in cubic feet.

If the specific gravity of the body is known, its cubic contents can be found by dividing its weight by its specific gravity, and then dividing again by either .03617 or 62.5 to find the volume either in cubic inches or in cubic feet, as may be desired.

EXAMPLE.—A certain body has a specific gravity of 4.38 and weighs 76 pounds; how many cubic inches are there in the body?

SOLUTION.— $\dfrac{76}{4.38 \times .03617} = 479.72$ **cu. in. Ans.**

Since the weight of a cubic foot of water varies at different temperatures, and with the amount of impurities it contains, it is necessary to have some standard when getting the specific gravity. This standard is pure distilled water at its maximum density, which occurs at a temperature of 39.1° F. Owing to the difficulty of maintaining the low temperature, however, other temperatures, as 60° or 62° F., are frequently used in practice, while in some instances a temperature of 32° F. is used. At 39.1° F. pure water weighs 62.425 pounds per cubic foot; but for ordinary calculations it is customary to take it as weighing 1,000 ounces, or 62.5 pounds, per cubic foot.

EXAMPLES FOR PRACTICE

1. If a certain quantity of red lead weighs 5 pounds in air, and 4.441 pounds in water, what is its specific gravity? Ans. 8.94+

2. A piece of iron weighs 1 pound in air and .861 pound in water. A piece of wood weighing 1 pound in air is attached to the iron and together they weigh .2 pound in the water. What is the specific gravity (*a*) of the iron? (*b*) of the wood? Ans. $\begin{cases} (a) & 7.194 \\ (b) & .602 \end{cases}$

3. An empty flask weighed 13 ounces; when filled with water, it weighed 22 ounces; and when filled with sulphuric acid, 29.56 ounces. What was the specific gravity of the acid? Ans. 1.84

4. How many cubic feet of brick, having a specific gravity of 1.9, are required to weigh 260 pounds? Ans. 2.19 cu. ft., nearly

CAPILLARITY

25. Capillarity is the mutual attraction or repulsion between a liquid and a solid that causes the fluid in contact with the solid to rise above or sink below the level of the surrounding liquid. If a clean glass rod is placed vertically in water, the water will be drawn up around the rod, as shown at *a* in Fig. 20. If the same rod is placed in mercury, the liquid will be depressed instead of raised. On examination, it will be found that water wets the glass, while mercury does not. If the rod is greased and placed in water, the surface of the water will be depressed about the rod. If a

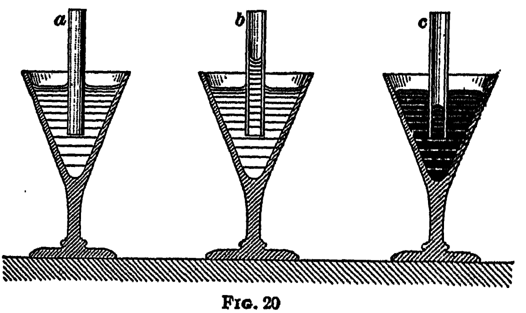


FIG. 20

clean lead or zinc strip is placed in mercury, the surface of the mercury will be raised about the strip. The greased rod when removed from the water will be dry, no water adhering to it, while the mercury will adhere to the lead or zinc strip, which will be wet.

In general, *all liquids that will wet the solids placed in them will be lifted, while those that do not wet them will be pushed down.*

These phenomena are due to a force known as capillarity, because they are best shown in very fine or hair-like tubes called *capillary tubes*. When the surface of the liquid rises at the surface of the rod or tube, the force causing the

phenomenon is called **capillary attraction,** and when the surface of the liquid is depressed the force is called **capillary repulsion.** At *b*, Fig. 20, is shown a glass tube with one end immersed in water, and at *c* is shown a glass tube immersed in mercury. The surface of the water in the tube *b* is concave, while the surface of the mercury in the tube *c* is convex.

The amount to which a liquid will ascend or be depressed, varies inversely as the diameter of the tube. Thus, water will rise twice as far in a tube $\frac{1}{32}$ inch in diameter as in one $\frac{1}{16}$ inch in diameter.

There are many illustrations of capillary action. It causes the oil to rise between the fibers of a lamp wick to the place of combustion. It enables cloth and sponges to take up moisture, and causes blotting paper to absorb ink. When paper is sized, however, so that its pores are filled, the ink dries by evaporation.

PNEUMATICS

PROPERTIES OF AIR AND GASES

ATMOSPHERIC PRESSURE

1. Pneumatics is the branch of mechanics that treats of the properties of gases.

2. Weight of Air.—As water is the most common of liquids, so air is the most common of gases. It was supposed by the ancients that air was imponderable—that is, that it weighed nothing—and it was not until about the year 1650 that it was proved that air really had weight. The ratio of the weight of a volume of air at 60° F., under atmospheric pressure, to that of an equal volume of water under the same conditions, is about 1 : 816; that is, under these conditions, air is only about $\frac{1}{816}$ as heavy as water. If a body is immersed in water and weighs less than the volume of water displaced, the body will rise and extend partly out of the water. The same is true to a certain extent of air. If a vessel made of light material is filled with a gas lighter than air and the total weight of vessel and gas together is less than the weight of the volume of air that they displace, the vessel will rise; the construction and operation of balloons are based on this principle.

3. Mercury Column Equivalent to Atmospheric Pressure.—Since air has weight, it is evident that the enormous quantity of air that constitutes the atmosphere must exert a considerable pressure on the earth. This is easily proved by taking a long glass tube, closed at one end, and filling it with mercury. If the finger is placed over the open

end, so as to keep the mercury from running out, and the tube is inverted and placed in a cup of mercury, as shown in Fig. 1, the mercury will fall, then rise, and after a few oscillations will come to rest at a height of 29.92, or roughly 30, inches above the top of the mercury in the cup. The height will always be the same under the same atmospheric conditions, allowance being made for the effects of capillary attraction.

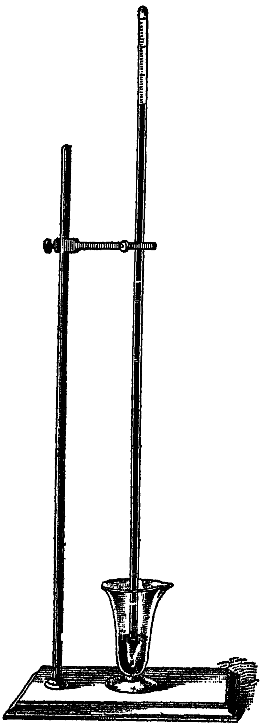


FIG. 1

Now, if the atmosphere has weight, it must press on every square unit of the surface of the mercury in the cup with equal intensity, except on that part of the surface covered by the tube. According to Pascal's law, which is given in *Hydrostatics*, this pressure is transmitted equally in all directions. There being nothing in the tube, except the mercury, to counterbalance the upward pressure of the air, the mercury falls in the tube until it exerts a downward pressure on the upper surface of the mercury in the cup sufficiently great to counterbalance the upward pressure produced by the atmosphere. In order that there shall be equilibrium, the pressure of the air per unit of area on the upper surface of the mercury in the cup must be equal to the pressure exerted per unit of area by the mercury inside the tube at the level of the surface of the mercury in the cup.

Suppose that the area of the inside of the tube is 1 square inch; then, since mercury is 13.59 times as heavy as water, and water weighs .03613 pound per cubic inch, the weight of the mercurial column is $.03613 \times 13.59 \times 29.92 = 14.69$ pounds. More accurate determinations make the average value at sea level 14.696 pounds per square inch at a temperature of 32° F.

Since this weight, exerted on 1 square inch of the liquid in the cup, just produces equilibrium, it is plain that the pressure of the outside air is 14.696 pounds on every square inch of surface. In engineering practice, however, a value of 14.7 is generally used. Since a column of mercury 29.92 inches high corresponds to a pressure of 14.696 pounds per square inch, a height of 1 inch corresponds to a pressure of $\frac{14.696}{29.92}$ $= .4911$ pound per square inch. For ordinary calculations, this value is taken at .49 pound.

4. **Vacuum.**—The space between the upper end of the tube and the upper surface of the mercury inside the tube is absolutely devoid of pressure and contains no substance, solid, liquid, or gaseous. This total absence of pressure constitutes a perfect **vacuum.** If there were a gas of some kind in this space, no matter how small its quantity might be, it would expand, filling the space, and its pressure would cause the column of mercury to fall still lower and become shorter, according to the amount of gas present; a **partial vacuum** would then exist in this space. That is, there would be only a partial absence of pressure. If the mercury falls 1 inch, so that the column is only 29 inches high, it is said, in ordinary language, that there is a vacuum of 29 inches. If it falls 8 inches, it is said that there is a vacuum of 22 inches. If it falls 16 inches, it is said that there is a vacuum of 14 inches, and so on. Suppose that the vacuum gauge of a condensing engine shows a vacuum of 26 inches; this indicates that there is enough air in the condenser to depress the mercury column $30 - 26 = 4$ inches, and to produce a pressure of $\frac{4}{30} \times 14.7 = 1.96$ pounds per square inch inside the condenser. Written as a formula, this would be

$$p = \frac{14.7 \times (30 - r)}{30}$$

in which p = absolute pressure in condenser, in pounds per square inch;

r = reading of vacuum gauge, in inches of mercury.

In all cases where the mercury column is used to measure a vacuum, the height of the column, in inches, gives the number of inches of vacuum. Thus, if the column is 5 inches high, or the vacuum gauge reads 5 inches, the vacuum is 5 inches. That is, the difference between the pressure in the condenser and the pressure of the atmosphere outside the condenser is equivalent to 5 inches of mercury. The height of the column of mercury is therefore a measure of the difference of pressures existing inside and outside the tube.

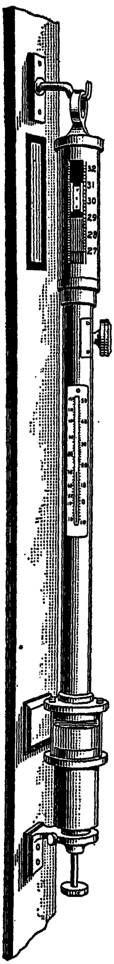


FIG. 2

5. Water Column Equivalent to Atmospheric Pressure.—If the tube had been filled with water instead of mercury, the height of the column of water necessary to balance the pressure of the atmosphere would have been $30 \times 13.59 = 407.7$ inches $= 33.975$ feet, generally taken in practice as 34 feet. This means that if a tube is filled with water, inverted, and placed in a dish of water in the same way as in the experiment made with the mercury, the resulting height of the column of water will be about 34 feet.

BAROMETERS

6. The **barometer** is an instrument used for measuring the pressure of the atmosphere. There are two kinds in general use—the *mercurial* and the *aneroid* The **mercurial barometer** is shown in Fig. 2. The principle is the same as in the case of the inverted tube shown in Fig. 1. The tube and cup at the bottom are protected by a brass or iron casing. At the top of the tube is a graduated scale that can be read to $\frac{1}{100}$ inch by means of a vernier. Attached to the casing is an accurate thermometer for determining the temperature of the outside air at the time the barometric observation is taken. This is necessary, since mercury expands when

the temperature is increased, and contracts when the temperature falls; for this reason, a standard temperature is assumed and all barometer readings are reduced to this temperature. This standard temperature is usually taken at 32° F., at which temperature the height of the column of mercury is about 30 inches. Another correction is made for the altitude of the place above sea level, and a third correction for the effects of capillary attraction.

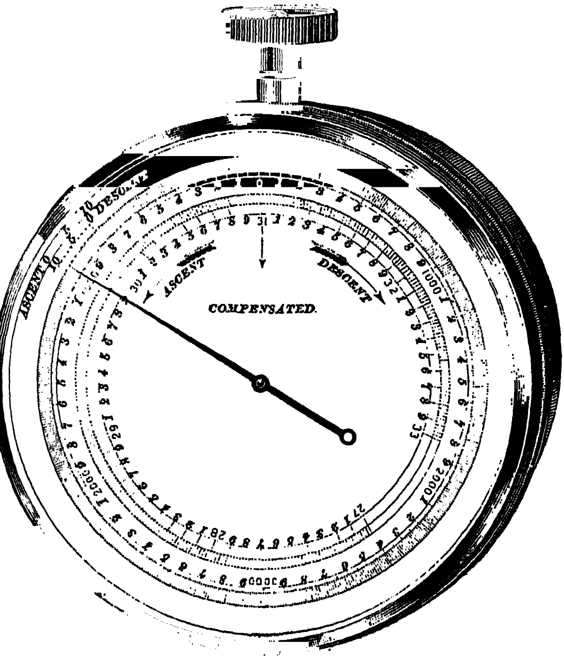


FIG. 3

7. An **aneroid barometer** is shown in Fig. 3. This instrument is made in various sizes, from the size of a large watch up to one with an 8- or 10-inch face. The barometer consists of a cylindrical box of metal, with a top of thin, elastic, corrugated metal. The air is removed from the box. When the atmospheric pressure increases, the top is pressed inwards; and when it is diminished the top is pressed outwards by its own elasticity aided by a spring beneath.

These movements of the cover are transmitted and multiplied by a combination of delicate levers that act on an index hand and cause it to move either to the right or to the left over a graduated scale. These barometers are self-correcting; that is, compensated for variations in temperature. They are portable, occupy but a small space, and are so delicate that they are said to show a difference in the atmospheric pressure when transferred from a table to the floor. They must be handled with care, as they are easily injured. The mercurial barometer is the standard.

It will be observed that in Fig. 3 the zero of the outer scale does not coincide with the 30 mark on the inner scale, as would seem consistent. The reason for placing the zero opposite the 31 division is thus explained. The average reading at sea level, or zero, is 29.92 inches. If the zero were set at 30, therefore, the reading would be on the scale part of the time, and off the scale the remainder of the time, according to the variation of pressure while working at sea level. Consequently, the zero is placed at the highest reading obtained at sea level, which is about 31 inches.

8. With air as with water, the lower the location the greater the pressure, and the higher the location the less the pressure. At the level of the sea, the height of the mercurial column is about 30 inches; at 5,000 feet above the sea, it is 24.7 inches; at 10,000 feet above the sea, it is 20.5 inches; at 15,000 feet above the sea, it is 16.9 inches; at 3 miles, it is 16.4 inches; and at 6 miles above the sea level it is 8.9 inches.

The heaviness also varies with the altitude; that is, a cubic foot of air at an elevation of 5,000 feet above sea level will not weigh as much as a cubic foot at sea level. This is proved conclusively by the fact that at a height of 3½ miles the mercurial column measures but 15 inches, indicating that half the weight of the entire atmosphere is below that. It is known that the height of the earth's atmosphere is at least 50 miles; hence, the air just before reaching the limit must be in an exceedingly rarefied state. It is by means

of barometers that great heights are measured. The aneroid barometer has the heights marked on the dial, so that they can be read directly. With the mercurial barometer, the heights must be calculated from the reading.

9. Atmospheric pressure is everywhere present and presses on all objects in all directions with equal intensity. If a book is laid on the table, the air presses on it in every direction with an average force of 14.7 pounds per square inch. It would seem as if it would take considerable force to raise a book from the table, since, if the size of the book were 8 inches by 5 inches, the pressure on it is $8 \times 5 \times 14.7 = 588$ pounds; but there is an equal pressure beneath the book to counteract the pressure on the top. It would now seem as if it would require a great force to open the book, since there are two pressures of 588 pounds each, acting in opposite directions, and tending to crush the book. This would be so but for the fact that there are layers of air between the leaves acting upwards and downwards with a pressure of 14.7 pounds per square inch.

If two metal plates are made as smooth and flat as it is possible to get them, and the face of one is laid on the face of the other, so that the air is almost entirely excluded from between them, it will take an immense force, compared with the weight of the plates, to separate them. This is because the full pressure of 14.7 pounds per square inch is then exerted on each of the plates with no counteracting equal pressure between them.

If a piece of flat glass be laid on a flat surface that has been previously moistened with water, it will require considerable force to separate them; this is because the water helps to fill up the pores in the flat surface and glass, and thus creates a partial vacuum between the glass and the surface, thereby reducing the counter pressure beneath the glass.

PROPERTIES OF GASES

10. Pressure of Gases.—According to modern and now generally accepted theories, a gas consists of molecules that are relatively far apart and are moving incessantly. The molecules in their motion frequently strike each other, and those near the walls of the vessel containing the gas frequently strike the walls and rebound. It is this continual striking of molecules against the containing walls that gives rise to what we call **pressure.**

Suppose that the cylinder, Fig. 4, contains any gas, as air. There is a definite number of molecules in the enclosed space, all moving to and fro. The number of molecules striking any portion of the containing wall per second remains on an average the same and produces a constant pressure that is the same at all points of the enclosing walls. If the piston be pushed into the dotted position, the molecules are crowded closer together and, provided that they move with the same average velocity as before, more of them must strike any given part of the enclosing walls, say 1 square inch, in a given interval of time than in the first case. This means that the pressure is greater in the second case than in the first.

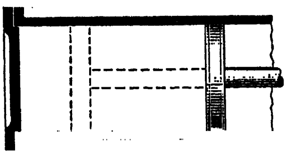


FIG. 4

Conversely, if the piston be moved to the right so as to increase the space filled by the molecules, the molecules will be farther apart along the enclosing walls and each square inch of wall will receive fewer impacts than before; thus the pressure per square inch will be less. In general, therefore, with a given number of molecules, or, what is the same thing, with a given weight of gas, the pressure is greater the smaller the space into which the gas is crowded.

11. Expansiveness of Gases.—No matter how large the enclosing vessel may be, the gas will fill it. Thus, if the cylinder in Fig. 4 is considered indefinitely long and the piston is moved a great distance to the right, the molecules

will still move in all parts of the enlarged space. They will, of course, be farther apart and will move greater distances before colliding with each other; and any part of the bounding wall will be struck less frequently.

If a bladder or a football, partly filled with air, is placed under a glass jar, called a **receiver**, and the air is then exhausted from the receiver, the bladder or football will immediately expand, as shown in Fig. 5. The pressure being removed from the outside of the bladder by exhausting the air from the receiver, the striking against the inner walls of the bladder by the molecules of contained air causes the bladder to distend.

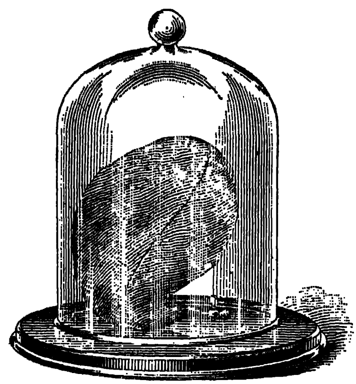


Fig. 5

12. Measurement of Pressure.—There are two ways of measuring the pressure of a gas; by means of an instrument called a **manometer**, and by means of a **gauge**. The manometer generally used is practically the same as a mercurial barometer, except that the tube is much longer, so that pressures equal to several atmospheres may be measured; and it is enlarged and bent into a **U** shape at the lower end. Both lower and upper ends are open, the lower end being connected to the vessel containing the gas whose pressure it is desired to measure.

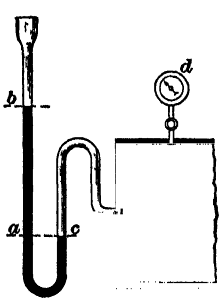


Fig. 6

The pressure is measured by the change of level of the mercury in the tube. In Fig. 6, suppose the pressure of the gas is such as to depress the level of the mercury to *c*, while in the other branch of the tube it rises to *b*. Now, the points *a* and *c* being at the same level, the pressure at those points must be equal. The pressure at *a* is due partly to

the weight of the column of mercury $a\,b$ above a and partly to the pressure of the atmosphere on the top of the column at b. In Art. **3**, it was shown that 1 inch of mercury corresponds to .49 pound per square inch; hence, if h denotes the difference in level, in inches, between the points b and a, .49 h is the pressure per square inch due to the weight of the column $a\,b$. Add to this the atmospheric pressure, 14.7 pounds per square inch, and the total pressure at a, and therefore at c, is $p = .49\,h + 14.7$. The pressure thus obtained is the pressure above vacuum, or the **absolute pressure.**

The ordinary pressure gauge d, Fig. 6, measures not the absolute pressure but the *excess* of the pressure of the gas over atmospheric pressure. The pressure thus measured is called the **gauge pressure.** Evidently, the column $a\,b$ measures the gauge pressure, for since $p = .49\,h + 14.7$, it follows that $.49\,h = p - 14.7$, which is the excess of pressure of the gas over atmospheric pressure.

In speaking of the pressure of a gas, the gauge pressure is usually meant; thus, to say the pressure of steam in a boiler is 70 pounds per square inch, means that that is the pressure shown by the gauge and is the excess of the steam pressure over the pressure of the atmosphere.

To obtain the absolute pressure when the gauge pressure is given, the atmospheric pressure is added. Thus, if the gauge pressure is 70 pounds per square inch, the absolute pressure is $70 + 14.7 = 84.7$ pounds per square inch. Conversely, the atmospheric pressure is subtracted from the absolute pressure to obtain the gauge pressure.

13. Units of Pressure.—Pressures are ordinarily expressed in *pounds per square inch*. Thus, in speaking of the pressure of steam in a boiler or steam cylinder, it is said to be 60, 80, or 100 pounds per square inch, as the case may be. Pressures may also be expressed in *pounds per square foot*. To reduce pounds per square inch to pounds per square foot, multiply by 144; thus, atmospheric pressure, which is 14.7 pounds per square inch, is $14.7 \times 144 = 2{,}116.8$

pounds per square foot. Conversely, pressures per square foot are divided by 144 to get pressures per square inch.

Small pressures, such as are produced by fans, are often measured in *ounces per square inch*, and sometimes in inches of water. It has been shown that a head of 1 foot of water gives a pressure of .434 pound per square inch, and hence 1 inch of water is $\frac{1}{12}$ of .434 = .0362 pound per square inch = 5.2 pounds per square foot.

Another unit of pressure is the *inch of mercury*, which equals .49 pound per square inch, as explained in Art. **3**. The relations between these five units of pressure are shown in Table I. The pressures given in any horizontal line are

TABLE I

RELATIVE UNIT PRESSURES

Pounds per Square Foot	Pounds per Square Inch	Ounces per Square Inch	Inches of Water	Inches of Mercury
1	$\frac{1}{144}$	$\frac{1}{9}$	.192	.0142
144	1	16	27.7	2.04
9	$\frac{1}{16}$	1	1.73	.13
5.2	.0362	.58	1	.074
70.56	.49	7.84	13.6	1
2,116.8	14.7	235.2	408 = 34 ft.	30

the same, using the units at the tops of the columns. Thus, 1 inch of water equals 5.2 pounds per square foot, equals .0362 pound per square inch, equals .58 ounce per square inch, etc. The last line gives atmospheric pressure expressed in the various units. These values deviate slightly in some cases from the theoretically exact values, but they are the values generally used in practical problems.

EXAMPLE.—The pressure of the forced draft of a marine boiler is 4.3 inches of water; what is the pressure expressed in the other units?

SOLUTION.— 1 in. of water = 5.2 lb. per sq. ft.; hence,

p = 4.3 × 5.2 = 22.36 lb. per sq. ft.
= 4.3 × .0362 = .1557 lb. per sq. in.
= 4.3 × .58 = 2.494 oz. per sq. in.
= 4.3 × .074 = .3182 in. of mercury. Ans.

14. Temperature.—With solids and liquids the **temperature**, or *hotness*, of a body has little or no influence on its behavior, and usually need not be taken into account. In the case of gases, however, the temperature exerts a marked influence and must always be considered.

Consider the cylinder, Fig. 4, to be filled with gas at some definite temperature as shown by a thermometer. Now, keeping the piston in the same position, let the gas be heated, say by a flame applied to the cylinder walls. The gas will grow hotter, that is, the temperature indicated by the thermometer will rise; and at the same time it will be found that the pressure of the gas, as shown by a gauge, will also rise. Now, since the volume of the gas has not changed, there will be the same number of molecules per cubic inch as before. But since the pressure has risen, the number of impacts in a second must have increased, which means that the molecules are moving at a greater speed than before.

According to the modern theory of heat, the temperature depends directly on the speed with which the molecules of a body are moving. When the temperature of a body rises, its molecules move faster; when the temperature falls, the molecules move more slowly. This subject will be treated more fully in *Heat*, Part 1.

15. Absolute Temperature.—Suppose that the gas in the cylinder shown in Fig. 4 is at the temperature of melting ice, 32° F., and has some definite pressure denoted by p. If the gas is heated, so that its temperature rises 1°, that is, to 33° F., the pressure will increase, provided that the piston is fixed so that the gas cannot change in volume. It has been found, by experiment, that the change of pressure is $\frac{1}{492}$ of the pressure at 32°; therefore the pressure at 33° is $p + \frac{1}{492}p = p\left(1 + \frac{1}{492}\right)$. If the temperature rises another degree, that is, to 34° F., the pressure increases by the same amount, $\frac{1}{492}p$, and at 34° the pressure is therefore $p + \frac{2}{492}p$.

Suppose that the pressure at 32° is 100 pounds per square inch; then the rise of pressure for a rise in temperature of 1° F. is $100 \times \frac{1}{492} = .20325$ pound per square inch. If the temperature of the gas is raised to 212°, the boiling point of water, the rise in temperature is $212° - 32° = 180°$, the change in pressure is $.20325 \times 180 = 36.585$ pounds per square inch, and the new pressure is $100 + 36.585 = 136.58$ pounds per square inch.

The change of pressure during a change of temperature may be represented graphically, as shown in Fig. 7. Through some point *a*, a vertical line is drawn and on it are marked points *b*, *c*, *d*, etc., corresponding to the temperatures shown. The vertical line thus corresponds to a thermometer scale. From the point *b*, which corresponds to 32°, the horizontal distance *be* is laid off to represent a pressure of 100 pounds per square inch, and from *d*, which represents a temperature of 212°, *df* is laid off, representing to the same scale the new pressure of 136.58 pounds per square inch.

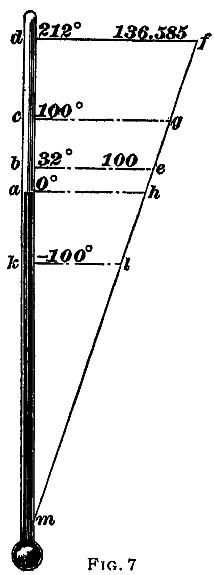


FIG. 7

Let *f* and *e* be joined by a straight line; then the horizontal distance between *bd* and *ef* at any point gives the pressure for the corresponding temperature. The point *c*, for example, corresponds to 100° and *cg* represents the pressure at 100° to the same scale that *be* represents 100 pounds per square inch.

If the temperature is lowered below 32° the pressure decreases at the same rate; that is, for each degree it decreases $\frac{1}{492}$ of the pressure at 32°. Hence, if the line *fe* is prolonged, the horizontal lines below *be* give the pressures for temperatures below 32°. Thus, *ah* represents the pressure at 0° and *kl* that at −100°, that is, 100° below 0°.

As the temperature lowers, the horizontal distances between the lines grow smaller, and at *m*, where the lines intersect, the

distance becomes zero. This means that if in some way the temperature could be lowered to that represented by the point m, the gas would exert no pressure on the enclosing walls. The molecules would be motionless and would no longer strike the walls, and the gas would be entirely without heat. The temperature corresponding to m is called the **absolute zero of temperature,** and temperatures reckoned from this zero as a starting point are called **absolute temperatures.**

Now let the position of the point m be determined. Starting at 32°, the decrease of pressure per degree is $\frac{1}{492}$ of the pressure at 32°. For a cooling of 10°, the decrease is $\frac{10}{492}$; for a cooling of 100°, $\frac{100}{492}$; and so on. Let the temperature be lowered 492° below 32°; then the decrease in pressure is $\frac{492}{492}$ of the pressure at 32°, or the whole of the original pressure. Hence, the gas exerts no pressure at 492° below 32° F., and this temperature is the absolute zero indicated by the point m.

Since from b to m, Fig. 7, is 492° and from b to a is 32°, m must be $492° - 32° = 460°$ below 0°. Hence, when the Fahrenheit scale is used, the absolute zero is 460° below the ordinary zero, and 460 must be added to the ordinary temperature to obtain the corresponding absolute temperature.

Let t = ordinary temperature as given by Fahrenheit thermometer;

T = corresponding absolute temperature.

Then, $$T = t + 460°$$

and $$t = T - 460°$$

The temperature at which water boils is 212°. The corresponding absolute temperature is $212° + 460° = 672°$. The ordinary temperature corresponding to an absolute temperature of 900° is $900° - 460° = 440°$. When the word temperature is used alone, the ordinary temperature is meant.

16. Gases and Vapors.—When liquids are heated to sufficiently high temperatures, they are converted into gases. When water, for example, is heated, it boils, giving off steam. If the heating is continued long enough, all the water will be

evaporated. Steam is simply a gas formed by boiling wat The water, however, does not change immediately into perfect gas. It first takes the form of **vapor,** in which state it is readily condensed. As the heating is continued, it takes a more stable gaseous form, and eventually becomes a perfect gas.

LAWS RELATING TO CHANGE OF STATE

17. The **state** of a gas is defined by three things: pressure, volume, temperature. Suppose a given quantity of gas has a definite pressure, volume, and temperature. If the volume is changed either by compressing the gas or by permitting it to expand, either the pressure or the temperature or both will change at the same time. The change from the original to the new pressure, volume, and temperature is called a *change of state.* If p_1, v_1, and t_1 denote the original, and p_2, v_2, and t_2 the final pressure, volume, and temperature, respectively, the gas is said to change from the state p_1, v_1, t_1 to the state p_2, v_2, t_2. The laws governing these changes of state are here given and explained.

BOYLE'S LAW

18. Pressure and Volume of a Gas.—*If the temperature of a given quantity of gas remains the same and the volume is changed, the pressure varies inversely as the volume.*

This is known as **Boyle's law.** If the piston, Fig. 4, be moved to the left until the gas occupies one-half of its original volume, double the number of molecules will strike a square inch of the wall in 1 second, provided that they have the same speed as at first; that is, provided that the gas remains at the same temperature; but this means that the new pressure is double the original pressure. If the piston is moved still farther until the new volume is one-third of the original volume, the new pressure will be three times the original pressure. If, on the other hand, the piston is moved to the right until the new volume is three times the original volume, the new pressure is one-third of the original

pressure; and so on. This relation may also be expressed as follows: *The volume of a given quantity of gas at constant temperature varies inversely as the pressure.*

Suppose 3 cubic feet of air to be under a pressure of 60 pounds per square inch in a cylinder fitted with a movable piston; then, the product of the volume and pressure is $3 \times 60 = 180$. Let the pressure be decreased to 30 pounds per square inch; then the volume will be 6 cubic feet, and $30 \times 6 = 180$, as before. Let the pressure be decreased to $7\frac{1}{2}$ pounds per square inch, the volume will then be increased to $60 \div 7\frac{1}{2} = 8$ times the original, or $8 \times 3 = 24$ cubic feet, and $24 \times 7\frac{1}{2} = 180$, as in the two preceding cases. It will now be noticed that if a gas is enclosed within a confined space and allowed to expand without change of temperature, *the product of the pressure and the corresponding volume for one position of the piston is the same as for any other position of the piston.* If the piston were to compress the air, the same result would be obtained.

Let p_0 and v_0 be the original pressure and volume of gas, and p_1 and v_1 any other pressure and corresponding volume. Then, if the temperature does not change,

$$p_0 v_0 = p_1 v_1 = \text{a constant quantity} \qquad (1)$$

In general, if p_1, v_1 and p_2, v_2 denote the pressures and volumes for any two states,

$$p_1 v_1 = p_2 v_2 \qquad (2)$$

19. Graphic Representation of Boyle's Law.—The change of state of a gas according to Boyle's law may be represented graphically, as shown in Fig. 8. On cross-section paper take two section lines OP and OV intersecting at O; let the spaces along OV represent volumes, and let those along OP represent pressures. In the figure, one space on OV represents 1 cubic foot, and on OP a pressure of 5 pounds per square inch. Where each horizontal line cuts OP is marked the pressure, as *5*, *10*, *15*, etc., and on the intersections of the vertical lines with OV are marked the proper volumes represented by the lines, as *2*, *4*, *6*, etc.

Suppose, now, that the gas in its original state has a pressure of 50 pounds per square inch and a volume of 2 cubic feet. This state is represented by the point A, opposite *50*, on OP, and above *2* on OV. The product of the pressure and volume is $50 \times 2 = 100$, and this is the constant quantity of formula **1** of Art. **18.** As the volume increases to 3, 4, 5, etc. cubic feet, the corresponding pressures are found as follows: Let p_0 and v_0 be the initial pressure and volume, and p and v the pressure and volume at any other point. Then $pv = p_0 v_0 = 50 \times 2 = 100$.

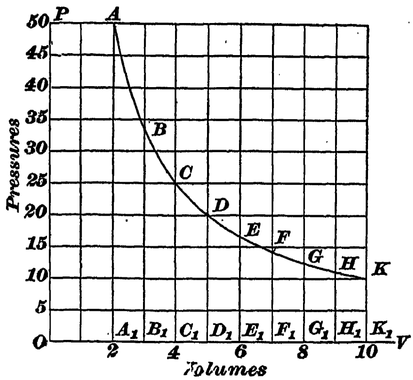


FIG. 8

For $v = 3$, $p \times 3 = 100$, or $p = \frac{100}{3} = 33\frac{1}{3}$ lb. per sq. in.
For $v = 4$, $p \times 4 = 100$, or $p = \frac{100}{4} = 25$ lb. per sq. in.
For $v = 5$, $p \times 5 = 100$, or $p = \frac{100}{5} = 20$ lb. per sq. in.
For $v = 6$, $p \times 6 = 100$, or $p = \frac{100}{6} = 16\frac{2}{3}$ lb. per sq. in.
For $v = 7$, $p \times 7 = 100$, or $p = \frac{100}{7} = 14\frac{2}{7}$ lb. per sq. in.
For $v = 8$, $p \times 8 = 100$, or $p = \frac{100}{8} = 12\frac{1}{2}$ lb. per sq. in.
For $v = 9$, $p \times 9 = 100$, or $p = \frac{100}{9} = 11\frac{1}{9}$ lb. per sq. in.
For $v = 10$, $p \times 10 = 100$, or $p = \frac{100}{10} = 10$ lb. per sq. in.

These different states are represented by the points B, C, D, etc.; thus, for $v = 5$ and $p = 20$, find the intersection of the section line through *20* on OP and that through *5* on OV; this is the point D. A curve drawn through these points, as shown, will represent the pressures and volumes at any positions.

In problems that involve Boyle's law and in which formula **1** of Art. **18** is used, the pressures must be absolute, not gauge, pressures. Throughout the remainder of this Section, all pressures will be understood as absolute pressures unless the contrary is distinctly stated.

20. The following examples illustrate the use of Boyle's law, as expressed in formula **1** of Art. **18.**

EXAMPLE 1.—If 10 cubic feet of air at atmospheric pressure, 14.7 pounds per square inch, is compressed at constant temperature until the gauge pressure reaches 36 pounds per square inch, what is the new volume?

SOLUTION.—The original pressure p_0 is 14.7, the volume v_0 is 10, and the final absolute pressure p is $36 + 14.7 = 50.7$. From the formula $p\,v = p_0\,v_0$,

$$v = \frac{p_0\,v_0}{p} = \frac{14.7 \times 10}{50.7} = 2.9 \text{ cu. ft., nearly. Ans.}$$

EXAMPLE 2.—An engine uses compressed air. When the flow of air to the cylinder is stopped, the volume of air in the cylinder is .84 cubic foot and the pressure is 60 pounds gauge; when the piston reaches the end of its stroke the air has expanded to a volume of 1.35 cubic feet. If the temperature remains constant, what is the gauge pressure of the air at the end of the expansion?

SOLUTION.—The initial absolute pressure p_0 is $60 + 14.7 = 74.7$, $v_0 = .84$, and $v_1 = 1.35$. From formula **1** of Art. **18**, $p_0\,v_0 = p_1\,v_1$, or

$$p_1 = \frac{p_0\,v_0}{v_1} = \frac{74.7 \times .84}{1.35} = 46.48 \text{ lb. per sq. in.}$$

This is the absolute pressure; hence the gauge pressure is $46.48 - 14.7 = 31.78$ lb. per sq. in. Ans.

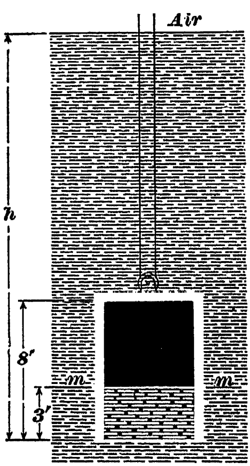


FIG. 9

EXAMPLE 3.—A cylindrical diving-bell 8 feet high, shown in Fig. 9, is lowered into sea-water until the water rises in the bell to a height of 3 feet above the bottom edge of the bell; what is the depth of the bottom of the bell below the surface of the water?

SOLUTION.—The head of water above the level $m\,m$ of the water in the bell is $(h - 3)$ ft. The weight of a column of sea-water 1 ft. high and 1 sq. in. in area is .445; hence the pressure at $m\,m$ per square inch is $.445\,(h - 3) + 14.7$, the 14.7 being the atmospheric pressure on the surface, which is transmitted without loss. Now, the original pressure of air in the bell was 14.7 lb. per sq. in., and denoting by A the area in square feet of the horizontal circular section of the bell, the original volume was $8\,A$ cu. ft. The air is now compressed so that it occupies only $(8 - 3)\,A$ cu. ft.

$= 5A$ cu. ft. From formula **1** of Art. **18**, the pressure is

$$p_1 = \frac{p_0 v_0}{v_1} = \frac{14.7 \times 8A}{5A} = 23.52 \text{ lb. per sq. in.}$$

Since the pressure of the air on the surface mm inside the bell mu. just balance the upward pressure of the water, $.445(h - 3) + 14.7 = 23.52$, and

$$h = \frac{23.52 - 14.7}{.445} + 3 = 22.82 \text{ ft. Ans.}$$

GAY-LUSSAC'S LAW

21. Pressure and Temperature of Gas.—The **law of Gay-Lussac,** or of **Charles,** is essentially that stated in Art. **15** and illustrated in Fig. 7. *If the volume of a gas remains constant, the increase of pressure per degree rise in temperature is $\frac{1}{492}$ of the pressure at the temperature of melting ice, that is, 32° F.*

Let p_0 = pressure at 32° F.;
p = pressure at some other temperature t° F.;
$T = t + 460$, absolute temperature corresponding to temperature t;
$T_0 = 32 + 460 = 492$, absolute temperature of melting ice.

In raising the temperature from 32° to t°, the change is $(t - 32)$ degrees, and since for each degree the increase of pressure is $\frac{1}{492}$ of p_0, the increase for $(t - 32)$ degrees is $\frac{t-32}{492}$ of p_0; hence, $p = p_0 + \frac{t-32}{492}p_0 = p_0\left(1 + \frac{t-32}{492}\right)$. Now, $1 + \frac{t-32}{492} = \frac{492 + t - 32}{492} = \frac{t + 460}{492} = \frac{T}{T_0}$, and $p = p_0\frac{T}{T_0}$, or,

$$\frac{p}{p_0} = \frac{T}{T_0} \qquad (1)$$

Let p_1 and T_1 equal pressure and absolute temperature for one state of a gas and p_2 and T_2 equal pressure and absolute temperature for a second state of the same gas; from the formula just given, $\frac{p_1}{p_0} = \frac{T_1}{T_0}$, also $\frac{p_2}{p_0} = \frac{T_2}{T_0}$. Dividing one equation by the other, member by member,

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} \qquad (2)$$

The following statement of Gay-Lussac's law follows from formulas **1** and **2**: *The volume remaining the same, the pressure of a gas varies directly as the absolute temperature.*

EXAMPLE 1.—A certain weight of air is heated from 60° to 144° at constant volume; if the initial absolute pressure was 20 pounds per square inch, what is the final pressure?

SOLUTION.— $p_1 = 20$, $T_1 = 60 + 460 = 520$, and $T_2 = 144 + 460 = 604$; using formula **2**, $\frac{20}{P_2} = \frac{520}{604}$, or

$$P_2 = \frac{20 \times 604}{520} = 23.23 \text{ lb. per sq. in., absolute.} \quad \text{Ans.}$$

EXAMPLE 2.—If 10 cubic feet of air has a pressure of 30 pounds per square inch, absolute, at a temperature of 70° F., to what temperature must it be raised in order that the final pressure may be 50 pounds per square inch, absolute, the volume being unchanged?

SOLUTION.—Here $p_1 = 30$, $p_2 = 50$, and $T_1 = 70 + 460 = 530$. From formula **2**, $\frac{30}{50} = \frac{530}{T_2}$, whence

$$T_2 = \frac{530 \times 50}{30} = 883\tfrac{1}{3}°. \quad t_2 = T_2 - 460° = 883\tfrac{1}{3}° - 460° = 423\tfrac{1}{3}° \text{ F.} \quad \text{Ans.}$$

COMBINATION OF BOYLE'S AND GAY-LUSSAC'S LAWS

22. Pressure, Volume, and Temperature of a Gas. Consider a certain weight of gas contained in a vessel, and in its first state *a*, Fig. 10, let the pressure, volume, and absolute temperature be p_1, v_1, and T_1, respectively. Now, keeping the temperature at T_1, let the pressure and volume change to p' and v_2, so that in the state *b* it is p', v_2, and T_1. Finally, let the state change from *b* to *c* during which the volume v_2 remains the same, but p' changes to p_2 and T_1 to T_2. Comparing state *c* with state *a*, it is seen that pressure, volume, and temperature have changed.

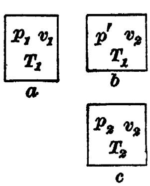


FIG. 10

There is a relation between the p_1, v_1, and T_1 of state *a* and the p_2, v_2, and T_2 of state *c*. In passing from state *a* to state *b*, the temperature remains constant and the change follows Boyle's law; hence, from formula **2**, Art. **18**,

$$p_1 v_1 = p' v_2$$

In passing from state b to state c, the volume is constant and the change is made according to Gay-Lussac's law; hence,

$$\frac{p'}{p_2} = \frac{T_1}{T_2}$$

From the first equation, $p' = \frac{p_1 v_1}{v_2}$; and from the second, $p' = p_2 \frac{T_1}{T_2}$. Placing these two values of p' equal to each other,

$$\frac{p_1 v_1}{v_2} = p_2 \frac{T_1}{T_2}$$

or,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

This formula is very important. It shows that with a given weight of a gas, the quotient obtained by dividing the product of the pressure and volume by the absolute temperature is the same for any state of the gas.

EXAMPLE.—A certain quantity of air has a volume of 40 cubic feet, a pressure of 30 pounds per square inch, absolute, and a temperature of 80° F.; the air expands until its volume is 56 cubic feet and the temperature falls to 40° F. What is the absolute pressure at the end of expansion?

SOLUTION.—In the first state, $p_1 = 30$, $v_1 = 40$, and $T_1 = 80 + 460 = 540$. In the second state, p_2 is unknown, $v_2 = 56$, and $T_2 = 40 + 460 = 500$. Using the formula given above,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}, \quad \frac{30 \times 40}{540} = \frac{p_2 \times 56}{500}$$

whence, $p_2 = \frac{30 \times 40 \times 500}{540 \times 56} = 19.84$ lb. per sq. in. Ans.

23. Expansion at Constant Pressure.—Let a gas change its state in such a way that the pressure remains the same; then, in the formula derived in Art. **22**, $p_1 = p_2$ and therefore

$$\frac{v_1}{T_1} = \frac{v_2}{T_2}, \text{ or } \frac{v_1}{v_2} = \frac{T_1}{T_2}$$

That is, *the pressure remaining the same, the volume of a given weight of gas varies directly as the absolute temperature.*

EXAMPLE.—A quantity of gas having a volume of 6.4 cubic feet is heated at constant pressure from 62° F. to 315° F.: what is the volume after heating?

SOLUTION.— $v_1 = 6.4$, v_2 is unknown, $T_1 = 62 + 460 = 522$, and $T_2 = 315 + 460 = 775$. From the above formula,

$$\frac{6.4}{v_2} = \frac{522}{775}, \text{ or } v_2 = \frac{6.4 \times 775}{522} = 9.5 \text{ cu. ft.} \quad \text{Ans.}$$

24. General Equations.—Consider a unit weight of gas, say 1 pound, and let v denote its volume in any given state. Then, by the formula in Art. 22,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3} = \frac{p v}{T}$$

where p_1, v_1, T_1, and p_2, v_2, T_2, etc. denote different states. The quantity $\frac{p v}{T}$ is therefore a constant; that is, it remains the same for all states of the pound of gas. Let R denote this constant quantity; then, $\frac{p v}{T} = R$, or $p v = R T$.

The value of R varies for different gases. For air, it may be determined if the pressure, volume, and temperature are known at some standard state. Let a pound of air be taken at atmospheric pressure and at the temperature of melting ice; the volume of 1 pound of air under these conditions is 12.387 cubic feet, the pressure is 14.696 pounds per square inch, and the absolute temperature is $460° + 32° = 492°$. Substituting these values in the formula above,

$$14.696 \times 12.387 = R \times 492, \text{ or } R = \frac{14.696 \times 12.387}{492} = .37$$

The exact values are used here because this value for R will be used in future problems. With this value of R, p in this formula must be expressed in pounds per square inch, and v in cubic feet.

EXAMPLE 1.—What is the volume of 1 pound of air at a pressure of 63 pounds per square inch absolute and a temperature of 80° F.?

SOLUTION.—The absolute temperature is $460 + 80 = 540$; from the formula $p v = R T$,

$$v = \frac{R T}{p} = \frac{.37 \times 540}{63} = 3.17 \text{ cu. ft.} \quad \text{Ans.}$$

EXAMPLE 2.—At what temperature will 1 pound of air occupy a volume of 16 cubic feet at a pressure of 22 pounds per square inch absolute?

SOLUTION.—Using the formula given above, $22 \times 16 = .3$ whence $T = \frac{22 \times 16}{.37} = 951.35°$. The ordinary temperature is there-fore $951.35° - 460° = 491.35°$. Ans.

25. If the weight of gas considered is not precisely 1 pound, the formula in Art. **24** is modified as follows:

Let G = weight of gas, in pounds;
v = volume of the G pounds, in cubic feet.

Then the volume of 1 pound is $\frac{v}{G}$, and this value takes the place of v in the formula in Art. **24.** Thus,

$$p\frac{v}{G} = R\,T, \text{ and } p\,v = G\,R\,T$$

EXAMPLE 1.—In an air compressor is 2.7 cubic feet of air at a pressure of 54 pounds per square inch gauge and a temperature of 210° F.; what is the weight of the air?

SOLUTION.—The absolute pressure is $54 + 14.7 = 68.7$, and the absolute temperature is $460° + 210° = 670°$. Substituting in the formula $p\,v = G\,R\,T$,

$$68.7 \times 2.7 = G \times .37 \times 670, \text{ or } G = \frac{68.7 \times 2.7}{.37 \times 670} = .748 \text{ lb. Ans.}$$

EXAMPLE 2.—A hot-air balloon has a capacity of 200 cubic feet; the temperature of the air inside is 280° F., and of that outside is 70° F. Find the weight of air in the balloon, the weight of air at 70° displaced by the balloon, and the lifting power of the balloon.

SOLUTION.—Both inside and outside the balloon the pressure is that of the atmosphere, 14.7 lb. From the formula, the weight of the hot air is

$$G = \frac{p\,v}{R\,T} = \frac{14.7 \times 200}{.37 \times (460 + 280)} = 10.738 \text{ pounds}$$

and the weight of the same volume of air at 70° is

$$G = \frac{14.7 \times 200}{.37 \times (460 + 70)} = 14.992 \text{ pounds}$$

The lifting power is the difference between these weights, or $14.992 - 10.738 = 4.254$ lb. Ans.

26. Heaviness.—The heaviness of a gas, that is, the weight of a cubic foot of it, is the reciprocal of the volume of 1 pound. For example, if 1 pound of air has a volume of 13 cubic feet, then the weight of 1 cubic foot is evidently

$\frac{1}{13}$ pound. Let H denote the heaviness, and v the volume of 1 pound, then

$$H = \frac{1}{v}, \text{ or } v = \frac{1}{H}$$

If this expression for v be substituted for the volume in the formula in Art. 22, the resulting equation is,

$$\frac{p_1 \frac{1}{H_1}}{T_1} = \frac{p_2 \frac{1}{H_2}}{T_2}, \text{ or } \frac{p_1}{T_1 H_1} = \frac{p_2}{T_2 H_2}$$

This may be written

$$\frac{H_1}{H_2} = \frac{p_1 T_2}{p_2 T_1} \qquad (1)$$

Let the temperature be constant; then, in formula 1, $T_2 = T_1$ and

$$\frac{H_1}{H_2} = \frac{p_1}{p_2} \qquad (2)$$

Let the pressure remain constant; then, in formula 1, $p_1 = p_2$ and

$$\frac{H_1}{H_2} = \frac{T_2}{T_1} \qquad (3)$$

Formulas 2 and 3 may be expressed, in words, as follows: *The heaviness of a given weight of gas varies directly as the absolute pressure when the temperature remains constant, and inversely as the absolute temperature when the pressure remains constant.*

Since $\frac{1}{v}$ expresses the heaviness when the weight of the gas is 1 pound, it is apparent that $\frac{G}{v}$ expresses the heaviness when the weight is greater or less than 1 pound, G being the weight of gas in pounds, and v the corresponding volume in cubic feet. Thus, for any states, $H_1 = \frac{G}{v_1}$, $H_2 = \frac{G}{v_2}$, etc.

Dividing, $H_1 \div H_2 = \frac{G}{v_1} \div \frac{G}{v_2}$.

$$\left.\begin{aligned} &\text{That is,} & \frac{H_1}{H_2} &= \frac{v_2}{v_1} \\ &\text{and} & v_1 H_1 &= v_2 H_2 \end{aligned}\right\} \qquad (4)$$

For any change of state, therefore, *the heaviness of a given quantity of gas is inversely as the volume.*

EXAMPLE 1.—The weight of 1 cubic foot of air at a temperature of 60° F. and under a pressure of one atmosphere, or 14.7 pounds per square inch, is .0764 pound; what will be the weight per cubic foot if the volume is compressed until the pressure is 73.5 pounds per square inch, or five atmospheres, the temperature still being 60° F.?

SOLUTION.—Applying formula **2**, $\frac{H_1}{H_2} = \frac{p_1}{p_2}$, $\frac{.0764}{H_2} = \frac{14.7}{73.5}$,

and, $H_2 = .0764 \times 5 = .382$ lb. per cu. ft. Ans.

EXAMPLE 2.—If 6.75 cubic feet of air, at a temperature of 60° F., and a pressure of one atmosphere is compressed to 2.25 cubic feet, the temperature remaining the same, what is the weight of 1 cubic foot of the compressed air?

SOLUTION.—From formula **4**, $v_1 H_1 = v_2 H_2$, $6.75 \times .0764 = 2.25 \times H_2$;

then, $$H_2 = \frac{6.75 \times .0764}{2.25} = .2292 \text{ lb. Ans.}$$

27. Homogeneous Formulas.—The formulas in the preceding articles, except those derived in Arts. **24** and **25**, are **homogeneous**; that is, they hold good whatever units are used for pressures and volumes. For volumes, cubic feet, cubic inches, or cubic meters may be used; for pressures, pounds per square inch, pounds per square foot, inches of water, or inches of mercury. However, the same units must be used for both pressures and for both volumes; thus, if p_1 is expressed in inches of mercury, p_2 must also be expressed in inches of mercury; and if v_1 is expressed in cubic inches, v_2 must also be expressed in cubic inches.

The formulas derived in Arts. **24** and **25** are not homogeneous. Using .37 as the value of R, the formulas hold good only when p is expressed in pounds per square inch, v in cubic feet, and G in pounds. If other units are used, R must have a different value. Furthermore, while the general statements of these formulas are true for any gas, the character of the constant R is such that, when determined, it limits the application of the formulas not only to certain units, but also to a particular gas, while the other formulas hold for all gases.

EXAMPLE.—A quantity of hydrogen occupies a volume of 60 cubic inches at a temperature of 70° when the barometer stands at 30 inches; what volume will be occupied when the temperature is 44° and the barometer stands at 29.4 inches?

SOLUTION.—The volumes are expressed in cubic inches and the pressures in inches of mercury. Using the formula in Art. **22**, $\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$, and substituting,

$$\frac{30 \times 60}{460 + 70} = \frac{29.4 \times v_2}{460 + 44}; \text{ hence, } v_2 = \frac{30 \times 60 \times 504}{530 \times 29.4} = 58.22 \text{ cubic inches.}$$

Ans.

EXAMPLES FOR PRACTICE

1. A vessel contains 25 cubic feet of gas at a pressure of 18 pounds per square inch, absolute; if 125 additional cubic feet of gas having the same pressure are forced into the vessel, what will be the resulting pressure? Ans. 108 lb. per sq. in.

2. A pound of air has a temperature of 126°, and a pressure of one atmosphere; what volume does it occupy? Ans. 14.75 cu. ft.

3. The volume of steam in the cylinder of a steam engine at cut-off is 1.35 cubic feet, and the absolute pressure is 85 pounds per square inch; if the absolute pressure at the end of the stroke is 25 pounds per square inch, what is the new volume, assuming that the expanding steam follows Boyle's law? Ans. 4.59 cu. ft.

MIXTURES OF GASES

28. Diffusion of Gases.—If two liquids that do not act chemically on each other are mixed together and allowed to stand, it will be found that after a time the liquids have separated and that the heavier has fallen to the bottom. If two equal vessels, containing gases of different heaviness, are put in communication with each other, the gases will be found to have mixed in equal proportions after a short time. If one vessel is higher than the other, and the heavier gas is in the lower vessel, the result will be the same. The greater the difference in heaviness of the two gases, the quicker they will mix. It is assumed that no chemical action takes place between the two gases. When the two gases have the same temperature and pressure, the pressure of the mixture will be the same; this is evident, since the total

volume has not been changed, and unless the volume or temperature changes, the pressure cannot change. This property of the mixing of gases is a very valuable one; for if gases acted like liquids, carbonic-acid gas, the result of combustion, which is $1\frac{1}{2}$ times as heavy as air, would remain next to the earth instead of dispersing into the atmosphere. and, in consequence, no animal life could exist.

29. Mixture of Equal Volumes of Gases Having Unequal Pressures.—*If two gases having equal volumes and temperatures, but different pressures, are mixed in a vessel whose volume is equal to one of the equal volumes of the gas, the pressure of the mixture will be equal to the sum of the two pressures, provided the temperature remains the same as before.*

EXAMPLE.—Each of two vessels contains 3 cubic feet of gas, subjected to pressures of 40 pounds and 25 pounds per square inch, respectively, and at a temperature of 60° F. The vessels are placed in communication with each other, and all the gas is compressed into one vessel. If the temperature of the mixture is also 60°, what is the pressure?

SOLUTION.—According to the rule just given, the pressure will be $40 + 25 = 65$ lb. per sq. in. This may be proved by applications of Boyle's law; thus, compress the gas whose pressure is 25 lb. per sq. in. until its pressure is 40 lb.; its volume may be found thus: $p\,v = p_1\,v_1$, or $25 \times 3 = 40 \times v$; whence, $v = 1.875$ cu. ft. Let communication be established between the two vessels; the pressure will evidently be 40 lb. and the total volume $3 + 1.875 = 4.875$ cu. ft. If this is compressed until the volume is 3 cu. ft., the temperature remaining at 60° throughout the whole operation, the final pressure may be found by the formula, $p\,v = p_1\,v_1$. Thus, $40 \times 4.875 = p_1 \times 3$, and $p_1 = \frac{40 \times 4.875}{3}$ $= 65$ lb. per sq. in., as before.

30. Mixture of Two Gases Having Unequal Volumes and Pressures.—Let v_1 and p_1 be the volume and pressure, respectively, of one of the gases; v_2 and p_2 be the volume and pressure, respectively, of the other gas; and V and P be the volume and pressure, respectively, of the mixture. Then, if the temperature remains the same,

$$P\,V = p_1\,v_1 + p_2\,v_2$$

That is, *if the temperature is constant, the product of the pressure and volume after mixing is equal to the sum of the*

products obtained by multiplying each volume by its corresponding pressure previous to mixing.

EXAMPLE.—Two gases of the same temperature, having volumes of 7 cubic feet and $4\frac{1}{2}$ cubic feet, and whose pressures are 27 pounds and 18 pounds per square inch, respectively, are mixed together in a vessel whose volume is 10 cubic feet. The temperature of the two gases and of the mixture being 60° F., what is the resulting pressure?

SOLUTION.—Applying the formula $P V = p_1 v_1 + p_2 v_2$,

$$P \times 10 = 27 \times 7 + 4\tfrac{1}{2} \times 18$$

$$P = \frac{189 + 81}{10} = 27 \text{ lb. per sq. in.} \quad \text{Ans.}$$

31. Mixture of Two Bodies of Air Having Unequal Pressures, Volumes, and Temperatures.—If a body of air having a temperature t_1, a pressure p_1, and a volume v_1 is mixed with another volume of air having a temperature t_2, a pressure p_2, and a volume v_2, to form a volume V, having a pressure P and a temperature t, then, either the new temperature t, the new volume V, or the new pressure P may be found, if the other quantities are known, by the following formula, in which T_1, T_2, and T are the absolute temperatures corresponding to t_1, t_2, and t;

$$\frac{P V}{T} = \frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}$$

EXAMPLE.—Five cubic feet of air having a pressure of 30 pounds per square inch, absolute, and a temperature of 80° F., is to be compressed together with 11 cubic feet of air having a pressure of 21 pounds per square inch, absolute, and a temperature of 45° F., in a vessel whose cubical capacity is 8 cubic feet; if the resulting pressure is 45 pounds per square inch, what is the temperature of the mixture?

SOLUTION.—Substituting in the formula,

$$\frac{45 \times 8}{T} = \left(\frac{30 \times 5}{540} + \frac{21 \times 11}{505}\right), \text{ or } \frac{360}{T} = .7352. \text{ Hence, } T = \frac{360}{.7352}$$
$$= 489.66°, \text{ nearly, and } t = 29.66°. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Two vessels contain air at pressures of 60 and 83 pounds per square inch, absolute; the volume of each vessel is 8.47 cubic feet. If all of the air in both vessels is removed to another vessel, and the new

pressure is 100 pounds per square inch, absolute, what is the volume of the vessel, the temperature being the same throughout?

Ans. 12.11 cu. ft.

2. A vessel contains 11.83 cubic feet of air at a pressure of 33.3 pounds per square inch, absolute; it is desired to increase the pressure to 40 pounds per square inch, absolute, by supplying air from a second vessel which contains 19.6 cubic feet of air at a pressure of 60 pounds per square inch, absolute. What will be the pressure in the second vessel after the pressure in the first has been raised to 40 pounds per square inch? Ans. 55.96 lb. per sq. in.

3. If 4.8 cubic feet of air having a pressure of 52 pounds per square inch, absolute, and a temperature of 170° is mixed with 13 cubic feet having a pressure of 78 pounds per square inch, absolute, and a temperature of 265°, what must be the volume of the vessel containing the mixture in order that the pressure of the mixture may be 30 pounds per square inch, absolute, and the temperature 80°?

Ans. 32.31 cu. ft.

PNEUMATIC MACHINES AND DEVICES

THE AIR PUMP

32. Action of the Air Pump.—The **air pump** is an instrument for removing air from an enclosed space. A sec-

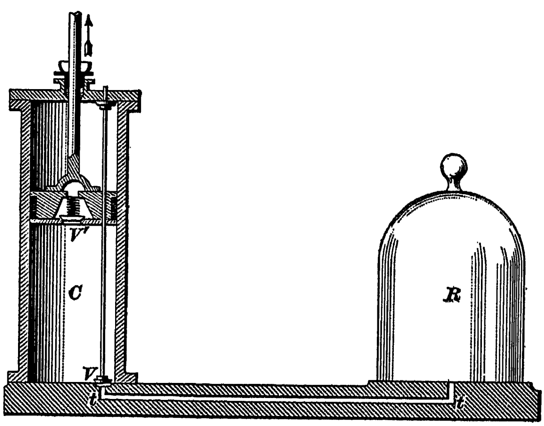


FIG. 11

tion of the principal parts is shown in Fig. 11, and the complete instrument in Fig. 12. The closed vessel R is called

the **receiver**, and the space that it encloses is that from which it is desired to remove the air. The receiver is usually made of glass, and the edges are ground so as to make a perfectly air-tight joint with the plate on which it rests. When made in the form shown, it is called a **bell-jar receiver.** The receiver rests on a horizontal plate in the center of which is an opening that communicates with the pump cylinder C by means of the air passage t. The pump piston fits the cylinder accurately, and has a valve V' opening upwards. At the junction of the air passage with the cylinder is another valve V, also opening upwards. When the piston is raised, the valve V' closes; and, since no air can get into the cylinder from above, the piston leaves a partial vacuum behind it. The pressure on top of V being now partially removed, the pressure of the air in the receiver R causes V to rise; the air in the receiver then expands and occupies the space behind the piston and the space in the passage t and the receiver R. The piston is now pushed down, the valve V closes, the valve V' opens, and the air in C escapes. The lower valve V is sometimes supported, as shown in Fig. 11, by a metal rod passing through the piston and fitting it tightly. When the piston is raised or lowered, this rod moves with it. A button near the upper end of the rod confines its motion to very narrow limits, the piston sliding on the rod during the greater part of the motion.

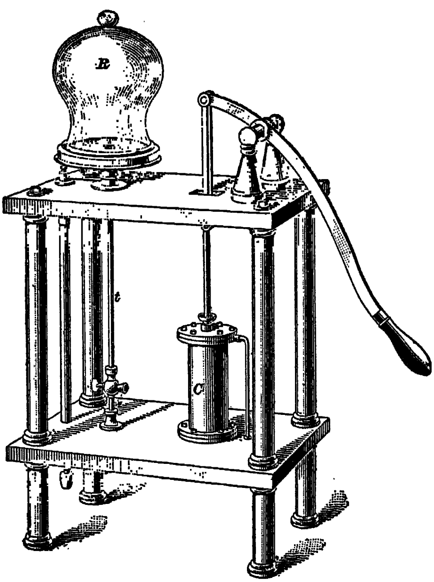


FIG. 12

33. Degrees and Limits of Exhaustion.—Suppose that the volume of R and t together is four times that of C, and that there are, say, 200 grains of air in R and t, and 50 grains in C, when the piston is at the top of the cylinder. At the end of the first stroke, when the piston is again at the top, 50 grains of air in the cylinder C will have been removed, and the 200 grains in R and t will occupy the spaces R, t, and C. The ratio between the sum of the spaces R and t and the total space $R + t + C$ is $\frac{4}{5}$; hence, $200 \times \frac{4}{5} = 160$ grains = the weight of air in R and t after the first stroke. After the second stroke, the weight of the air in R and t will be $200 \times \frac{4}{5} \times \frac{4}{5} = 200 \times (\frac{4}{5})^2 = 200 \times \frac{16}{25} = 128$ grains. At the end of the third stroke, the weight will be $[200 \times (\frac{4}{5})^2] \times \frac{4}{5} = 200 \times (\frac{4}{5})^3 = 200 \times \frac{64}{125} = 102.4$ grains. At the end of n strokes, the weight will be $200 \times (\frac{4}{5})^n$. It is evident that *it is impossible to remove all the air that is contained in R and t by this method.* It requires an exceedingly good air pump to reduce the pressure of the air in R to $\frac{1}{50}$ inch of mercury. When the air has become so rarefied as this, the valve V' will not lift, and no more air can be exhausted.

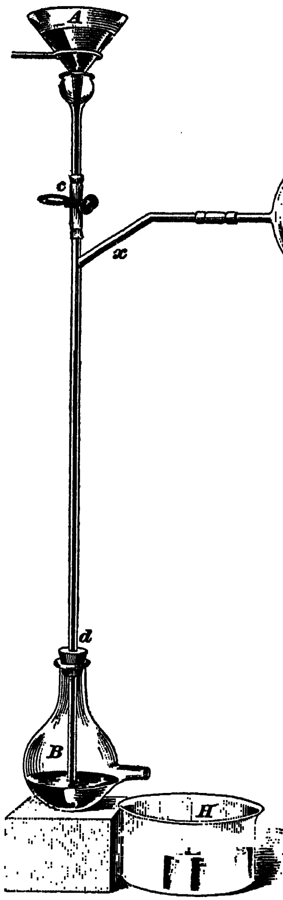


FIG. 18

34. Sprengel's Air Pump.—In Fig. 13 is shown a glass tube longer than 30 inches, open at both ends, and connected by means of rubber tubing with a funnel A filled with mercury and supported by a stand. Mercury is allowed to fall into this tube at a rate regulated by a clamp at c. The lower end of the tube $c\,d$ fits in the flask B, which has a spout at the side a little higher than the lower end of $c\,d$; the upper part has a branch at x to which a receiver R can be tightly fixed. When the clamp at c is opened, the first portions of the mercury that run out close the tube and prevent air from entering from below. These drops of mercury act like little pistons, carrying the air in front of them and forcing it out through the bottom of the tube. The air in R expands to fill the tube every time that a drop of mercury falls, thus creating a partial vacuum in R, which becomes more nearly complete as the process goes on. The escaping mercury falls into the dish H, from which it can be poured back into the funnel from time to time. As the exhaustion from R goes on, the mercury rises in the tube $c\,d$ until, when the exhaustion is complete, it forms a continuous column about 30 inches high.

This instrument necessarily requires a great deal of time for its operation, but the results are very complete. The pressure in R has been reduced to $\frac{1}{43000}$ inch of mercury. By the use of chemicals in addition to the above, a vacuum of $\frac{1}{650000}$ inch of mercury has been obtained.

APPARATUS SHOWING WEIGHT AND PRESSURE OF THE ATMOSPHERE

35. Magdeburg Hemispheres.—By means of the two hemispheres shown in Fig. 14, it can be proved that the atmosphere presses on a body equally in all directions. Such hemispheres were invented by Otto Von Guericke, of Magdeburg, Germany, and are called the **Magdeburg hemispheres.** One of the hemispheres is provided with a stop-cock, by which it can be attached to an air pump. The rims fit accurately and are well greased, so as to make an air-tight

joint. As long as the hemispheres contain air, they can be separated with ease; but when the air in the interior is pumped out by means of an air pump, they can be separated only with great difficulty. The force required to separate them will be equal to the area of the largest circle of the hemisphere, which is the projected area, in square inches, multiplied by 14.7 pounds. This force will be the same in whatever position the hemisphere may be held, thus proving that the pressure of air on it is the same in all directions.

36. The Weight Lifter.—The pressure of the atmosphere is very clearly shown by means of an apparatus like that illustrated in Fig. 15. Here, a cylinder fitted with a piston is held in suspension by a chain. At the top of the cylinder is a plug *a* that can be taken out. This plug is removed and the piston pushed up, the force necessary being equal to the weight of the piston and rod *b*, until it touches the cylinder head. The plug is then screwed in, and assuming that no air is left in the cylinder, the piston will remain at the top until a weight has been hung on the rod, equal to the area of the piston multiplied by 14.7 pounds, less the weight of the piston and rod. If a force just great enough to move the piston downwards were applied to the rod, the pressure of the air would raise any weight less than this to the top of the cylinder.

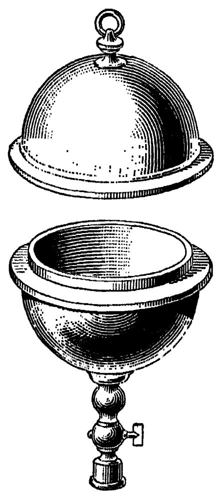


FIG. 14

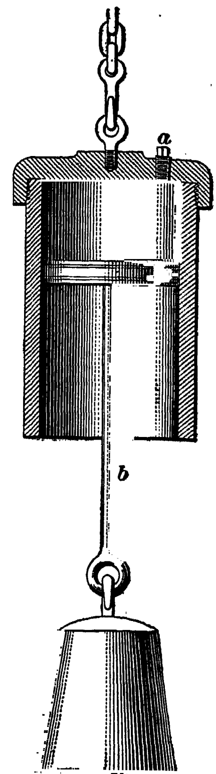


FIG. 15

Suppose the weight to be removed, and the piston to be supported, say at the middle of the length of the cylinder. Let the plug be removed and air admitted above the piston; then, with the piston in the middle of the cylinder, screw the plug back into its place; if the piston be forced upwards, the farther up it goes the greater will be the force necessary to push it, on account of the compression of the air. If the piston is of large diameter, it will also require a great force to pull it out of the cylinder. For example, let the diameter of the piston be 20 inches, the length of the cylinder 36 inches plus the thickness of the piston, and the weight of the piston and rod 100 pounds. If the piston is at the middle of the cylinder, there will be 18 inches of space above it and 18 inches of space below it. The area of the piston is $20^2 \times .7854 = 314.16$ square inches, and with the atmospheric pressure on it, there will be $314.16 \times 14.7 = 4{,}618$ pounds, nearly, pressing on each side of the piston. In order to hold the piston central, an upward force of 100 pounds must be exerted to balance the weight of the piston and rod. In order to move the piston upwards 9 inches, reducing the volume one-half and doubling the pressure, the upward pressure on it must be twice the atmospheric pressure, plus 100 pounds, provided that the temperature remains constant. The total upward force is therefore $2 \times 4{,}618 + 100 = 9{,}336$ pounds. The force in excess of that of the atmosphere necessary to cause the piston to move upwards 9 inches will then be $9{,}336 - 4{,}618 = 4{,}718$ pounds.

Now suppose the piston to be moved downwards until it is just at the point of being pulled out of the cylinder. The volume above it will then be twice as great as before, and the pressure one-half as great, or $4{,}618 \div 2 = 2{,}309$ pounds. The total upward pressure will be the pressure of the atmosphere, or 4,618 pounds. The force necessary to pull the piston downwards to this point will be the difference between the total upward pressure and the downward pressure due to the pressure inside the cylinder and the weight of the piston and rod. This gives the force necessary to pull the piston down as $4{,}618 - (2{,}309 + 100) = 2{,}209$ pounds.

37. The Baroscope.—The buoyant effect of air is very clearly shown by means of an instrument called the **baroscope**, shown in Fig. 16. It consists of a scale beam, from one extremity of which is suspended a small weight, and from the other a hollow copper sphere. In air, they exactly balance each other; but when placed under the receiver of an air pump and the air exhausted, the sphere sinks, showing that it is really heavier than the small weight. Before the air is exhausted, each body is buoyed up by the weight of the air it displaces, and since the sphere displaces the more air, it loses more weight by reason of this displacement than the small weight. Suppose that the volume of the sphere exceeds that of the weight by 10 cubic inches; the weight of this volume of air is 3.1 grains. If this weight is added to the small weight, it will overbalance the sphere in air, but will exactly balance it in a vacuum.

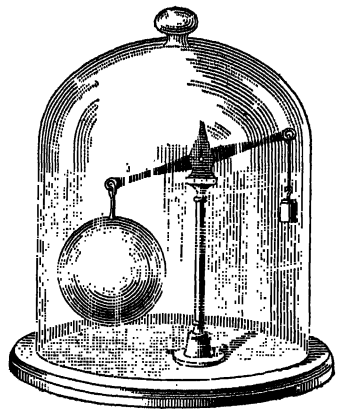


FIG. 16


FIG. 17

38. The Cartesian Diver.—The device shown in Fig. 17 illustrates the elasticity of air and the transference of pressure in all directions in water. It is called the **Cartesian diver** and consists of a glass jar nearly filled with water, having a rubber bulb at the top filled with air. The image in the jar is made of glass and is hollow, the weight being less than that of an equal volume of water, so that it will float at the top of the jar. The tail

of the image has a hole in it, the water being prevented from getting inside of the image by the pressure of the air within it. If the bulb is squeezed, the air in it will be forced out, creating a pressure on the water which, being transferred in all directions, causes the water to flow into the tail of the image, compressing the air inside and thus causing it to fall to the bottom of the jar. When the pressure on the bulb is released, the air flows back into the bulb, the pressure on the water is removed, and the air within the image expands; the image again becoming lighter than water, rises to the top of the jar.

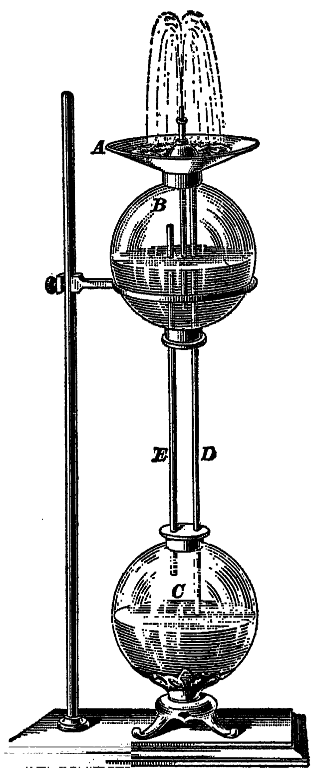


FIG. 18

39. Hero's Fountain. **Hero's fountain** derives its name from its inventor, Hero, who lived at Alexandria about 120 B. C. It depends for its operation on the elastic properties of air. It is shown in Fig. 18 and consists of a brass dish A and two glass globes B and C. The dish communicates with the lower part of the globe C by a long tube D, and another tube E connects the two globes. A third tube passes through the dish A to the lower part of the globe B. This last tube being taken out, the globe B is partially filled with water; the tube is then replaced and water is poured into the dish. The water flows downwards through the tube D into the lower globe, and expels the air, which is forced into the upper globe. The air thus compressed acts on the water and makes it shoot out through the shortest tube in the form of a jet, as represented

in the figure. Were it not for the resistance of the atmosphere and friction, the water would rise to a height above the water in the dish equal to the difference of the level of the water in the two globes.

THE SIPHON

40. The Principle of the Siphon.—The action of the **siphon** illustrates the effect of atmospheric pressure. The siphon is simply a bent tube with branches of unequal length open at both ends, and is used to convey a liquid from a higher point to a lower one, over an intermediate point higher than either. In Fig. 19, *a* and *b* are two vessels, *b* being lower than *a*, and *a c b* is the bent tube or siphon. Suppose this tube to be filled with water and placed in the vessels, as shown, with the short branch *a c* in the vessel *a*. The water will flow from the vessel *a* into the vessel *b*, so long as the level of the water in *b* is below the level of the water in *a*, and the level of the water in *a* is above the lower end of the tube *a c*. The atmospheric pressure on the surfaces of *a* and *b* tends to force the water up the tubes *a c* and *b c*. When the siphon is filled with water, each of these pressures is counteracted in part by the pressure of the water in that branch of the siphon which is immersed in the water on which the pressure is exerted. The atmospheric pressure opposed to the weight of the longer column of water will, therefore, be resisted with greater force than that opposed to the weight of the shorter column; consequently, the pressure exerted on the shorter column will be greater than that on the longer column, and this excess pressure will produce motion.

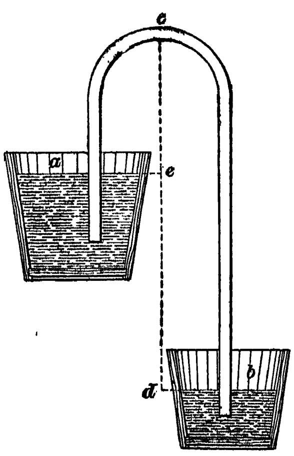


FIG. 19

Let $h_1 = dc =$ vertical distance, in feet, between surface of water in b and highest point of center line of tube;

$h_2 = ec =$ distance, in feet, between surface of water in a and highest point of center line of tube.

The downward pressure at the level ae due to the head ce is $.434\,h_2$ pounds per square inch, while the upward pressure due to the atmospheric pressure on the surface a is 14.7 pounds per square inch; then, the net upward pressure per square inch is $(14.7 - .434\,h_2)$ pounds. Similarly, at the level bd the net upward pressure per square inch is $(14.7 - .434\,h_1)$ pounds. The water column acb is urged from a toward b by a force of $(14.7 - .434\,h_2)$ pounds per square inch and in the opposite direction by a force of $(14.7 - .434\,h_1)$ pounds per square inch. The difference between the forces is $(14.7 - .434\,h_2) - (14.7 - .434\,h_1) = .434(h_1 - h_2)$ pounds per square inch and since the upward force at a is the greater, the water moves from a toward b, that is, upwards in the shorter column and downwards in the longer. The net head $(h_1 - h_2)$ producing the flow is de, the difference between the level of the surface of the water in the two vessels.

It will be noticed that the short column must not be higher than 34 feet for water, or the siphon will not work, since the pressure of the atmosphere will not support a column of water that is higher than 34 feet; 28 feet is about the greatest height at which a siphon will work well.

41. Intermittent Springs.—Sometimes a spring is observed to flow for a time and then cease; then, after an interval to flow again for a time. The generally accepted explanation of this is that there is an underground reservoir fed with water through fissures in the earth, as shown in Fig. 20. The outlet for the water is shaped like a siphon, as shown. When the water in the reservoir reaches the same point as the highest point of the outlet, it flows out until the level of the water in the reservoir falls below the mouth of

the siphon, the water flowing out of the reservoir faster than it is supplied to it. This flow then ceases until the water in

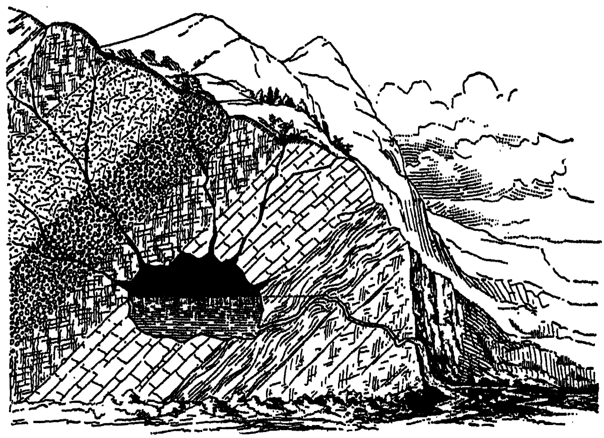


FIG. 20

the reservoir has again reached the level of the highest point of the siphon.

AIR COMPRESSORS

42. For many purposes, compressed air is preferable to steam or any other gas for use as a motive power. In such cases, **air compressors** are used to compress the air. These are made in many forms, but the most common one consists of a cylinder, called the *air cylinder*, placed in front of the crosshead of a steam engine, so that the piston of the air cylinder can be driven by attaching its piston rod to the crosshead, in a manner similar to a direct-acting steam pump. A cross-section of the air cylinder of a compressor of this kind is shown in Fig. 21, in which a is the piston and b is the piston rod, driven by the crosshead of a steam engine not shown in the figure. Both ends of the lower half of the cylinder are fitted with inlet valves d and d', which allow the air to enter the cylinder, and both ends of the upper half are fitted with discharge valves f and f', which allow the air to escape from the cylinder after it has been compressed to the required pressure.

Suppose the piston *a* to be moving in the direction of the arrow; then the inlet valves *d* in the left-hand end of the cylinder from which the piston is moving will be forced inwards by the pressure of the atmosphere, which overcomes the resistance of the light spring *c*, thus allowing the air to flow in and fill the cylinder. On the other side of the piston, the air is being compressed, and, consequently, it acts with

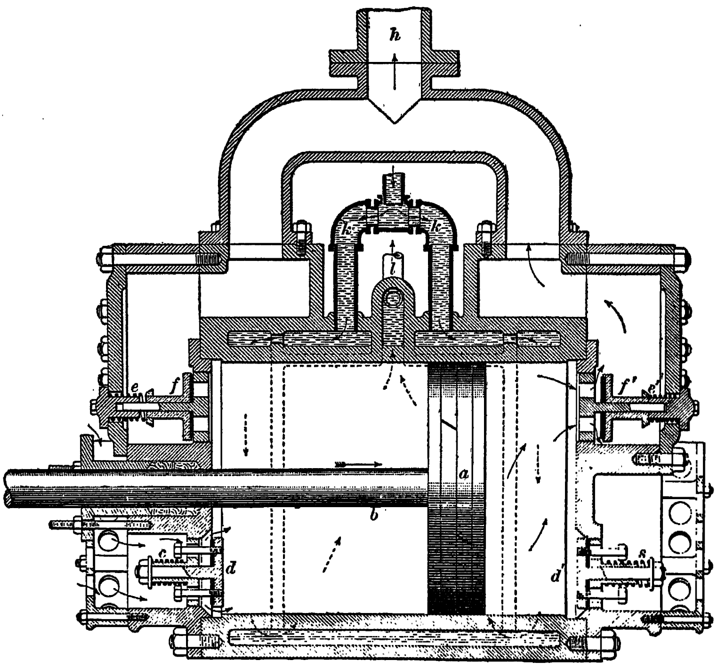


FIG. 21

the springs *s* to force the inlet valves *d'* in the right-hand end of the cylinder to their seats. In the right-hand end of the cylinder, the discharge valves *f'* are opened when the pressure of the air in the cylinder is great enough to overcome the resistance of the light springs *e'* and the pressure of the air in the passages leading to the discharge pipe *h*, and the discharge valves *f* are pressed against their seats by the springs *e* and the pressure of the air in the passages.

Suppose that it is desired to compress the air to 59 pounds per square inch, and it is necessary to find at what point of the stroke the discharge valves will open. Now, 59 pounds per square inch is equal to a pressure of four atmospheres, that is, four times the pressure of the atmosphere, very nearly; hence, when the pressure in the cylinder becomes great enough to force air out through the discharge valves, the volume must be one-quarter of the volume at atmospheric pressure, or, the valves will open when the piston has traveled three-quarters of its stroke, provided that the air is compressed at constant temperature. The air, after being discharged from the cylinder, passes out through the delivery pipe *h*, and is then conveyed to its destination.

When air or any other gas is compressed its temperature rises. For high pressures, this rise of temperature becomes a serious consideration, for two reasons: (1) When the air is discharged at a high temperature, the pressure falls considerably when it is cooled down to its normal temperature, and this represents a serious loss in the economical working of the machine. (2) The alternate heating and cooling of the compressor cylinder by the hot and cold air is very destructive to it, and increases the wear to a great extent. To prevent excessive heating, cooling devices are resorted to, the most common one being the so-called **water-jacket.** The cylinder walls are hollow, as shown in Fig. 21; the cold water enters this hollow space in the cylinder wall through the pipes *k*, *k*, flows around the cylinder, and finally passes out through the discharge pipe *l*. Much of the heat generated by the compression of the air passes through the cylinder walls and is absorbed and carried off by the water; thus the temperature of the air does not rise to the point at which it will be seriously detrimental.

HYDRAULICS
(PART 1)

FLOW OF WATER

FUNDAMENTAL PRINCIPLES

1. Flow in a Pipe of Uniform Diameter.—When water flows through a pipe of uniform cross-section, the quantity passing any section in a given interval of time depends on the area of the cross-section and the velocity with which the water moves. It is usually assumed that all the particles of water have the same velocity; but if they have different velocities the mean or average may be taken as the velocity of the flow.

Let A = area of cross-section of pipe, in square feet;
v = mean velocity of flow, in feet per second;
Q = quantity, in cubic feet, flowing past any section in 1 second.

Evidently, the quantity Q is equal to the volume of a column whose base is the area A and whose height is equal to v; hence,

$$Q = A v \qquad (1)$$

or

$$v = \frac{Q}{A} \qquad (2)$$

EXAMPLE.—With a velocity of 8 feet per second, what quantity of water will be discharged by a 3-inch pipe: (*a*) in 1 second? (*b*) in 1 hour?

SOLUTION.—(*a*) The area A is $\frac{.7854 \times 3^2}{144}$ sq. ft.; hence,

$$Q = A v = \frac{.7854 \times 3^2}{144} \times 8 = .3927 \text{ cu. ft. per sec.} \quad \text{Ans.}$$

(*b*) The flow per hour is

$$.3927 \times 60 \times 60 = 1{,}413.7 \text{ cu. ft.} \quad \text{Ans.}$$

2. Flow in a Pipe of Varying Diameter.—Since water is practically incompressible, the quantity flowing past any section must be the same as the quantity flowing past any other section in the same interval of time, provided that the pipe is full of water. Hence, if the cross-section of the pipe varies, the velocity also must vary. Where the cross-section is least, the velocity is greatest; and where the cross-section is greatest, the velocity is least. If, therefore, A_1, A_2, A_3 denote the areas of successive cross-sections and v_1, v_2, v_3 denote the corresponding velocities, then $Q = A_1 v_1$; $Q = A_2 v_2$, and $Q = A_3 v_3$; hence,

$$A_1 v_1 = A_2 v_2 = A_3 v_3$$

Assuming that the pipe always remains full, it appears that the velocities at various sections of the pipe vary inversely as the areas of those sections.

EXAMPLE.—The velocity through a part of a pipe that is 3 inches in diameter is 5 feet per second. (*a*) What is the velocity through an enlarged part of the pipe whose diameter is 4 inches? (*b*) What is the velocity through a valve that has an opening of $3\frac{1}{2}$ square inches?

SOLUTION.—(*a*) Since this formula applies when the areas are expressed in square inches, $A_1 = .7854 \times 3^2$, $A_2 = .7854 \times 4^2$, $v_1 = 5$. Substituting these values in the formula,

$$v_2 \times .7854 \times 4^2 = 5 \times .7854 \times 3^2$$

As the common factor .7854 cancels,

$$v_2 = 5 \times \frac{3^2}{4^2} = 2.8125 \text{ ft. per sec. Ans.}$$

(*b*)
$$v_3 \times 3.5 = 5 \times .7854 \times 3^2$$

or
$$v_3 = \frac{5 \times .7854 \times 9}{3.5} = 10.098 \text{ ft. per sec. Ans.}$$

3. Internal Pressure of Flowing Water.—The reservoir *a*, Fig. 1, is connected with a second reservoir *b* by a bent pipe of varying cross-section. In the first place, suppose the lower end of the pipe to be closed so that there is no flow. The water in *a* and in the pipe being at rest, the pressure at any section of the pipe is determined by the principles of hydrostatics. Thus, at the section s_1 the level *c d* of the water in *a* is at a distance h_1 above the center of the section, and the gauge pressure of the water at that point is therefore .434 h_1 pounds per square inch, when h_1 is

expressed in feet. For example, if a section of the pipe is 10 feet below the level cd, the gauge pressure at that section is $.434 \times 10 = 4.34$ pounds per square inch. To obtain the absolute pressure, the pressure of the atmosphere, 14.7 pounds per square inch, must be added to the gauge pressure.

Let a small tube t_1, open at both ends, be screwed into the pipe at s_1; when no water flows from the reservoir a to the reservoir b the water will rise in this tube to the level cd, and the height h_1 of the water in the tube therefore measures

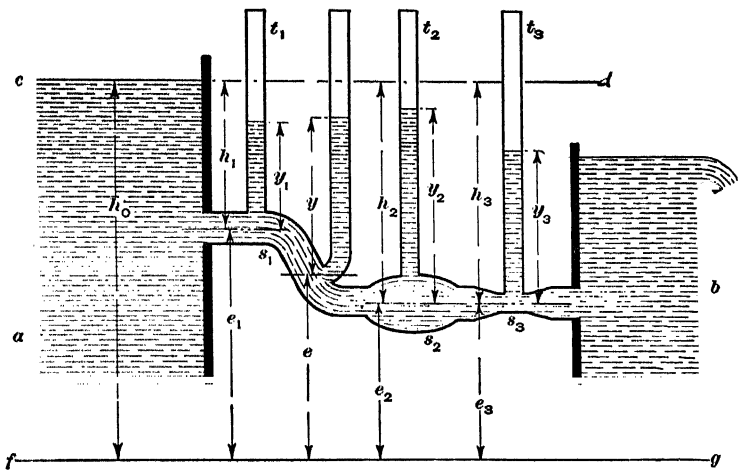


FIG. 1

the gauge pressure of the water at the section s_1. Similarly, if tubes t_2 and t_3 are screwed into the pipe at the sections s_2 and s_3, respectively, the water will rise in these to the level cd, and the heights h_2 and h_3 will measure the gauge pressures at these points.

The distances h_1, h_2, and h_3 are called the **hydrostatic heads** on the sections s_1, s_2, and s_3, and the horizontal line cd, to which the water in the tubes will rise when no water flows, is called the **hydrostatic level.**

When the water is flowing from the vessel a to the vessel b, it will be found that the water in the tube t_1 has a height y_1,

which is less than h_1; and likewise, in the tubes t_2 and t_3, the height y_2 is less than h_2 and y_3 is less than h_3. The decrease in height is found to be greatest in the tube t_3 inserted at s_3, the smallest section, and is least in the tube t_2 inserted at s_2, the largest section.

The heights y_1, y_2, and y_3 measure the gauge pressures at the sections s_1, s_2, and s_3, respectively, and are called the **pressure heads** on those sections.

In the case of water flowing in a pipe, *the pressure head at any section is less than the hydrostatic head, and the difference between the two is greater the smaller the section or the greater the velocity of flow.*

4. Energy of a Mass of Flowing Water.—Consider a mass of water flowing in the pipe of Fig. 1. The mass possesses a certain amount of energy, that is, it is capable of doing a definite amount of work, and this energy is composed of the three factors, kinetic energy, energy due to pressure, and potential energy.

1. *Kinetic Energy.*—If the mass weighs G pounds and has a velocity of v feet per second, the amount of the kinetic energy is $\frac{G v^2}{2g}$ foot-pounds, in which g is equal to 32.16, the acceleration due to gravity.

2. *Energy Due to Pressure.*—The mass of water possesses some energy because of its internal fluid pressure. To estimate the amount of this energy, proceed as follows: Suppose the water to enter a cylinder a, Fig. 2, from a reservoir, b, and to push against a piston on the other side of which is atmospheric pressure. Let p denote the absolute pressure, in pounds per square inch, of the water against

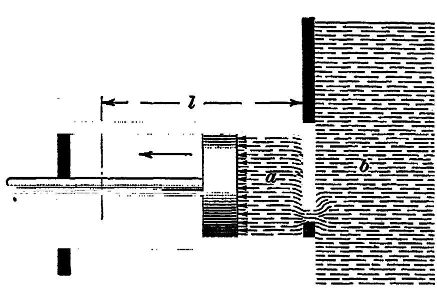


FIG. 2

the piston. When the piston has moved to the end of its stroke, let the water in the cylinder be shut off from the reservoir and opened to the atmosphere; the pressure of the water in the cylinder will then be that of the atmosphere—14.7 pounds per square inch. If A denotes the area of the piston, in square inches, the total net pressure urging the piston to the left, during the stroke, is $A\ (p - 14.7)$; hence, the work done is $A\ l\ (p - 14.7)$ foot-pounds, when l denotes the length of the stroke of the piston, in feet. The motion is assumed to take place very slowly, so that there is practically no change in the kinetic energy of the water; and as the water remains at the same level, there is no change of potential energy; hence, the work done is equal to the energy given up by the cylinder of water when its pressure is decreased to that of the atmosphere.

Now $p - 14.7$ is the gauge pressure of the water, and if y denotes the **pressure head,** in feet, corresponding to the gauge pressure, $p - 14.7 = .434\ y$. Further, since $\frac{A}{144}$ is the area of cylinder, in square feet, $\frac{A\ l}{144}$ is the volume of cylinder, in cubic feet, and $\frac{A\ l}{144} \times 62.5 = .434\ A\ l$ is the weight of water in the cylinder. Let G denote this weight; then $.434\ A\ l = G$, or $A\ l = \frac{G}{.434}$. Now, substituting the values thus found for the terms $p - 14.7$ and $A\ l$, in the expression $A\ l\ (p - 14.7)$, the energy due to pressure is

$$\frac{G}{.434} \times .434\ y = G y \text{ foot-pounds}$$

The pressure energy of a weight G of water is therefore equal to the product of the weight and the pressure head.

3. *Potential Energy.*—If the water is elevated a distance e feet above some assumed datum line $f g$, Fig. 1, it has a potential energy due to its position. A *datum line* is a reference line from which measurements are taken, and is usually assumed at the most convenient point for taking such measurements. The amount of this potential energy is the

work that would be required to raise the weight G through a height e feet, or Ge foot-pounds.

The *total energy* of the water is the sum of these three energies; denoting it by E,

$$E = \frac{Gv^2}{2g} + Gy + Ge = G\left(\frac{v^2}{2g} + y + e\right)$$

5. Equation of Energy.—If the flow through the pipe shown in Fig. 1 is assumed to be frictionless, the energy of a mass of water passing any section must be the same as the energy in passing any other section. The energy of 1 pound of water at the section s_1, with the velocity v_1 is

$$E_1 = \frac{v_1^2}{2g} + y_1 + e_1 \qquad (1)$$

For the section s_2, where the velocity is v_2, the energy is

$$E_2 = \frac{v_2^2}{2g} + y_2 + e_2 \qquad (2)$$

For the section s_3, it is

$$E_3 = \frac{v_3^2}{2g} + y_3 + e_3 \qquad (3)$$

Now, $E_1 = E_2 = E_3$, since no energy is lost in doing work against friction; hence,

$$\frac{v_1^2}{2g} + y_1 + e_1 = \frac{v_2^2}{2g} + y_2 + e_2 = \frac{v_3^2}{2g} + y_3 + e_3 \qquad (4)$$

It will be observed that y and e are distances or heights. To the height e of the water above the assumed datum line, the name **potential head** is given. It is apparent that the quantity $\frac{v^2}{2g}$ may also represent a height; in fact, it is the height that a body will rise when projected upwards with an initial velocity v. To this quantity, therefore, the name **velocity head** is given.

According to formula **4**, in frictionless steady flow, the sum of the velocity head, pressure head, and potential head is the same at all sections of the pipe. This statement is known as **Bernoulli's law**.

6. The Velocity Head.—Let h_0, Fig. 1, denote the height of the water level $c\,d$ above the datum line fg. One

pound of water at the level cd has no kinetic energy, being at rest, and no pressure energy; hence, its total energy is its potential energy with reference to fg, and this energy is $1 \times h_0 = h_0$ foot-pounds. The energy of this pound on the surface must be the same as that of a pound passing through the pipe at the section s_1; hence,

$$h_0 = \frac{v_1^2}{2g} + y_1 + e_1 \qquad (1)$$

But $h_0 = e_1 + h_1$; hence, $\frac{v_1^2}{2g} + y_1 + e_1 = e_1 + h_1$, or $\frac{v_1^2}{2g} = h_1 - y_1$. In general,

$$\frac{v^2}{2g} = h - y; \text{ or } y = h - \frac{v^2}{2g} \qquad (2)$$

That is, *in frictionless flow, the velocity head at any section is equal to the difference between the hydrostatic head and the pressure head.* Evidently, therefore, the distances of the water levels below cd in the tubes t_1, t_2, and t_3, Fig. 1, are the velocity heads for the sections s_1, s_2, and s_3, respectively.

EXAMPLE 1.—In Fig. 1, $h_1 = 15$ feet, $h_2 = h_3 = 20$ feet, and the areas at s_1, s_2, and s_3 are $A_1 = 3$ square inches, $A_2 = 5$ square inches, and $A_3 = 1$ square inch. The velocity of the water as it passes s_1 is 8 feet per second. Find the velocity heads and pressure heads for the three sections; also the absolute pressure of the water as it passes each section.

SOLUTION.—Using the formula of Art. **2**, $A_1 v_1 = A_2 v_2$, $3 \times 8 = 5 \times v_2$; whence,

$$v_2 = \frac{3 \times 8}{5} = 4.8 \text{ ft. per sec.;}$$

and, similarly, $$v_3 = \frac{3 \times 8}{1} = 24 \text{ ft. per sec.}$$

The velocity heads are:

$$\frac{v_1^2}{2g} = \frac{8^2}{2 \times 32.16} = .995 \text{ ft. at } s_1,$$

$$\frac{v_2^2}{2g} = \frac{4.8^2}{2 \times 32.16} = .358 \text{ ft. at } s_2,$$

and $$\frac{v_3^2}{2g} = \frac{24^2}{2 \times 32.16} = 8.95 \text{ ft. at } s_3.$$

The pressure heads are now found by formula **2**.

$$y_1 = h_1 - \frac{v_1^2}{2g} = 15 - .995 = 14.005 \text{ ft.;}$$

likewise, $$y_2 = 20 - .358 = 19.642 \text{ ft.;}$$

$$y_3 = 20 - 8.95 = 11.05 \text{ ft.}$$

The absolute pressure at the section s_1 is

$$p_1 = .434\, y_1 + 14.7 = .434 \times 14.005 + 14.7 = 20.778 \text{ lb. per sq. in.}$$

At s_2,

$$p_2 = .434 \times 19.642 + 14.7 = 23.225 \text{ lb. per sq. in.}$$

And at s_3,

$$p_3 = .434 \times 11.05 + 14.7 = 19.496 \text{ lb. per sq. in.} \quad \text{Ans.}$$

EXAMPLE 2.—If the hydrostatic head is 20 feet and the pressure head at a given section is 12 feet, what is the velocity at the section?

SOLUTION.—By formula **2**, $\frac{v^2}{2g} = h - y = 20 - 12 = 8$ ft.

Therefore,

$$v^2 = 2g \times 8, \text{ or } v = \sqrt{2g \times 8} = \sqrt{2 \times 32.16 \times 8}$$
$$= 22.684 \text{ ft. per sec.} \quad \text{Ans.}$$

7. Application of Bernoulli's Law.—The following example shows how Bernoulli's law may be used in the solution of actual problems.

EXAMPLE.—Water flows from a vessel a and through an inclined pipe p, Fig. 3. At the section s_1, the pipe diameter is 5 inches, while at the end s_2 the diameter is 3 inches. The tube inserted at s_1 shows a pressure head y_1 of 22 feet, and the vertical distance between sections s_1 and s_2 is $e_1 = 14$ feet. Required, the flow in cubic feet per second.

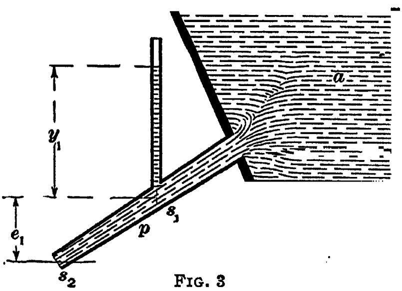


FIG. 3

SOLUTION.—Friction is neglected. Taking the line through s_2 as the datum line, the potential head for the section s_1 is 14 ft. and for the section s_2 is 0. Since, at s_2, the water is subjected only to atmospheric pressure, the pressure head is 0, while for the section s_1 it is 22 ft. Denoting by v_1 and v_2 the velocities at s_1 and s_2, the velocity heads are, respectively, $\frac{v_1^2}{2g}$ and $\frac{v_2^2}{2g}$. Applying formula **4** of Art. **5** to the sections s_1 and s_2,

$$\frac{v_1^2}{2g} + y_1 + e_1 = \frac{v_2^2}{2g} + y_2 + e_2,$$

or

$$\frac{v_1^2}{2g} + 22 + 14 = \frac{v_2^2}{2g} + 0 + 0;$$

whence,

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = 36,$$

or

$$v_2^2 - v_1^2 = 36 \times 2 \times 32.16 = 2{,}315.5$$

By the formula of Art. **2**, $A_1 v_1 = A_2 v_2$, or

$$v_2 = v_1 \times \frac{A_1}{A_2} = v_1 \times \frac{.7854 \times 5^2}{.7854 \times 3^2} = \frac{25}{9} v_1$$

Hence, $$v_2^2 = \left(\frac{25}{9} v_1\right)^2 = \frac{625}{81} v_1^2$$

Substituting this value of v_2^2 in the preceding equation,

$$\frac{625}{81} v_1^2 - v_1^2 = 2{,}315.5,$$

or $$v_1^2 = \frac{2{,}315.5}{\frac{625}{81} - 1} = \frac{2{,}315.5 \times 81}{625 - 81} = 344.77,$$

and $$v_1 = \sqrt{344.77} = 18.57 \text{ ft. per sec.}$$

$$v_2 = \frac{25}{9} v_1 = 51.583 \text{ ft. per sec.}$$

By formula **1** of Art. **1**,

$$Q = A_1 v_1 = \frac{.7854 \times 5^2}{144} \times 18.57 = 2.532 \text{ cu. ft. per sec.} \quad \text{Ans.}$$

8. Piezometers.—A gauge or tube inserted in a pipe to show the pressure of the water is called a **piezometer.** When a piezometer is to be placed on a pipe through which water is flowing, the tube should always be so inserted as to be at right angles to the current in the pipe, as shown at *a*, Fig. 4. If the tube is so inclined that the current flows against the end, as shown at *b*, the action of the current will force the water into the tube and cause it to rise higher than the head due to the pressure; and if inclined in the opposite direction, as at *c*, the action of the current will reduce the indicated pressure.

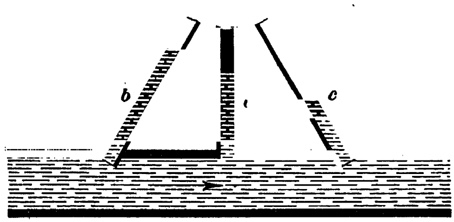


FIG. 4

If a pipe running full of water is equipped with a number of piezometers at various points along its length, the line joining the tops of the columns of water in the several piezometers is called the *hydraulic grade line.* In the case of a pipe of uniform diameter discharging from one reservoir into another, the hydraulic grade line is a straight line

drawn from a point on the surface of the water vertically above the inlet to a point on the surface vertically above the outlet.

EXAMPLES FOR PRACTICE

1. How much water per minute will pass any section of an 8-inch water main if the velocity is $6\frac{1}{2}$ feet per second? Ans. 136.14 cu. ft.

2. In example 1, what will be the velocity through a part of the main that is contracted to a diameter of $6\frac{1}{2}$ inches?

Ans. 9.846 ft. per sec.

3. The hydrostatic head on a given section of a pipe is 13 feet, the velocity of the water passing through the section is 12 feet per second, and friction is neglected. (*a*) What is the velocity head? (*b*) What is the pressure head? (*c*) What is the absolute fluid pressure at the section?

Ans. $\begin{cases} (a) & 2.239 \text{ ft.} \\ (b) & 10.761 \text{ ft.} \\ (c) & 19.37 \text{ lb. per sq. in.} \end{cases}$

4. In Fig. 3 suppose that the height of water shown by the piezometer is 15 feet and the vertical distance between the sections s_1 and s_2 is 10 feet. Let the pipe at s_1 be 6 inches in diameter, and at s_2 4 inches in diameter. Compute the discharge Q in cubic feet per second. Ans. 3.9 cu. ft.

5. When there is no flow, the piezometer inserted at a certain point of a pipe shows a height of 39 inches, but when the flow is started this height drops to 23 inches. Neglecting friction, what is the velocity of the flow at the section? Ans. 9.26 ft. per sec.

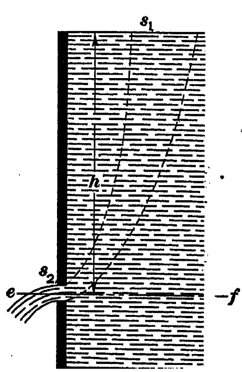


FIG. 5

FLOW OF WATER THROUGH ORIFICES

9. Theoretical Velocity of Efflux. Let a small aperture be made in the side of a vessel, Fig. 5, containing water and let the distance of the aperture from the upper level of the water be denoted by h. To determine the velocity with which the water will flow from the aperture, Bernoulli's law may be applied. A small mass of water may be imagined as starting at the surface s_1 and moving downwards to the orifice s_2 just as though enclosed in a

frictionless tube, as shown by the dotted outlines. At s_1, the velocity is practically zero, and the velocity head is therefore zero; the pressure head is likewise zero. If the line ef is taken as the datum line for potential heads, the potential head for s_1 is h. For s_2, let v denote the velocity with which the water flows through the aperture, called the **velocity of efflux**; then $\frac{v^2}{2g}$ is the velocity head. The pressure head is zero, the orifice being open to the atmosphere, and the potential head is also zero. Hence, applying Bernoulli's law,

$$0 + 0 + h = \frac{v^2}{2g} + 0 + 0;$$

whence, $v^2 = 2gh$, and

$$v = \sqrt{2gh}$$

The theoretical velocity of efflux is the same as the velocity the water would attain in falling through the height h.

EXAMPLE 1.—A small orifice is made in a pipe 50 feet below the water level; what is the velocity of the issuing water?

SOLUTION.—By the formula,

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 50} = 56.71 \text{ ft. per sec.} \quad \text{Ans.}$$

If the velocity of efflux is known, the head h required to produce the velocity can be found as shown by the following example:

EXAMPLE 2.—An issuing jet of water has a velocity of 60 feet per second; what is the head that causes it to flow with this velocity?

SOLUTION.—By transposing the formula,

$$h = \frac{v^2}{2g} = \frac{60^2}{2 \times 32.16} = 55.97 \text{ ft.} \quad \text{Ans.}$$

10. Flow Under Pressure.—Suppose that the surface of the water in the vessel of Fig. 5 is subjected to a pressure in addition to the pressure of the atmosphere. This extra pressure may be due to a loaded piston placed on the surface of the water, or perhaps to steam pressure.

Let p denote this extra pressure in pounds per square inch. The height of a column of water that will cause the pressure p is $h_1 = \frac{p}{.434} = 2.304\,p$; hence, under these conditions, the water at the surface s_1 has a pressure head $h_1 = 2.304\,p$.

If water issues from an orifice h feet below the surface that sustains the pressure h_1, Bernoulli's law gives the equation

$$0 + h_1 + h = \frac{v^2}{2g} + 0 + 0;$$

whence, $$v = \sqrt{2g(h_1 + h)} \qquad (1)$$

The total head, $h_1 + h$, is called the **equivalent head,** and must, in all cases, be reduced to feet before substituting in the formula.

EXAMPLE 1.—The area of a piston fitting a vertical vessel filled with water is 27.36 square inches. The total pressure on the piston is 80 pounds; the weight of the piston is 25 pounds; and the head of the water at the level of the orifice is 6 feet 10 inches; what is the velocity of the efflux, assuming that there are no resistances?

SOLUTION.— $80 + 25 = 105$ lb., the total pressure on the upper surface of the liquid.

$$105 \div 27.36 = 3.8377 \text{ lb. per sq. in.}$$

Then, $$h_1 = 3.8377 \div .434 = 8.8426 \text{ ft.},$$

the head, in feet, due to the pressure of 105 lb.;

$$h = 6 \text{ ft. } 10 \text{ in.} = 6.8333 \text{ ft.}$$

Using formula **1**,

$$v = \sqrt{2g(h_1 + h)} = \sqrt{2g(8.8426 + 6.8333)} = \sqrt{2 \times 32.16 \times 15.6759}$$
$$= 31.75 \text{ ft. per sec.} \quad \text{Ans.}$$

Because the atmospheric pressure is the same at both the upper surface and the point of efflux, its effect at these two points is neutralized and pressures above the atmosphere are used instead of absolute pressures. When the fluid is discharged into a vacuum, however, absolute pressures must be used.

In deriving the formulas of Art. **9** and formula **1** of this article, it is assumed that the orifice opens to the atmosphere. If the orifice is subjected to an external pressure, which may be denoted by p_2, the corresponding pressure head on the orifice is $h_2 = 2.304\, p_2$. Using this pressure head for s_2, Bernoulli's law gives the equation

$$0 + h_1 + h = \frac{v^2}{2g} + h_2 + 0;$$

whence, $\frac{v^2}{2g} = h + h_1 - h_2$, and

$$v = \sqrt{2g(h + h_1 - h_2)} \qquad (2)$$

EXAMPLE 2.—Water under a head of 175 feet flows through an orifice into a tank containing compressed air having a gauge pressure of 35 pounds per square inch; find the velocity of efflux.

SOLUTION.—Use formula 2, $v = \sqrt{2g(h + h_1 - h_2)}$. $h = 175$, $h_1 = 0$, and $h_2 = 2.304 \times 35 = 80.64$ ft. Then, substituting,

$$v = \sqrt{2 \times 32.16 \times (175 - 80.64)} = 77.9 \text{ ft. per sec. Ans.}$$

11. Flow Through a Large Orifice.—If, in Fig. 5, the area of the orifice is greater than about one-twentieth of the area of the cross-section of the vessel, the velocity of the water at the surface becomes appreciable and must be taken into account.

Let a = area of orifice in any unit, as square feet or square inches;
A = area of surface, in the same unit as a;
v = velocity of efflux, in feet per second;
V = velocity with which surface s_1 sinks, in feet per second.

By the formula of Art. **2**, $A V = a v$, or $V = \frac{a}{A} v$. The velocity head at s_1 is $\frac{V^2}{2g}$, and, using this instead of zero, Bernoulli's law gives

$$\frac{V^2}{2g} + 0 + h = \frac{v^2}{2g} + 0 + 0$$

Transposing, $\frac{v^2}{2g} - \frac{V^2}{2g} = h$; but $V^2 = \frac{a^2}{A^2} v^2$; hence

$$\frac{v^2}{2g} - \frac{a^2 v^2}{2g A^2} = h,$$

or

$$\frac{v^2}{2g}\left(1 - \frac{a^2}{A^2}\right) = h$$

Solving for v,

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}}$$

When the area at the orifice is more than one-twentieth of the area of the cross-section of the vessel, this formula should be used; when less than one-twentieth of that area, the formula of Art. **9** gives results that are sufficiently accurate.

EXAMPLE.—An orifice 4 inches square is cut in the bottom of a vessel having a rectangular cross-section 11 inches by 14 inches; the water level is 14 feet above the bottom. Compute: (a) the velocity of efflux; (b) the discharge per second.

SOLUTION.—(a) Area of cross-section is $14 \times 11 = 154$ sq. in. Area of orifice is $4 \times 4 = 16$ sq. in. Since $154 \div 16 = 9\frac{5}{8}$, the area of the surface is less than twenty times the area of the orifice; hence, using the above formula and substituting,

$$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}} = \sqrt{\frac{2 \times 32.16 \times 14}{1 - \frac{16^2}{154^2}}} = 30.17 \text{ ft. per sec. Ans.}$$

(b) Using the formula of Art. **1**,

$$Q = Av = \frac{16}{144} \times 30.17 = 3.352 \text{ cu. ft. per sec. Ans.}$$

If the formula of Art. **9** is used instead of the more exact formula just given, the velocity is found to be

$$v = \sqrt{2 \times 32.16 \times 14} = 30.008 \text{ ft. per sec.,}$$

which is slightly less than the value obtained by the formula of this article.

STANDARD ORIFICES

12. Edges of Standard Orifices.—An orifice in the side or bottom of a vessel or reservoir, and at a distance below the surface of the water, is called a **standard orifice** when the flow through it takes place in such a manner that the jet touches the opening on the inside edge only. A hole in a thin plate, as shown in Fig. 6, is such an orifice, as is also a square-edged hole in the side of a vessel, as shown in Fig. 7, when the thickness of the side is not so great that the jet touches it beyond the inner edge. If the sides of the reservoir are very thick, a standard orifice can be made by beveling the outer edges, as shown in Fig. 8.

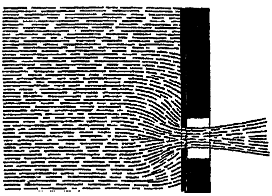


FIG. 6

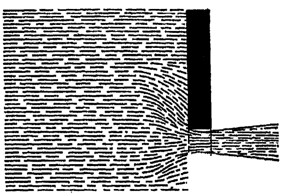


FIG. 7

13. Contraction of the Jet.—When a jet issues from a circular orifice, it contracts so that the diameter is least at a distance from the edge equal to about one-half the diameter of the orifice. Beyond this point, the jet gradually enlarges and becomes broken by the resistance of the air. Orifices that are not circular also cause contractions.

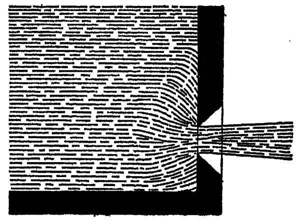


FIG. 8

The **coefficient of contraction** is the number by which the area of the orifice is multiplied in order to obtain the least cross-section of the jet. Experiments on jets from standard orifices have given values for this coefficient varying from .57 to .71. The probable mean value is about .62.

14. Coefficient of Velocity.—The number by which the theoretical velocity must be multiplied in order to obtain the actual maximum velocity, or the velocity where the cross-section of the jet is least, is called the **coefficient of velocity.**

If v = theoretical velocity;
v' = actual velocity;
c' = coefficient of velocity;

the formula of Art. **9** becomes,

$$v' = c'v = c'\sqrt{2gh}$$

It is found that c' is greater for high heads than for low, and values ranging from .975 to nearly 1 have been obtained by different experimenters. An average value usually taken is .98.

EXAMPLE 1.—What is the actual velocity of discharge from a small standard orifice in the side of a vessel, if the head is 20 feet?

SOLUTION.—

$v' = c'\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 20} = 35.15$ ft. per sec. Ans.

Most of the problems occurring in hydraulics involve the operations of multiplication, division, involution, and evolution in such a way that they are most readily solved by the

use of logarithms. *Logarithms*, therefore, should be reviewed very carefully and thoroughly in order that its principles may be readily applied to the solutions of the problems here given.

EXAMPLE 2.—Solve example 1 by means of logarithms.

SOLUTION.—First find the logarithm of the product of the numbers under the radical sign from the table of logarithms by adding the logarithms of the individual numbers, as follows:

$$\begin{aligned} \log\ 2 &= .30103 \\ \log 32.16 &= 1.50732 \\ \log 20 &= \underline{1.30103} \\ &\ 3.10938 = \log\,(2 \times 32.16 \times 20) \end{aligned}$$

The logarithm of the square root of this product is found by dividing its logarithm by 2; thus,

$$\log \sqrt{2 \times 32.16 \times 20} = 3.10938 \div 2 = 1.55469$$

Finally, the logarithm of the product of .98 multiplied by the quantity under the radical sign is the sum of the logarithm of .98 and 1.55469, or $\bar{1}.99123 + 1.55469 = 1.54592$. From the table of logarithms, the number corresponding to this logarithm is found to be 35.15. Ans.

15. Coefficient of Discharge.—It is evident that the contraction of the jet issuing from an orifice and the reduction of the theoretical velocity at the smallest cross-section both tend to reduce the quantity of water flowing through an orifice as calculated from the formulas $Q = a\,v$ and $v = \sqrt{2gh}$. Let Q' denote the theoretical discharge and Q the actual discharge; then the ratio $\frac{Q}{Q'}$ is called the **coefficient of discharge.** Denoting this coefficient by k,

$$Q = k\,Q'$$

The values of k vary with the shape of the orifice, the head, and the velocity of discharge. These values have been determined by experiment, and it is found that they vary between .59 and .63 for the most practical cases. For ordinary computations, an average value of k may be taken as .61. Therefore,

$$Q = k\,Q' = k\,A\,v = k\,A\sqrt{2gh}$$

or

$$Q = .61\,A\sqrt{2gh}$$

where A = area of orifice, in square feet;
Q = discharge, in cubic feet per second;
h = head on center of orifice, in feet.

EXAMPLE.—What will be the actual discharge from a circular standard orifice 3 inches in diameter under a head of 25 feet?

SOLUTION.—The area of a 3-in. circle is

$$.7854 \times 3^2 = 7.0686 \text{ sq. in.} = 7.0686 \div 144 = .049 \text{ sq. ft.}$$

$$Q = k\,A\sqrt{2gh} = .61 \times .049\sqrt{2 \times 32.16 \times 25} = 1.1986 \text{ cu. ft. per sec}$$

Ans

EXAMPLES FOR PRACTICE

1. What is the discharge, in cubic feet per minute, from a standard circular orifice whose diameter is $2\frac{1}{2}$ inches, if the head is 20 feet?
Ans. 44.75 cu. ft. per min.

2. A square orifice in the side of a reservoir measures .2 foot on each side, and the head on the center is 22 feet; what is the discharge in cubic feet per second?
Ans. .9178 cu. ft. per sec.

3. What is the discharge from a rectangular orifice 1 foot wide, if the head on the upper edge is $2\frac{1}{2}$ feet and the depth of the orifice $10\frac{1}{2}$ inches?
Ans. 7.337 cu. ft. per sec.

WEIRS

16. Use of Weirs.—A **weir** is a vertical obstruction placed across a stream or channel and containing a notch (or a number of them) in its upper edge through which water is allowed to flow for purposes of measurement. It has been found that, when properly constructed and carefully managed, a weir forms one of the most convenient and accurate devices for measuring the discharge of streams. Many careful experiments have been made to determine the quantity of water that will flow through different forms of weirs under varying conditions. As a result of these experiments, rectangular weirs have come to be generally used, and the amount of flow in any particular case may be calculated by simple formulas with tabulated coefficients that depend on observed conditions. Triangular weirs are occasionally used for experimental purposes.

17. Rectangular Weirs.—There are two types of rectangular weirs; those with and those without end contractions. A **weir with end contractions** is shown in Fig. 9. The notch is narrower than the channel

through which the water flows and this causes a contraction at the bottom and at the two sides of the issuing stream. A **weir without end contractions** is shown in Fig. 10. In

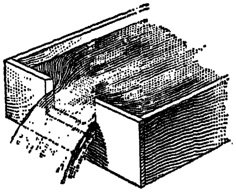


FIG. 9

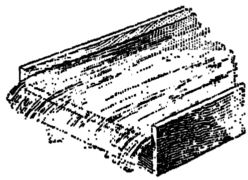


FIG. 10

this case, the notch is as wide as the channel leading to it; consequently, the issuing stream is contracted at the bottom only.

The edge *a* of the notch, in either Fig. 11 or Fig. 12, is called the **crest of the weir.** The inner edges of the notch are made sharp, so that the water in passing through it touches only along a line. For very accurate work, the edges, both vertical and horizontal, should be made with a thin plate of metal having a sharp inner edge, as shown in Fig. 11; but for ordinary work, the edges of the board in which the notch is cut may be chamfered off to an angle of about 30°, as shown in Fig. 12.

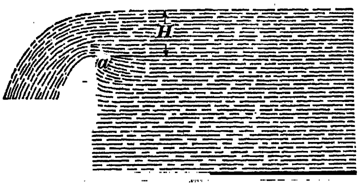


FIG. 11

The bottom edge of the notch must be straight and set perfectly level, and the sides must be set carefully at right angles to the bottom.

The head *H* producing the flow, Figs. 11 and 12, is the vertical distance from the crest to the surface of the water. It must be measured at a point so far from the crest that the curvature of the flowing water will not affect the measurement.

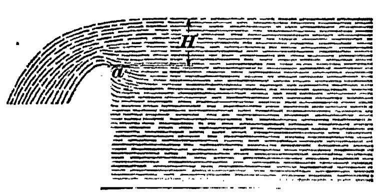


FIG. 12

The distance from the crest of the weir to the bottom of the feeding canal or reservoir should be at least three times the head; and, with a weir having end contractions, th tance from the vertical edges to the sides of the canal s also be at least three times the head.

The water must approach the weir quietly, and with little velocity. It is sometimes necessary to provide means for reducing the velocity of the water as it approaches the weir.

18. Discharge of Weirs.—When the dimensions of the notch and the head on the crest of a weir are known, the discharge can be computed by means of the following formulas and tables of coefficients:

Let l = length of weir in feet;
H = measured head, in feet;
v = velocity, in feet per second, with which water approaches weir;
h = head equivalent to velocity with which water approaches weir;
k = coefficient of discharge;
Q' = theoretical discharge, in cubic feet per second;
Q = actual discharge, in cubic feet per second.

The formula for the theoretical discharge per second is obtained by the use of higher mathematics and is

$$Q' = \tfrac{2}{3}\sqrt{2g}\,l\,(H+h)^{\frac{3}{2}} \qquad (1)$$

If there is no velocity of approach, this becomes

$$Q' = \tfrac{2}{3}\sqrt{2g}\,l\,H^{\frac{3}{2}} \qquad (2)$$

The actual discharge for weirs with end contractions, and considering the velocity of approach, is given by the formula

$$Q = \tfrac{2}{3}k\sqrt{2g}\,l\,(H+1.4\,h)^{\frac{3}{2}} = 5.347\,k\,l\,(H+1.4\,h)^{\frac{3}{2}} \qquad (3)$$

If there is no velocity of approach, this becomes

$$Q = \tfrac{2}{3}k\sqrt{2g}\,l\,H^{\frac{3}{2}} = 5.347\,k\,l\,H^{\frac{3}{2}} \qquad (4)$$

For weirs without end contractions, and considering the velocity of approach, the formula is

$$Q = \tfrac{2}{3}k\sqrt{2g}\,l\,(H+\tfrac{4}{3}h)^{\frac{3}{2}} = 5.347\,k\,l\,(H+\tfrac{4}{3}h)^{\frac{3}{2}} \qquad (5)$$

If there is no velocity of approach, this becomes

$$Q = \tfrac{2}{3}k\sqrt{2g}\,l\,H^{\frac{3}{2}} = 5.347\,k\,l\,H^{\frac{3}{2}} \qquad (6)$$

EXAMPLE.—What is the discharge of a stream, if the length of the weir is 5 feet, the head $10\frac{1}{2}$ inches, the coefficient of discharge .603, and the velocity of approach = 0, the weir having end contractions?

SOLUTION.—Applying formula **4**, and substituting,

$$Q = 5.347 \times .603 \times 5 \times .875^{\frac{3}{2}} = 13.195 \text{ cu. ft. per sec.} \quad \text{Ans.}$$

19. The value of the coefficient k varies with the effective head and width of the weir. Tables I and II give fairly accurate values for these coefficients for the given conditions.

TABLE I

COEFFICIENTS FOR WEIRS WITH END CONTRACTIONS

Effective Head Feet	Length of Weir, in Feet						
	.66	1	2	3	5	10	19
.1	.632	.639	.646	.652	.653	.655	.656
.15	.619	.625	.634	.638	.640	.641	.642
.20	.611	.618	.626	.630	.631	.633	.634
.25	.605	.612	.621	.624	.626	.628	.629
.30	.601	.608	.616	.619	.621	.624	.625
.40	.595	.601	.609	.613	.615	.618	.620
.50	.590	.596	.605	.608	.611	.615	.617
.60	.587	.593	.601	.605	.608	.613	.615
.70		.590	.598	.603	.606	.612	.614
.80			.595	.600	.604	.611	.613
.90			.592	.598	.603	.609	.612
1.00			.590	.595	.601	.608	.611
1.2			.585	.591	.597	.605	.610
1.4			.580	.587	.594	.602	.609
1.6				.582	.591	.600	.607

NOTE.—The head given is the effective head, $H + 1.4\,h$. When the velocity of approach is small, h is neglected.

Table I, Coefficients for Weirs with End Contractions, gives values of the coefficient of discharge k for weirs with end contractions and different values of H and l. Table II, Coefficients for Weirs Without End Contractions, gives values for k for weirs without end contractions. Weirs with end

contractions are more often used and are to be recommended in most cases. Values of k for values of H and l between those given in the tables can be found by interpolating, assuming that the variation is uniform between the values given.

TABLE II

COEFFICIENTS FOR WEIRS WITHOUT END CONTRACTIONS

Effective Head Feet	Length of Weir, in Feet						
	19	10	7	5	4	3	2
.10	.657	.658	.658	.659			
.15	.643	.644	.645	.645	.647	.649	.652
.20	.635	.637	.637	.638	.641	.642	.645
.25	.630	.632	.633	.634	.636	.638	.641
.30	.626	.628	.629	.631	.633	.636	.639
.40	.621	.623	.625	.628	.630	.633	.636
.50	.619	.621	.624	.627	.630	.633	.637
.60	.618	.620	.623	.627	.630	.634	.638
.70	.618	.620	.624	.628	.631	.635	.640
.80	.618	.621	.625	.629	.633	.637	.643
.90	.619	.622	.627	.631	.635	.639	.645
1.00	.619	.624	.628	.633	.637	.641	.648
1.2	.620	.626	.632	.636	.641	.646	
1.4	.622	.629	.634	.640	.644		
1.6	.623	.631	.637	.642	.647		

NOTE.—The head given is the effective head, $H + \frac{4}{3}h$. When the velocity of approach is small, h may be neglected.

20. The **velocity of approach** is the mean velocity with which the water flows through the canal leading to the weir. If A is the area of the cross-section of the water in this canal, $v = \frac{Q}{A}$, and the equivalent head is

$$h = \frac{v^2}{2g} = .01555\ v^2$$

The velocity v may be measured approximately by means of a float on the water in the canal or stream, but a better

method is to compute the flow Q by formula **4** or formula **6** of Art. **18**, assuming that $v = 0$, and then calculate an approximate value of v from $v = \frac{Q}{A}$. However, since v is small with a properly constructed weir, it is usually neglected, unless great accuracy is required.

EXAMPLE 1.—What is the discharge from a weir with end contractions under the following conditions: the length of the weir is 4 feet $1\frac{1}{2}$ inches, and the measured head $10\frac{1}{16}$ inches? Assume that there is no velocity of approach.

SOLUTION.—The length l of the weir is 4 ft. $1\frac{1}{2}$ in. = 4.125 ft., and the head H is $10\frac{1}{16}$ in. = .84 ft., nearly. From Table I, the coefficient k = .600 for a weir 3 ft. long and a head of .8 ft. and k = .604 for a weir 5 ft. long with the same head. There is an increase in the coefficient of (.604 − .600) ÷ 2 = .002 for each increase of 1 ft. in length. The coefficient for a weir 4.125 ft. long is, therefore,

$$.600 + (1.125 \times .002) = .60225$$

The coefficient k = .603 for a weir 5 ft. long with a head of .9 ft. and k = .598 for a weir 3 ft. long with the same head. There is an increase in the coefficient of (.603 − .598) ÷ 2 = .0025 for each increase of 1 ft. in length.

The coefficient for a weir 4.125 ft. long with a head of .9 ft. is, therefore,

$$.598 + (1.125 \times .0025) = .60081$$

The decrease in coefficient for an increase in head of .1 ft. is

$$.60225 - .60081 = .00144$$

and for an increase in head of .04 ft. the decrease is

$$.00144 \times \frac{.04}{.1} = .000576$$

This subtracted from the coefficient for .8 ft. gives .60225 − .000576 = .601674 as the coefficient of discharge for a weir 4.125 ft. long and a head of .84 ft. Using but four decimal places, the discharge, by formula 4 of Art. **18**, is

$$Q = 5.347 \times .6017 \times 4.125 \times .84^{\frac{3}{2}} = 5.347 \times .6017 \times 4.125 \times \sqrt{.84^3} = 10.22$$

cu. ft. per sec. Ans.

EXAMPLE 2.—If the canal leading to the weir of example 1 is 10 feet wide and 3 feet deep below the crest of the weir, what is the head equivalent to the velocity of approach?

SOLUTION.—The depth of water in the canal is the depth below the crest plus the head = 3.84 feet. The area of the cross-section of the water in the canal is $A = 3.84 \times 10 = 38.4$ sq. ft.,

and the velocity is

$$v = 10.22 \div 38.4 = .266 \text{ ft. per sec.}$$

The head h equivalent to the velocity v is

$$h = \frac{v^2}{2g} = \frac{.266^2}{64.32} = .0011 \text{ ft.} \quad \text{Ans.}$$

NOTE.—This value of h is so small that its influence on the discharge is much less than the probable errors in measuring the head H, and so need not be considered in finding the discharge.

EXAMPLES FOR PRACTICE

1. What is the discharge per second from a weir with end contractions when the length of the weir is 24 inches, and the measured head is $10\frac{1}{2}$ inches? Assume that there is no velocity of approach.
Ans. 5.188 cu. ft.

2. With the same conditions as in example 1, except that the weir is without end contractions, what is the discharge per second?
Ans. 5.641 cu. ft.

3. What is the discharge per second from a weir with end contractions 36 inches long, with a head of 9 inches and a velocity of approach of 3 feet per second?
Ans. 8.80 cu. ft.

FLOW THROUGH TUBES

21. The Standard Tube.—A **standard tube, or an adjutage,** is a tube whose length is two and one-half or three times its diameter. When water flows from a reservoir through such a tube, as shown in Fig. 13, the jet contracts when it first leaves the reservoir, then expands again until it fills the tube near its outer end, this contraction and expansion resembling that of the jet from a standard orifice.

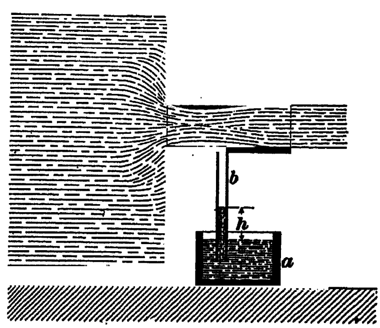


FIG. 13

Owing to the contraction of the jet, there is a space between the jet and the tube at the point where the jet is smallest. When the tube extends far enough beyond the contraction, the jet again fills the tube; the swiftly moving water of the jet carries some of the air, from the space, along with it, thus producing a partial vacuum around the contracted portion of the jet.

If a small branch tube as shown at b, Fig. 13, is carried down into a cup of mercury a, the pressure of the atmosphere will force mercury into b to a height h that depends on the vacuum in the space around the jet, and if a small hole is made in the tube it will be found that air is drawn in through the hole.

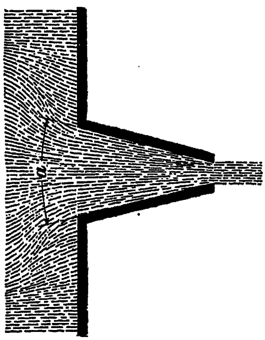


FIG. 14

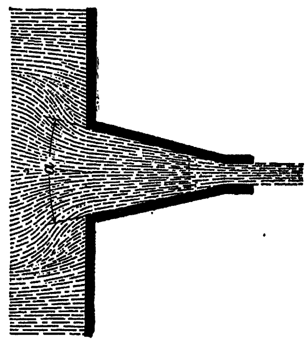


FIG. 15

On account of the difficulty in maintaining uniform conditions, which makes the value of the coefficient of discharge uncertain, tubes are seldom used for measuring the flow of water.

The coefficient of discharge for a standard tube is greater than for a standard orifice. An average value is $k = .82$.

The coefficient of velocity for cylindrical tubes is the same as the coefficient of discharge.

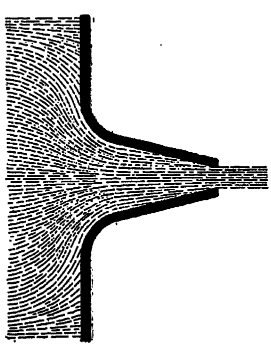


FIG. 16

22. Conical Tubes.—For **conical tubes**, as shown in Figs. 14 and 15, the coefficient of discharge reaches a maximum value of .946 when the angle a of the cone is 13° 24′. If the inner edge of the tube is well rounded, as shown in Fig. 16, the coefficient of discharge is still further increased and may be made nearly 1.

The coefficient of velocity for conical tubes increases with the angle of the cone until at an angle of about 40° it becomes

approximately the same as the coefficient of velocity for the standard orifice, which, it will be remembered, is .98.

23. Compound Tubes.—Examples of **compound tubes** are shown in Figs. 17 and 18. Experiments have shown that the velocity through the minimum section *a* is greater

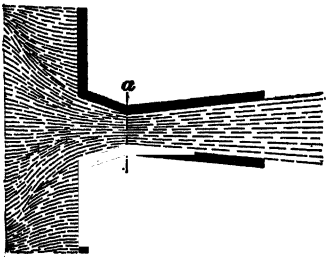


FIG. 17

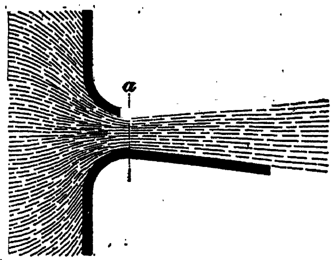


FIG. 18

than the theoretical velocity due to the head. The values of the coefficient of discharge for the section *a* vary greatly under different conditions of head and proportions of tubes. Under certain conditions, values as high as 2.43 have been obtained.

24. Inward Projecting Tubes.—It has been demonstrated by experiment that when a tube projects into a reservoir, as shown in Figs. 19 and 20, the contraction is increased and

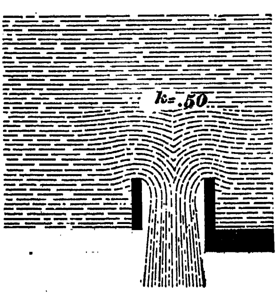


FIG. 19

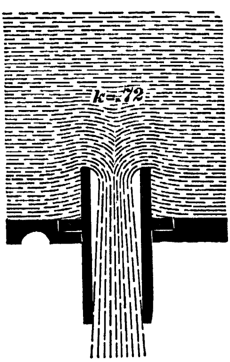


FIG. 20

the discharge is greatly reduced. For the tube shown in Fig. 19, the coefficient of discharge is about .50; and for that shown in Fig. 20, about .72.

FLOW OF WATER IN PIPES

LOSSES OF HEAD

25. Preliminary Statement.—In considering the flow of water through pipes of considerable length, one important factor hitherto neglected must receive attention; namely, the friction between the water and the interior surface of the pipe.

Referring to Fig. 21, the level of the water in the reservoir is at a height h above the end of the pipe. If the pipe discharges into the atmosphere, h is the head that causes the flow; but if the pipe discharges into a second reservoir, the head h is the difference between the water levels in the two reservoirs. If G pounds of water are discharged per second, the theoretical work of this quantity of water falling through the height h is $G h$ foot-pounds. Now, if v is the velocity of discharge, $\frac{G v^2}{2 g}$ is the kinetic energy of the outflowing water; and if there is no work done in overcoming friction, the two expressions are equal, or $G h = G \frac{v^2}{2 g}$, or $h = \frac{v^2}{2 g}$, as in the case of a theoretical flow from an orifice. In the case of long pipes, it is always found that $\frac{G v^2}{2 g}$ is less than $G h$, the difference being the work done against the various frictional resistances. Hence, the head $\frac{v^2}{2 g}$ is less than the total head h, and the difference $h - \frac{v^2}{2 g}$ is the loss of head due to frictional resistance. In the case of very long pipes, the

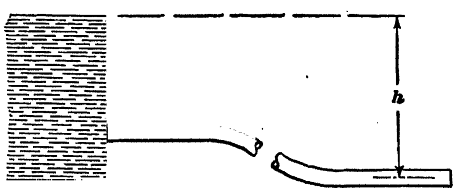


FIG. 21

velocity v may become very small, showing that nearly all the head h has been lost in overcoming resistance.

In the following paragraphs, the losses of head occasioned by resistances of various kinds will be considered.

26. Loss of Head From Friction in the Pipe. Experiments have shown that the friction of water flowing through a pipe depends, approximately, on the following laws:

I. *The loss in friction is proportional to the length of the pipe.*
II. *It varies nearly as the square of the velocity.*
III. *It varies inversely as the diameter of the pipe.*
IV. *It increases with the roughness of the pipe.*
V. *It is independent of the pressure in the pipe.*

The following formula is based on experimental data, and is in accord with the above laws:

Let l = length of pipe, in feet;
d = diameter of pipe, in feet;
v = velocity, in feet per second, of water flowing in pipe;
f = a coefficient depending on roughness of pipe; the value .02 is usually taken where very accurate results are not required;
h_f = head lost by friction;
g = 32.16.

Then,
$$h_f = f\frac{l v^2}{d\,2g}$$

Example.—What is the loss of head due to friction in a 10-inch pipe 1,000 feet long, if the mean velocity of flow is 8 feet per second and $f = .0197$?

Solution.—Using the formula and substituting,
$$h_f = f\frac{l v^2}{d\,2g} = .0197 \times \frac{1{,}000}{.83\frac{1}{3}} \times \frac{8^2}{2 \times 32.16} = 23.522 \text{ ft.} \quad \text{Ans.}$$

27. Loss of Head at Entrance.—Water, on entering a pipe from a reservoir, meets with resistances due to friction and contraction, and there is a loss of head similar to

the loss due to friction. The usual form of the equation expressing this loss of head is

$$h_e = m\frac{v^2}{2g}$$

where v = velocity, in feet per second;
m = coefficient depending on form of end of pipe;
h_e = head lost at entrance, in feet.

For long water mains the value of m is usually taken as .5.

FIG. 22

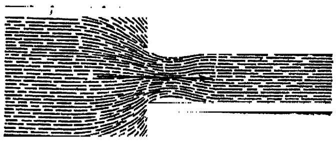


FIG. 23

28. Loss of Head Due to Change of Section.—When water flows from a small section to a larger one, as shown in Fig. 22, energy is absorbed in producing eddies among the

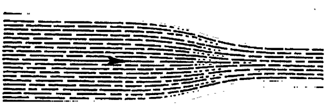


FIG. 24

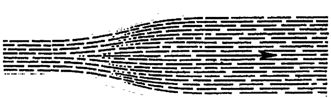


FIG. 25

water particles just at the enlargement. The change from a large to a smaller section, as shown in Fig. 23, causes a contraction in the mouth of the smaller section. The result in both cases is a loss of head. If the change in section in the pipe is made gradually, as in Figs. 24 and 25, the loss is small and may be neglected when computing the flow. In practice, a change in section is usually made by means of a **reducer**, like that shown in Fig. 26.

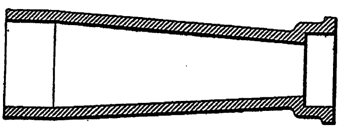


FIG. 26

29. Loss of Head Due to Bend.—When there are sudden bends in the pipe, there will be a loss, due partly to shock and eddies, and partly to the contraction in the flow,

as shown in Figs. 27 and 28. Experiments made with bends like that shown in Fig. 27 show that the loss of

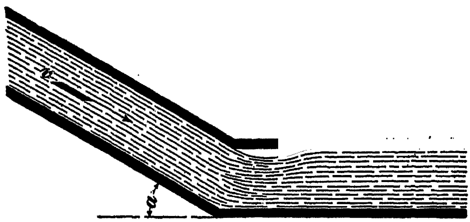


FIG. 27

head h_b may be expressed in terms of the mean velocity by the formula

$$h_b = c\frac{v^2}{2g} \qquad (1)$$

Table III gives values of c for different values of the angle a.

For a 90° bend like that shown in Fig. 28, the loss of head h_b' is expressed by the formula

$$h_b' = c'\frac{v^2}{2g} \qquad (2)$$

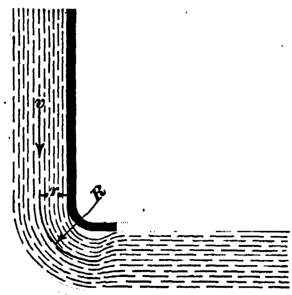


FIG. 28

in which c' depends on the ratio between the radius r of the pipe and the radius R of the bend.

TABLE III

VALUES OF c FOR ANGLES a

$a =$	10°	20°	40°	60°	80°	90°	100°	110°	120°	130°	140°	150°
$c =$	.017	.046	.139	.364	.74	.984	1.26	1.56	1.86	2.16	2.43	2.81

Table IV gives values of c' corresponding to various values of the ratio $\frac{r}{R}$.

From Table IV, it is seen that when R is made large in comparison to r, the value of c', and hence the loss in head, is small.

There may be other resistances, such as valves, that change the direction of flow of the water or suddenly change the area through which the water flows. If the pipe is carefully designed and laid, however, these losses may be made so small in comparison with the other losses just named that they may be neglected in the formulas for head and velocity.

TABLE IV

VALUES OF c' FOR RATIOS $\frac{r}{R}$

$\frac{r}{R}$ =	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
c' =	.131	.138	.158	.206	.294	.440	.661	.977	1.408	1.978

GENERAL FORMULAS FOR FLOW IN PIPES

30. Formulas for Velocity of Flow.—As stated in Art. **25**, the difference $h - \frac{v^2}{2g}$, between the total head and velocity head, is the loss of head due to the various frictional resistances; hence, if a pipe has n bends, like that shown in Fig. 28,

$$h - \frac{v^2}{2g} = h_f + h_e + n\,h_b';$$

whence,
$$v = \sqrt{2g\left[h - (h_f + h_e + n\,h_b')\right]} \qquad (1)$$

Using the values of h_f, h_e, and h_b' from the formulas of Arts. **26** and **27**, and formula **2** of Art. **29**,

$$h - \frac{v^2}{2g} = f\frac{l}{d}\frac{v^2}{2g} + m\frac{v^2}{2g} + n\,c'\,\frac{v^2}{2g}$$

Solving for v, the following formula is obtained for the velocity of flow:

$$v = \sqrt{\frac{2gh}{1 + f\frac{l}{d} + m + n\,c'}} = 8.02\sqrt{\frac{h}{1 + f\frac{l}{d} + m + n\,c'}} \qquad (2)$$

When the form of bend shown in Fig. 27 is used, the coefficient c from Table III must be used instead of c'.

Giving m the value .5 for the common case of a pipe a bell end, and assuming that there are no sharp be similar resistances, the formula for v becomes

$$v = \sqrt{\frac{2gh}{1.5 + f\frac{l}{d}}} = 8.02\sqrt{\frac{h}{1.5 + f\frac{l}{d}}} \qquad (3)$$

31. Flow in Long Pipes.—When the pipe is very long, compared with its diameter, say when l exceeds 4,000 d, the term $f\frac{l}{d}$ becomes so much larger than the 1.5 to which it is added in formula **3** of Art. **30**, that the 1.5 may be disregarded, and the formula becomes

$$v = \sqrt{\frac{2gh}{f\frac{l}{d}}} = \sqrt{\left(\frac{2g}{f}\right)\left(\frac{hd}{l}\right)} \qquad (1)$$

The factor $\frac{2g}{f}$ may be replaced by a new symbol $\frac{1}{c}$, in which case the formula takes the form

$$v = \sqrt{\frac{hd}{cl}} \qquad (2)$$

This is D'Arcy's formula for the flow in long pipes.

Since the quantity Q discharged is given by the formula $Q = Av$, the following formula is obtained for the discharge, in cubic feet per second:

$$Q = A\sqrt{\frac{hd}{cl}} \qquad (3)$$

or, since $A = .7854\ d^2$ for circular pipes, $Q = .7854\ d^2\sqrt{\frac{hd}{cl}}$, which may also be written

$$Q = \sqrt{\frac{.617\ d^5 h}{cl}} \qquad (4)$$

32. Table V gives the values of the coefficient c based on the experiments of D'Arcy.

It will be observed that the coefficient for smooth pipes is in all cases half that of rough ones. As all pipes, no matter how clean and smooth they may be when first laid,

become in process of time more or less coated and foul, it is safer in practice to always use the coefficient for rough pipes when a permanent system is being laid down.

TABLE V

TABLE OF COEFFICIENTS

Diameters Inches	Value of c for Rough Pipes	Value of c for Smooth Pipes
3	.00080	.00040
4	.00076	.00038
6	.00072	.00036
8	.00068	.00034
10	.00066	.00033
12	.00066	.00033
14	.00065	.00033
16	.00064	.00032
24	.00064	.00032
30	.00063	.00032
36	.00062	.00031
48	.00062	.00031

33. By carefully studying Table V, it will be seen that the coefficients for pipes from 8 to 48 inches in diameter do not vary greatly. Moreover, from formula **3** of Art. **31**, it appears that, all other conditions being equal, the quantity discharged is affected by only the square root of the coefficient, so that slight differences in its value are insignificant in reference to the volume of water discharged. Formula **4** of Art. **31** contains the factor .617; hence, by taking .000617 as an approximate coefficient for pipes within the limits of 8 and 48 inches in diameter, the formula becomes

$$Q = \sqrt{\frac{.617\, d^5 h}{.000617\, l}};$$

whence,

$$Q = \sqrt{\frac{1{,}000\, d^5 h}{l}} \qquad (1)$$

Now, let H denote the head per thousand feet, so that $\frac{h}{l} = \frac{H}{1{,}000}$. Substituting this in the formula just given,

$$Q = \sqrt{d^5 H} \qquad (2)$$

In this formula, it must be borne in mind that H is the fall in feet per thousand feet.

For pipes of smaller diameter, from 3 to 8 inches, it is well to assume a coefficient of .000785. Then for such pipes, from formula **4** of Art. **31**,

$$Q = \sqrt{\frac{.617\, d^5 h}{.000785\, l}} = .887\sqrt{d^5 H} \qquad (3)$$

That is to say, for these smaller diameters the delivery will be, in round numbers, 90 per cent. of that given by formula **2.**

34. Formulas for Smooth Pipes.—While, in practice, the formulas for rough pipes should always be used, it is sometimes useful to know the probable discharge through smooth ones. Since the coefficients for the latter are always one-half of those for the former, for smooth pipes formula **2** of Art. **33** may be written,

$$Q = \sqrt{2\, d^5 H} = 1.4\sqrt{d^5 H}$$

In general, the discharge through a smooth pipe is 1.4 times that through a rough pipe of the same diameter; and, reciprocally, the discharge through a rough pipe is .7 that through a smooth one of the same diameter; these factors represent the practical limits between which the extremes of roughness and smoothness can affect the flow through long pipes.

35. Diameter of Pipe for Given Flow.—Formulas giving the diameter of a pipe for a desired discharge per second are readily obtained from the formulas in Arts. **33** and **34.** Solving formula **2** of Art. **33** for d gives for rough pipes,

$$d = \sqrt[5]{\frac{Q^2}{H}} \qquad (1)$$

For smooth pipes, by solving the formula in Art. **34,**

$$d = \sqrt[5]{\frac{Q^2}{2\, H}} \qquad (2)$$

For pipes of small diameters, up to about 8 inches, multiply by 1.05; that is, add 5 per cent. to the diameters given by these formulas.

36. Formulas for Velocity.—From the formula, $v = \frac{Q}{A}$, and as $A = .7854\, d^2$ for circular pipes $v = \frac{Q}{.7854\, d^2}$; from

formula **2** of Art. **33**, $Q = \sqrt{d^5 H}$; hence, for large rough pipes,

$$v = \frac{\sqrt{d^5 H}}{.7854\, d^2} = 1.27 \sqrt{d H} \qquad (1)$$

Similarly, for small rough pipes,

$$v = 1.13 \sqrt{d H} \qquad (2)$$

For large smooth pipes,

$$v = 1.78 \sqrt{d H} \qquad (3)$$

and for small smooth pipes

$$v = 1.6 \sqrt{d H} \qquad (4)$$

37. Head Required for a Given Flow.—The head required per 1,000 feet to produce a flow of Q cubic feet per second in a pipe d feet in diameter is found from formula **2** of Art. **33**. Squaring both members of the formula, it becomes,

$$Q^2 = d^5 H;$$

whence,

$$H = \frac{Q^2}{d^5} \qquad (1)$$

For pipes of small size, from formula **3** of Art. **33**,

$$H = \frac{Q^2}{.785\, d^5} = 1.27 \frac{Q^2}{d^5} \qquad (2)$$

The following examples show the application of the preceding formulas:

EXAMPLE 1.—A rough pipe, 16 inches in diameter and 3,700 feet long, connects two reservoirs, the difference of elevation between the two being 187 feet. With what velocity does the water flow through the pipe?

SOLUTION.—Substituting in formula **2** of Art. **31**, d = 16 in. = $1\frac{1}{3}$ ft., h = 187 ft., l = 3,700 ft., and, from Table V, c = .00064. Hence,

$$v = \sqrt{\frac{187 \times \frac{4}{3}}{.00064 \times 3,700}} = 10.26 \text{ ft. per sec. Ans.}$$

EXAMPLE 2.—What is the velocity through the pipe in example 1, calculated by formula **1** of Art. **36**?

SOLUTION.— $H = \dfrac{187 \times 1,000}{3,700} = 50.5$; $d = \frac{4}{3}$; hence, substituting,

$$v = 1.27\sqrt{\frac{4}{3} \times 50.5} = 10.42 \text{ ft. per sec. Ans.}$$

NOTE.—In approximate formulas, such as all those that apply to the flow of water through the pipes necessarily are, the results obtained in examples 1 and 2 are equivalent to an agreement, and in practice one might happen to be as nearly right as the other. It is obvious that, when the character of the pipe may vary so widely as to interior surface, a very close result can never be hoped for, and all that can be done is to keep within probable limits.

EXAMPLE 3.—A rough pipe, 10 inches in diameter, is laid with a fall of $7\frac{1}{2}$ feet per 1,000; what is the discharge?

SOLUTION.—Applying formula **2** of Art. **33**,

$$Q = \sqrt{d^5 H} = \sqrt{\left(\frac{10}{12}\right)^5 \times 7.5}$$

and using logarithms,

$$\begin{array}{lr} \log 10 \ldots\ldots\ldots & 1.00000 \\ \log 12 \ldots\ldots\ldots & 1.07918 \\ \hline \text{Subtracting, } \log \frac{10}{12} \;\ldots & \bar{1}.92082 \\ & 5 \\ \hline \text{log of fifth power} \;\ldots & \bar{1}.60410 \\ \log 7.5 \ldots\ldots\ldots & .87506 \\ \hline & 2)0.47916 \\ \hline \text{log of square root} \;\ldots & 0.23958 \\ \text{Corresponding number} = & 1.736 \end{array}$$

Therefore, the discharge is 1.736 cu. ft. per sec. Ans.

EXAMPLE 4.—It is desired to discharge 3 cubic feet per second from a pipe line having a fall of 5 feet per 1,000; what diameter of rough cast-iron pipe will be required?

SOLUTION.—Inserting the data in formula **1** of Art. **35**,

$$d = \sqrt[5]{\frac{Q^2}{H}} = \sqrt[5]{\frac{9}{5}}$$

$$\begin{array}{lr} \log 9 \ldots\ldots\ldots & .95424 \\ \log 5 \ldots\ldots\ldots & .69897 \\ \hline & 5)\ .25527 \\ \hline \text{log of fifth root} \;\ldots & 0.05105 \\ \text{Corresponding number} = & 1.125 \end{array}$$

Therefore, the diameter is 1.125 ft. = $13\frac{1}{2}$ in. Ans.

As cast-iron pipes are made to standard sizes, and there are no $\frac{1}{2}$ inches, the nearest appropriate size would be a 14-in. pipe.

EXAMPLE 5.—It is desired to discharge $\frac{1}{2}$ cubic foot per second from a 4-inch pipe; what head per 1,000 feet of length is necessary to accomplish this?

SOLUTION.—From formula **2**, $H = 1.27\,\frac{Q^2}{d^5}$, in which $Q = \frac{1}{2}$ and $d = \frac{1}{3}$; hence, substituting, it becomes,

$$H = 1.27 \times \frac{(\frac{1}{2})^2}{(\frac{1}{3})^5} = 77.15 \text{ ft.} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1. A rough pipe 20 inches in diameter connects two reservoirs 2 miles apart; the difference in level is 375 feet. With what velocity will the water flow through the pipe? Ans. 9.617 ft. per sec.

2. What should be the diameter of a rough pipe 1 mile long, with a uniform drop of 52.8 feet, to discharge 5 cubic feet per second?

Ans. 14.41 in. A 15-in. pipe would be used

3. What is the discharge per second from a pipe 14 inches in diameter with a fall of 9 feet per 1,000? Ans. 4.41 cu. ft.

FLOW OF GASES

FLOW THROUGH ORIFICES AND SHORT TUBES

38. Preliminary Statement.—Owing to the compressibility of gases, the laws governing their flow through orifices and in pipes are not so simple as those relating to the flow of liquids. Different writers and experimenters give different formulas, and the results obtained by the use of these formulas frequently disagree; hence, in practical problems, considerable judgment is required in selecting the formula that suits most closely the conditions of the case under consideration.

It is believed that the formulas given in the following paragraphs are as reliable as any. In each case, the conditions under which the formula is applicable are stated.

39. Water Formula.—Suppose that a gas that has been enclosed in a vessel is allowed to flow through an orifice into the atmosphere. Let p_1 denote the pressure per square inch of the gas in the reservoir, and p_a the atmospheric pressure. Now, if p_1 is but slightly greater than p_a, the density of the gas changes but little during the flow, and in consequence the same formula as that used for water may be used without serious error, namely, $v^2 = 2gh$.

In the case of the flow of water, h denotes the head, in feet, on the orifice; likewise, in the case of a gas, h denotes the head or height, in feet, of a column of gas that corresponds to the difference of pressure $p_1 - p_a$. To illustrate this point, suppose that the gas in the reservoir is air and that the pressure is 16 pounds per square inch, absolute, and the temperature 60° F. At this pressure and temperature, 1 cubic foot

of air weighs .08316 pound. The difference in pressure $p_1 - p_a$, is $16 - 14.7 = 1.3$ pounds per square inch, or $1.3 \times 144 = 187.2$ pounds per square foot. Evidently, a column having an area of 1 square foot and weighing 187.2 pounds will produce this pressure and the column will have to contain $187.2 \div .08316 = 2{,}251$ cubic feet; the column will, therefore, be 2,251 feet high. Hence, in this case, $h = 2{,}251$, and the velocity of flow will be

$$v = \sqrt{2\,g\,h} = \sqrt{2 \times 32.16 \times 2{,}251} = 380.5 \text{ feet per second.}$$

In general, if w_1 is the weight of 1 cubic foot of gas in the reservoir at the pressure p_1,

$$h = \frac{144\,(p_1 - p_a)}{w_1} \qquad (1)$$

Substituting the value of h in the formula $v^2 = 2\,g\,h$,

$$v^2 = \frac{2\,g \times 144\,(p_1 - p_a)}{w_1};$$

whence,

$$v = 96.24\sqrt{\frac{p_1 - p_a}{w_1}} \qquad (2)$$

This formula is termed the **water formula**, and holds good only when the pressure difference is small.

40. Formula for Weights Discharged.—Just as in the case of water, $Q' = A\,v$, where Q' denotes the theoretical volume discharged in cubic feet per second and A is the area of the orifice. In the case of air, as with water, there is a coefficient of discharge by which the quantity Q' must be multiplied to obtain the actual discharge Q.

Denoting the coefficient by k, $Q = k\,Q' = k\,A\,v$; and by substituting the value of v in formula **2** of Art. **39**,

$$Q = 96.24\,k\,A\sqrt{\frac{p_1 - p_a}{w_1}} \qquad (1)$$

If a represents the area of the orifice, in square inches $A = \frac{a}{144}$, and the formula becomes

$$Q = \frac{96.24}{144}\,k\,a\sqrt{\frac{p_1 - p_a}{w_1}}$$

or

$$Q = .668\,k\,a\sqrt{\frac{p_1 - p_a}{w_1}} \qquad (2)$$

If G denotes the weight of the Q cubic feet of air, that is, the number of pounds discharged per second,

$$G = w_1 Q = .668\, k\, a\, w_1 \sqrt{\frac{p_1 - p_a}{w_1}}$$

or

$$G = .668\, k\, a \sqrt{w_1 (p_1 - p_a)} \qquad (3)$$

Now, from the general formula $p v = G R T$, from *Pneumatics*, and the weight w_1 of 1 cubic foot of air at a pressure of p_1 and an absolute temperature T_1, the following formula is obtained, in which v_1 is the volume of 1 cubic foot of air and R is .37.

$$p_1 v_1 = w_1 R T_1$$

and

$$w_1 = \frac{p_1 v_1}{R T_1} = \frac{p_1 \times 1}{.37\, T_1};$$

or

$$w_1 = 2.7 \frac{p_1}{T_1} \qquad (4)$$

Inserting this value of w_1 in formula **3**,

$$G = .668\, k\, a \sqrt{\frac{2.7\, p_1 (p_1 - p_a)}{T_1}} = 1.1\, k\, a \sqrt{\frac{p_1 (p_1 - p_a)}{T_1}}$$

From numerous experiments, Fliegner found that the coefficient k has the value .964; therefore, $1.1\, k = 1.06$. The final equation is, therefore,

$$G = 1.06\, a \sqrt{\frac{p_1 (p_1 - p_a)}{T_1}} \qquad (5)$$

Fliegner states that this formula may be used when the pressure in the reservoir is less than twice the atmospheric pressure, that is, when p_1 is less than $2 p_a$.

Example.—Air flows from a reservoir in which the pressure is 5 pounds per square inch, gauge, into the atmosphere; the temperature in the reservoir is 69° F. and the diameter of the orifice is 1 inch. Compute the flow.

Solution.—Here $p_1 = 5 + 14.7 = 19.7$ lb., which is less than twice 14.7 lb.; $a = .7854 \times 1^2 = .7854$ sq. in.; $T_1 = 69 + 460 = 529°$. Using formula **5**, and substituting,

$$G = 1.06 \times .7854 \times \sqrt{\frac{19.7 \times (19.7 - 14.7)}{529}} = .3593 \text{ lb. Ans.}$$

41. Fliegner's Second Formula.—For cases in which the reservoir pressure is more than double the atmospheric

pressure, Fliegner gives the following formula, in which the symbols have the same meanings as in formula **5** of Art. **40**:

$$G = .53\, a \frac{p_1}{\sqrt{T_1}}$$

This formula and formula **5** of Art. **40** were deduced from experiments made on the flow of air into the atmosphere; however, it is probable that they may be used for the flow from one reservoir into a second reservoir in which the pressure is different from atmospheric pressure.

EXAMPLE 1.—Air having a temperature of 60° F. and an absolute pressure of 63 pounds per square inch flows through an orifice $\frac{1}{2}$ inch in diameter into the atmosphere; what is the flow per second?

SOLUTION.—Since 63 is greater than 2×14.7, the formula just given is used. $a = .7854 \times (\frac{1}{2})^2$; $p_1 = 63$; and $T_1 = 460 + 60 = 520$. Substituting these values,

$$G = .53 \times \frac{63 \times .7854 \times (\frac{1}{2})^2}{\sqrt{520}} = .2875 \text{ lb. Ans.}$$

EXAMPLE 2.—Compressed air flows from a reservoir in which the pressure is 60 pounds per square inch, gauge, into a second reservoir containing air at a pressure of 45 pounds per square inch, gauge; the temperature is 65° F. What is the flow per second if the area of the orifice is $\frac{1}{4}$ square inch?

SOLUTION.—Since the higher pressure is less than twice the lower pressure, formula **5** of Art. **40** is used, replacing p_a by the lower pressure p_2. $p_1 = 60 + 14.7 = 74.7$; $p_2 = 45 + 14.7 = 59.7$; $T_1 = 460 + 65 = 525$; and $a = \frac{1}{4}$. Then, substituting,

$$G = 1.06 \times \tfrac{1}{4} \times \sqrt{\frac{74.7 \times (74.7 - 59.7)}{525}} = .387 \text{ lb. Ans.}$$

FLOW OF AIR IN PIPES

42. Phenomena of Flow in Pipes.—When air or any other compressible gas flows in a pipe, the same weight must pass any cross-section of the pipe in a given interval of time. It does not follow, however, from this that the same volume passes every cross-section; for since the gas is compressible, the volume of a given weight depends on the pressure, and, in general, this is different at different sections.

Suppose that the gas flows into a pipe from a reservoir in which the pressure is p_1 pounds per square inch. The velocity at the start is v_1 feet per second. The temperature is supposed to remain constant during the flow. As in the case of liquids, the friction of the gas against the sides of the pipe causes a loss of pressure, the amount of which depends on the diameter and length of pipe and the velocity of flow.

After traveling a distance of l feet, the pressure of the gas is p_2 pounds per square inch, which is less than the initial pressure p_1. Now, as the temperature has not changed, the drop of pressure must result in an increase of volume per pound of gas; for, according to Boyle's law, $p_1 V_1 = p_2 V_2$, or $V_2 = V_1 \frac{p_1}{p_2}$; hence, since the same weight is passing each cross-section, the gas must have a greater velocity when its pressure is p_2 than at the start, when its pressure was p_1.

It is found that, other conditions being equal, the drop of pressure is greater the higher the initial velocity v_1. It is advisable therefore to keep v_1 as low as possible consistent with a reasonable diameter of pipe. In long mains, v_1 should not exceed 20 to 25 feet per second.

43. General Formula.—The fundamental formula for the flow of air in a long pipe is derived by the use of higher mathematics. It is

$$v_1 = \sqrt{\frac{12\, g R T d}{4\, f l} \times \left(\frac{p_1^2 - p_2^2}{p_1^2}\right)} \qquad (1)$$

in which g = acceleration of gravity = 32.16;
R = .37, from *Pneumatics;*
T = absolute temperature of air, which is assumed constant;
d = diameter of pipe, in inches;
l = length of pipe, in feet;
f = coefficient of friction of air against pipe walls;
p_1 = initial absolute pressure, in pounds per square inch;
p_2 = final absolute pressure, in pounds per square inch;
v_1 = initial velocity of air, in feet per second.

Substituting the numerical values of g and R, the formula takes the simpler form

$$v_1 = 5.975\sqrt{\frac{Td}{fl} \times \left(\frac{p_1^2 - p_2^2}{p_1^2}\right)} \qquad (2)$$

Ordinarily, the temperature of the air is approximately 70° F., that is, $T = 460° + 70° = 530°$. Using this value of T, the formula becomes

$$v_1 = \frac{137.6}{p_1}\sqrt{\frac{d}{fl} \times (p_1^2 - p_2^2)} \qquad (3)$$

EXAMPLE.—Air at an initial gauge pressure of 60 pounds per square inch flows through a pipe 6 inches in diameter and 9,000 feet long; assuming that $f = .0045$, with what velocity must the air enter the pipe so that the drop of pressure shall not exceed 8 pounds per square inch?

SOLUTION.—Here, $p_1 = 60 + 14.7 = 74.7$; $p_2 = 74.7 - 8 = 66.7$; and $p_1^2 - p_2^2 = 1{,}131.2$; $d = 6$; $f = .0045$; $l = 9{,}000$. Then, substituting in formula **3**,

$$v_1 = \frac{137.6}{74.7}\sqrt{\frac{6 \times 1{,}131.2}{.0045 \times 9{,}000}} = 23.85 \text{ ft. per sec.}$$

Hence, the velocity should not exceed 23.85 ft. per sec. Ans. In solving examples of this kind, time may be saved by using logarithms.

44. Quantity of Air Delivered.—Let Q_1 denote the volume of air, in cubic feet, entering the pipe per second at the pressure p_1, and Q the volume of an equal weight of free air, that is, air at atmospheric pressure; then, since the volumes of equal weights of air are inversely as the absolute pressures,

$$\frac{Q_1}{Q} = \frac{14.7}{p_1}, \text{ or } Q = \frac{Q_1 p_1}{14.7}$$

Let a denote the area of the pipe in square inches, which, for a pipe of circular section, equals $.7854\ d^2$, when d is in inches; then,

$$Q_1 = \frac{a}{144} \times v_1$$

and

$$Q = \frac{Q_1 p_1}{14.7} = \frac{a v_1 p_1}{144 \times 14.7} = \frac{.7854\, d^2 v_1 p_1}{144 \times 14.7}$$

Usually, the delivery is expressed in cubic feet of free air per minute, and, using the minute instead of the second, the formula becomes

$$Q = \frac{60 \times .7854}{144 \times 14.7} d^2 v_1 p_1 \qquad (1)$$

For v_1, substitute the value in formula **3** of Art. **43,** and the formula becomes

$$Q = \frac{60 \times .7854 \times 137.6}{144 \times 14.7} d^2 \sqrt{\frac{d}{fl}(p_1^2 - p_2^2)}$$

or

$$Q = 3.063 \sqrt{\frac{d^5}{fl}(p_1^2 - p_2^2)} \qquad (2)$$

45. Values of Coefficient of Friction for Air.—The coefficient f is not constant, but varies with the diameter of the pipe. Values agreeing well with experiments are given by the formula

$$f = .003\left(1 + \frac{4}{d}\right)$$

EXAMPLE.—Air at an initial gauge pressure of 80 pounds per square inch flows through a pipe 8 inches in diameter and 8,000 feet long; the drop in pressure is 5 pounds per square inch. Compute the discharge in cubic feet of free air per minute.

SOLUTION.—From the formula

$$f = .003\left(1 + \frac{4}{8}\right) = .0045$$

Substituting in formula **2** of Art. **44,**

$$Q = 3.063 \sqrt{\frac{8^5}{.0045 \times 8{,}000}(94.7^2 - 89.7^2)} = 2{,}806 \text{ cu. ft. Ans.}$$

46. Formulas for Diameter, Length, and Drop in Pressure.—By transforming formula **2** of Art. **44,** the following formulas are readily derived:

$$d = .64 \sqrt[5]{\frac{Q^2 f l}{p_1^2 - p_2^2}} \qquad (1)$$

$$l = 9.38 \frac{d^5}{Q^2 f}(p_1^2 - p_2^2) \qquad (2)$$

$$p_2 = p_1 \sqrt{1 - \frac{.107\, Q^2 f l}{d^5 p_1^2}} \qquad (3)$$

When the drop in pressure is small, it may be calculated by the simple approximate formula

$$p_1 - p_2 = .0535 \frac{Q^2 f l}{d^5 p_1} \qquad (4)$$

If the drop, as calculated by formula **4,** is relatively large, say 5 per cent. of p_1, or more, formula **3,** which is more exact, should be used to find the lower pressure, p_2.

From formula **3** of Art. **43**, the following formula is also obtained:

$$p_2 = p_1\sqrt{1 - \frac{v_1^2 f l}{18{,}934\, d}} \qquad \textbf{(5)}$$

This formula may be used to compute the pressure p_2 when the initial velocity v_1 is known.

EXAMPLE 1.—A pipe 5 miles long is required to deliver the equivalent of 4,000 cubic feet of free air per minute with a final pressure of 230 pounds per square inch, gauge, and the drop in pressure is not to exceed 20 pounds per square inch; what must be the diameter of the main?

SOLUTION.—Since d is unknown, f must be assumed and afterwards corrected if necessary; therefore, assume $f = .0045$; $p_2 = 230 + 14.7 = 244.7$; $p_1 = 244.7 + 20 = 264.7$; $p_1^2 - p_2^2 = 10{,}188$; $Q = 4{,}000$; and $l = 5 \times 5{,}280$. Now, using formula **1** and substituting,

$$d = .64\sqrt[5]{\frac{4{,}000^2 \times .0045 \times 5 \times 5{,}280}{10{,}188}} = 7.25 \text{ in. Ans.}$$

For this value of d, $f = .003\left(1 + \frac{4}{7.25}\right) = .0047$, which is so near the assumed value, .0045, that a correction is unnecessary. The next larger commercial size of pipe, 8 in., should be used.

EXAMPLE 2.—Using the 8-inch pipe in example 1, compute the actual drop in pressure.

SOLUTION.—For an 8-inch pipe,

$$f = .003\left(1 + \frac{4}{8}\right) = .0045$$

From formula **3**,

$$p_2 = 264.7\sqrt{1 - \frac{.107 \times 4{,}000^2 \times .0045 \times 5 \times 5{,}280}{8^5 \times 264.7^2}} = 252.7 \text{ lb.}$$

Hence, the drop is $p_1 - p_2 = 264.7 - 252.7 = 12$ lb. Ans.

By the approximate formula **4**, the drop in pressure would be

$$p_1 - p_2 = \frac{.0535 \times 4{,}000^2 \times .0045 \times 5 \times 5{,}280}{8^5 \times 264.7} = 11.72 \text{ lb.}$$

EXAMPLE 3.—Through what length of 6-inch pipe can the equivalent of 1,500 cubic feet of free air per minute be discharged with a final pressure of 75 pounds per square inch, gauge, and a drop in pressure of 4 pounds per square inch?

SOLUTION.—For a 6-in. pipe, $f = .003\,(1 + \frac{4}{6}) = .005$; $p_2 = 75 + 14.7 = 89.7$; and $p_1 = 89.7 + 4 = 93.7$; hence, $p_1^2 - p_2^2 = 93.7^2 - 89.7^2 = 733.6$. Now, substituting in formula **2**,

$$l = 9.38 \times \frac{6^5 \times 733.6}{1{,}500^2 \times .005} = 4{,}756 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. With what theoretical velocity will air flow from a reservoir in which the pressure is 20 pounds per square inch, absolute, into the atmosphere? Assume the temperature to be 60°. Ans. 687.5 ft. per sec.

NOTE.—First use formula **4** of Art. **40** for finding the weight of 1 cubic foot of air, and then use formula **2** of Art. **39**.

2. Air flows from a reservoir, in which the pressure is 150 pounds per square inch, gauge, into the atmosphere through an orifice $\frac{3}{4}$ inch in diameter; the temperature of the air on the reservoir is 65° F. Compute the weight flowing per minute. Ans. 101 lb., nearly

NOTE.—Use the formula in Art. **41** and remember that the formula gives pounds per second.

3. A main to carry compressed air is 20,000 feet long and 10 inches in diameter; the initial pressure is 125 pounds per square inch, absolute. Compute the drop in pressure for the following initial velocities: (*a*) 20 feet per second; (*b*) 30 feet per second.

$$\text{Ans.}\begin{cases}(a)\ 11.63\text{ lb. per sq. in.}\\(b)\ 28.12\text{ lb. per sq. in.}\end{cases}$$

NOTE.—Use the formula in Art. **45** and then use formula **5** of Art. **46**.

4. What diameter of pipe is required for transmission of air under the following conditions: length, 1 mile; final pressure, 80 pounds per square inch, gauge; drop in pressure not to exceed 5 pounds per square inch; quantity discharged per minute, the equivalent of 7,000 cubic feet of free air? Ans. 10.4 in.; hence, use 11-in. pipe

NOTE.—Assume $f = .0042$ and use formula **1** of Art. **46**.

5. With an initial pressure of 300 pounds per square inch, absolute, and a drop of 20 pounds per square inch, what will be the discharge of free air per minute from a main 10 miles long and 15 inches in diameter? Ans. 20,295 cu. ft. in round numbers

NOTE.—Use the formula in Art. **45** to find f, and then use formula **2** of Art. **44**.

HYDRAULICS

(PART 2)

ENERGY OF FALLING WATER

1. Methods of Utilizing the Energy of Water. Water, in passing from a higher to a lower level, can do work, and the amount of work that can thus be done by a given weight *G* of water is the available energy of that weight of water. This energy may be used in one of three ways, as illustrated in Fig. 1: (1) The water may be per-

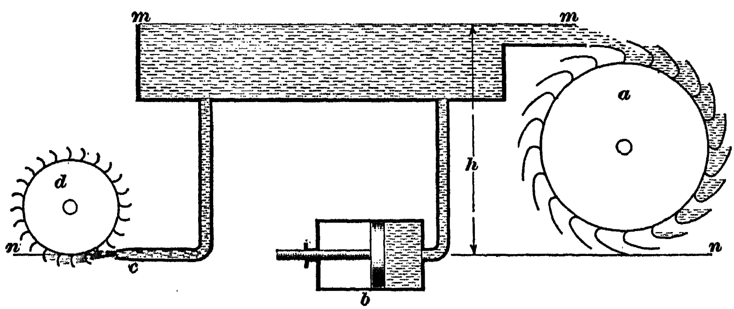


FIG. 1

mitted to flow from the upper level *m m* into the buckets of an overshot waterwheel *a*; the weight of the water acts as a motive force and causes the wheel to turn. (2) The water may enter a cylinder *b* placed at the lower level *n n*; the water in the cylinder has a pressure due to the hydrostatic head *h*, the distance between the two levels, and by virtue of this pressure may push a piston back and forth and thus do work. (3) The water may be allowed to escape from a

nozzle c at the level $n n$, and the jet of water may be made to impinge on the buckets of an impulse wheel d.

Corresponding to these three ways of utilizing the energy of the falling water, hydraulic motors are divided into three classes: (1) gravity motors, in which the weight of the water is used; (2) pressure motors, in which the pressure of the water is used; (3) velocity motors, in which the impulse of a moving jet is the motive force.

2. Available Energy.—In each case, let G be the weight of water in pounds used in some given interval of time. In case the water is used to turn the wheel a, the weight G simply descends a vertical distance h and the work done is $G h$ foot-pounds. Let the water used per stroke in the cylinder b be G pounds, and let A denote the piston area in square feet and l the length of stroke in feet. Then $A l$ is the volume of water used per stroke, in cubic feet, and $P A l$ is the work per stroke, where P denotes the pressure on the piston, in pounds per square foot. If, now, H denotes the weight of a cubic foot of water and G pounds of water are used per stroke, $G \div H$ is the volume used. Then,

$$\frac{G}{H} = A l; \text{ also, } P = H h$$

Hence,

$$\text{work per stroke} = P \times A l = H h \times \frac{G}{H} = G h \text{ foot-pounds}$$

Consequently, in discharging G pounds of water from the higher to the lower level through the cylinder b, the work done is also $G h$ foot-pounds. Finally, the water issues from the nozzle c with a velocity v, and the kinetic energy of G pounds moving at this speed is $\frac{G v^2}{2g}$ foot-pounds. But in the case of frictionless flow through the tube and nozzle, the theoretical velocity is $v = \sqrt{2 g h}$, whence $\frac{v^2}{2g} = h$ and the energy is again $G h$ foot-pounds. This leads, therefore, to the following important principle: *The available energy of a weight of water G falling through a height h is the product G h, whether the water acts by weight, pressure, or impulse.*

3. Power of a Fall of Water.—Let Q denote the quantity of water, in cubic feet, flowing in 1 second, and let h denote the fall, in feet. The work done in 1 second is $G h$ foot-pounds $= 62.5\ Q h$ foot-pounds, as water weighs approximately 62.5 pounds per cubic foot. Now, 1 horsepower is the performance of 33,000 foot-pounds of work per minute, or $33{,}000 \div 60 = 550$ foot-pounds per second; hence, the theoretical horsepower of a given fall of water may be expressed by the formula

$$\text{H. P.} = \frac{62.5\ Q h}{550} = .1136\ Q h$$

EXAMPLE.—A flume leading from a dam has a fall of 35 feet, and discharges 210 cubic feet of water per minute; what is the theoretical horsepower?

SOLUTION.— $Q = 210 \div 60 = 3.5$, and $h = 35$; hence,
$\text{H. P.} = .1136\ Q h = .1136 \times 3.5 \times 35 = 13.916$. Ans.

4. Efficiency.—No motor, however, can utilize all the power in the fall of a given weight of water. Part of the energy is lost in overcoming the resistances due to the friction of the water as it flows through the gates and channels leading to the motor; part is absorbed in shocks and eddies, and in the friction of the water as it passes through the motor; and part is lost in the form of velocity as the water leaves the motor, or as it falls from the motor to the lower level of the water. Besides the above losses, due to resistances to the motion of the water, the mechanical losses due to the friction of the motor itself must be taken into account.

The **efficiency** of a motor is the ratio of the actual work it will do to the theoretical work in the water used. Thus, if the actual work done by a waterwheel is equal to 750 horsepower, when the theoretical work that the water would do is equal to 1,000 horsepower, the efficiency of the wheel is $750 \div 1{,}000 = .75 = 75$ per cent.

EXAMPLE.—What is the efficiency of a waterwheel that delivers 24 horsepower when using 660 pounds of water per second with a head of 25 feet?

SOLUTION.—The theoretical power is $\frac{660 \times 25}{550} = 30$ H. P.; therefore, the efficiency is $24 \div 30 = .80 = 80$ per cent. Ans.

5. Power Transmitted by a Water Main.—When water flows through a main, part of the head h is lost because of the friction. If this lost head is denoted by h_f, the net head is $h - h_f$; hence, the horsepower delivered at the end of the main is

$$\text{H. P.} = .1136\,Q(h - h_f)$$

Without friction, the horsepower is that given by the formula of Art. **3**; hence, the efficiency of the transmission is

$$\frac{h - h_f}{h} = 1 - \frac{h_f}{h}$$

6. Energy of a Jet.—As stated in Art. **2**, the theoretical energy of G pounds of water moving with a velocity of v feet per second is $\frac{G v^2}{2g}$ foot-pounds. If h denotes the head on the orifice, Fig. 1, theoretically $v = \sqrt{2 g h}$; actually, however, $v = c\sqrt{2 g h}$, where c denotes a coefficient known as the coefficient of velocity; hence, $\frac{v^2}{2g} = c^2 h$, and $\frac{G v^2}{2g} = c^2 G h$.

Let A denote the area of the cross-section of the jet in square feet, and H the weight of a cubic foot of water; then the weight of water discharged per second is $G = H A v$ and the energy E of this quantity per second is $\frac{G v^2}{2g}$, or $c^2 G h$. Substituting the value of G, the formulas become

$$E = \frac{H A v^3}{2g} \qquad (1)$$

and

$$E = c^2 H A v h \qquad (2)$$

Since E is the work the jet is capable of doing in 1 second, the horsepower of the jet is $\frac{c^2 G h}{550}$; and by substituting the value of G the formula becomes

$$\text{H. P.} = \frac{c^2 H A v h}{550} \qquad (3)$$

It should be noted that the area of the jet is not necessarily the area of the orifice from which the jet issues, as explained in *Hydraulics*, Part 1.

The efficiency of the jet, that is, the ratio of the actual energy to the theoretical energy, is $\frac{c^2 G h}{G h} = c^2$.

EXAMPLE.—In a test of an impulse waterwheel, the discharge from the nozzle was found to be 2.819 cubic feet per second and the pressure head was 384.7 feet. (*a*) Taking $c = .98$, calculate the horsepower of the jet. (*b*) The horsepower developed by the wheel was found to be 107.4; what was the efficiency of the wheel?

SOLUTION.—(*a*) Substituting in formula **3**,

$$\text{H. P.} = \frac{.98^2 \times 62.5 \times 2.819 \times 384.7}{550} = 118.36. \quad \text{Ans.}$$

(*b*) $\text{Efficiency} = \frac{\text{actual H. P}}{\text{theoretical H. P.}} = \frac{107.4}{118.36} = .9074 = 90.74$ per cent. Ans.

7. Pressure Due to Impact of a Jet.—Let a jet of water strike a surface inclined at an angle a with the original direction of the jet, as shown in Fig. 2. The surface is supposed to be perfectly smooth, so that there is no loss from shocks or friction, and the water is prevented from spreading sidewise. Under these conditions, the velocity v with which the water leaves the surface is equal to the velocity v that it had when it struck.

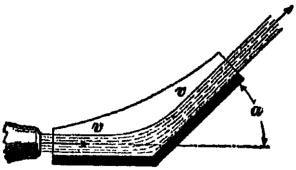


FIG. 2

The impact of the water produces a pressure on the surface that tends to move it. Let P denote the component of this pressure in the direction of the jet, and let M be the mass of the water that issues in 1 second. On leaving the nozzle, the momentum of this mass is Mv; but on leaving the surface the velocity component in the direction of the jet is $v \cos a$, and the momentum in the same direction is therefore $Mv \cos a$; the mass, therefore, has its momentum in the direction of the jet decreased by the amount $Mv - Mv \cos a = Mv(1 - \cos a)$. This change of momentum must be caused by the reaction of the surface against the jet in the direction of the jet. This reaction is of course equal to and opposite to the pressure P. From the principles of mechanics, the change of momentum is equal to the impulse,

which is Pt, in which t is the time in seconds through which the force acts. Then, $Pt = Mv(1 - \cos a)$. The time t is taken as 1 second, so that $P = Mv(1 - \cos a)$; but $M = \frac{G}{g}$; hence,

$$P = \frac{Gv}{g}(1 - \cos a)$$

EXAMPLE.—A jet whose cross-section is 1 square inch flows with a velocity of 75 feet per second, and strikes a surface that changes its direction 35°; what pressure is exerted on the surface in the direction of the jet before striking?

SOLUTION.—From the above formula, by substituting,

$$P = \frac{75 \times 1 \times 62.5}{144} \times \frac{75}{32.16} \times (1 - .81915) = 13.73 \text{ lb. Ans.}$$

8. Pressure on a Flat Surface at Right Angles to the Jet.—When the surface is at right angles to the jet, as shown in Fig. 3, $a = 90°$ and $\cos a = \cos 90° = 0$; in this case, therefore, the formula of Art. 7 reduces to

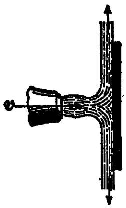


FIG. 3

$$P = \frac{Gv}{g} \qquad (1)$$

As the jet issues from the orifice, there is a reaction on the vessel from which it issues, which is just equal to the pressure that is produced by the jet as it strikes the vertical surface. The effect of the reaction and pressure of a jet may be shown by experiment

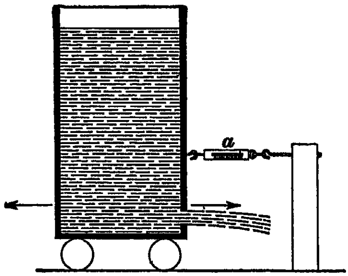


FIG. 4

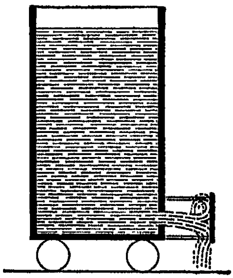


FIG. 5

as follows: Let the vessel be placed on rollers, as shown in Fig. 4, in such a way that a very slight pressure will produce motion. When the water issues from the orifice, as shown,

the vessel will begin to move in the opposite direction. If there were no friction, a spring balance attached to the vessel, as shown at *a*, would show a pull equal to $\frac{G v}{g}$. Now, if a plate is fastened to the vessel as shown in Fig. 5, so that the jet strikes it, the pressure exerted by the jet on the plate will equal the reaction of the jet on the vessel, and there will be no motion.

If the plate is perfectly smooth, so that there is no loss from friction, the velocity of the water as it leaves the plate will be the same as the velocity with which it struck, and there will be no change in the energy contained in the water. The velocity in the direction of the jet has been entirely overcome and changed to pressure, but since this pressure produces no motion, no work is done.

As in Art. 6, let $G = H A v$, and $\frac{v^2}{2g} = c^2 h$; then,

$$P = \frac{G v}{g} = \frac{H A v^2}{g} = 2 c^2 H A h \qquad (2)$$

The hydrostatic pressure exerted on an area A by a head h is equal to $H A h$; it appears therefore that, with c equal to one, the reaction of a jet, whose area is a and whose velocity of flow is produced by a head h, is twice the hydrostatic pressure that would be produced on the same area by the same head.

EXAMPLE.—The area of a jet from the side of a vessel is 2 square inches, the head on the center of the orifice is 10 feet, and the coefficient of velocity is .98. (*a*) What pressure will the jet exert when it impinges on a vertical plane surface? (*b*) What is the pressure on the vessel due to the reaction of the jet?

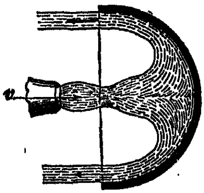
SOLUTION.—Using formula 2,

(*a*) $P = 2 \times .98^2 \times 62.5 \times \frac{2}{144} \times 10 = 16.67$ lb. Ans.

(*b*) The reaction is equal to the pressure P. Ans.

9. Pressure of a Jet on a Hemispherical Cup. If the water strikes into a hemispherical cup, as shown in Fig. 6, the direction in which it leaves the cup makes an angle of 180° with the direction of motion of the jet. The cup is supposed to be smooth, so that there is no loss of velocity

or energy. Then, since $a = 180°$, $\cos a = -1$ and $1 - \cos a = 1 - (-1) = 2$, which, substituted in the formula of Art. **7**, gives for the hemispherical cup



$$P = \frac{2\,G\,v}{g}$$

that is, *the pressure is twice as great as when the jet strikes a flat plate at right angles to the direction of its motion.*

FIG. 6

EXAMPLE.—If the jet in the example of Art. 7 strikes a hemispherical cup, so that its direction is changed 180°, what is the pressure exerted?

SOLUTION.—Substituting in the formula,

$$P = 2 \times \frac{75 \times 1 \times 62.5}{144} \times \frac{75}{32.16} = 151.83 \text{ lb. Ans.}$$

10. Pressure on Moving Surfaces Due to Jets. Suppose the plate, Fig. 3, to be moving away from the nozzle with a velocity of u feet per second. The absolute velocity of the jet, or its velocity relative to the fixed nozzle, being v, its velocity relative to the moving plate is $v - u$. The one plate considered will soon move beyond the influence of the jet. If, however, the plate is one of a series of plates that follow each other in rapid succession, as in waterwheels, the pressure of the water acts continuously. The effect then is the same as though the plates were stationary and the jet had the velocity $(v - u)$. The pressure on the plate in the direction of the jet is obtained by merely substituting the relative velocity $(v - u)$ for the velocity v in the formula of Art. **7**, thus:

$$P = \frac{G}{g}(v - u)(1 - \cos a) \qquad (1)$$

For a series of flat plates at right angles to the jet, $a = 90°$, $\cos a = 0$, and the formula becomes

$$P = \frac{G}{g}(v - u) \qquad (2)$$

For a series of hemispherical cups like that shown in Fig. 6, $a = 180°$, $\cos a = -1$, $1 - \cos a = 2$, and the formula becomes

$$P = \frac{2\,G}{g}(v - u) \qquad (3)$$

11. Work Done by Jets Impinging on Moving Surfaces.—The surface moves u feet per second and the force acting on it in the line of its motion is P. The work done per second is therefore Pu foot-pounds. Substituting for P the expressions given in formulas **2** and **3** of Art. **10**, and denoting by W the work done per second,

$$W = \frac{Gu}{g}(v - u) \text{ for flat plates}$$

and $$W = \frac{2Gu}{g}(v - u) \text{ for hemispherical cups}$$

From Art. **6**, the energy of the jet is $\frac{Gv^2}{2g}$ foot-pounds per second. If all this energy is utilized,

$$W = \frac{Gv^2}{2g};$$

but if the work done on the plate is less than the energy,

$$W = e\frac{Gv^2}{2g},$$

where e, the efficiency, is some proper fraction. The highest value of e for each of the two cases just mentioned can be found by taking $e\frac{Gv^2}{2g}$ equal to the values of the work done.

For a jet striking a flat plate at right angles, $\frac{Gu}{g}(v - u) = e\frac{Gv^2}{2g}$, or $uv - u^2 = \frac{ev^2}{2}$. Multiplying the last expression by 4, $4uv - 4u^2 = 2ev^2$, or $4u^2 - 4uv = -2ev^2$. Completing the square, $4u^2 - 4uv + v^2 = v^2 - 2ev^2$, and taking the square root, $2u - v = \sqrt{v^2 - 2ev^2} = v\sqrt{1 - 2e}$.

In order that the quantity under the radical shall not be negative, $2e$ must not exceed 1, or e must not exceed $\frac{1}{2}$. Hence, when a jet impinges on a series of flat plates, not more than one-half of the energy of the jet is utilized, or, in other words, the efficiency cannot exceed 50 per cent. When

$$e = \tfrac{1}{2},$$

$$v\sqrt{1 - 2e} = 0;$$

and, therefore, $$2u - v = 0;$$

or, $$u = \tfrac{1}{2}v.$$

For hemispherical cups, $\frac{2Gu}{g}(v-u) = e\frac{Gv^2}{2g}$, or $2uv - 2u^2 = \frac{ev^2}{2}$. Multiplying by 2, $4uv - 4u^2 = ev^2$; completing the square, $4u^2 - 4uv + v^2 = v^2 - ev^2 = v^2(1-e)$; extracting the square root, $2u - v = v\sqrt{1-e}$. With hemispherical cups, therefore, the highest possible efficiency is when $e = 1$, which makes $v\sqrt{1-e} = 0$; in which case, $2u - v = 0$, or $u = \frac{1}{2}v$. Hence, *when the velocity of the cups is one-half the velocity of the impinging jet, the efficiency is unity, that is, the entire energy of the jet is utilized in moving the cups.*

If the whole energy is thus utilized, the absolute velocity of the water leaving the cup must be zero. That this is true is readily seen; for the water leaves the cups with the velocity $v-u$ backwards and the cup is moving forwards with the velocity u; the absolute velocity of the water is therefore $u-(v-u) = 2u - v$, and this is zero when $u = \frac{1}{2}v$. Of course, in the actual case, there is friction, so that these high values cannot be fully attained.

EXAMPLES FOR PRACTICE

1. If a stream discharges 120 cubic feet of water per minute with a fall of 50 feet, what work is it theoretically able to do?
Ans. 375,000 ft.-lb. per min.

2. What is the horsepower corresponding to the work in example 1?
Ans. $11\frac{4}{11}$ H. P.

3. If 450 pounds of water is discharged each minute from an orifice under a head of 80 feet and the coefficient of velocity is .98, what is the horsepower equivalent to the energy in the jet? Ans. 1.048 H. P.

4. If the jet in example 3 impinges on a plane surface at right angles to its direction of motion, what pressure does it exert?
Ans. 16.4 lb.

5. A jet of water flows from a nozzle under a head of 100 feet with a coefficient of velocity of .98, and impinges on a series of moving hemispherical cups; what must be the velocity of a cup in order that the water will leave it with no absolute velocity? Ans. 39.3 ft. per sec.

6. If .25 cubic foot of water is discharged in each second from the nozzle in example 5, (*a*) what is the pressure exerted on the cup? (*b*) What is the work done by this pressure in one second?

Ans. { (*a*) 38.19 lb. (*b*) 1,500.9 ft.-lb.

HYDRAULIC MACHINES AND MOTORS

WATERWHEELS

12. Overshot Wheels.—**Overshot waterwheels** are most often applied to falls of from 10 to 50 feet. Higher falls are sometimes used, however. In an overshot wheel,

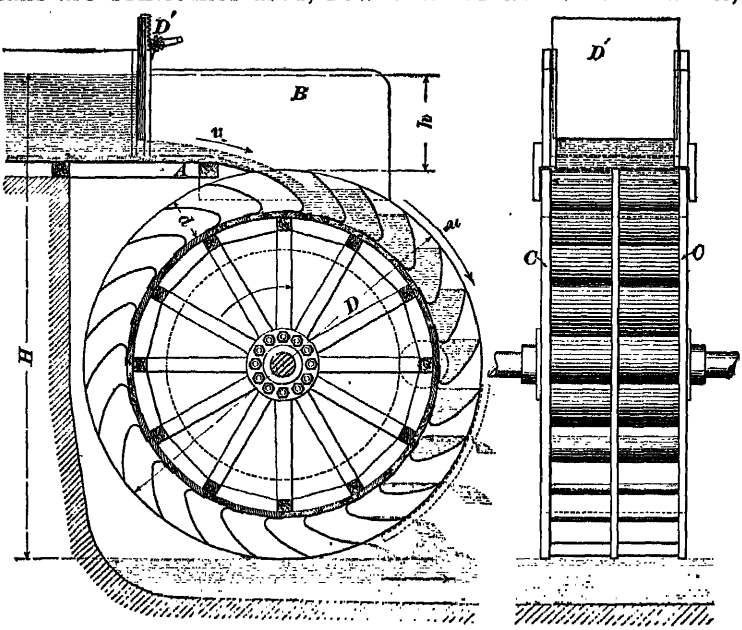


FIG. 7

a small amount of the work is done by the impact of the water as it enters the buckets, but much the greater part is done by the weight of the water as it descends in the buckets.

Fig. 7 shows two views of an overshot wheel with curved iron buckets. The water is brought out to the crown *C* by a

trough, or sluice, A, which may be curved toward the wheel. It should be so placed that the water will enter the first, second, or third bucket from the vertical center line of the wheel. The thickness of the sheet of water in the trough should not exceed 6 or 8 inches. The sides B of the trough are extended far enough beyond the vertical center line to insure the filling of several buckets when the wheel is to be started.

The supply of water to the wheel is regulated by a gate in the sluice, as shown at D'. This gate is generally operated by hand, but may be operated by an automatic governor.

An example will best illustrate the losses to which the overshot wheel is subject. Suppose the total fall H, Fig. 7, to be 20 feet. It may be assumed, as a first approximation, that the diameter D is 16 feet. In order that the centrifugal force acting on the water in the buckets will not cause too much spilling, the circumferential speed u should not be too great. A good value of u is given by the formula

$$u = \sqrt{2\,D};$$

whence, in this case, $u = \sqrt{2 \times 16} = 5.6$ feet per second. The velocity v of the water entering the buckets should be about double the velocity u of the buckets; hence, $v = 2 \times 5.6 = 11.2$ feet per second. The head to produce this velocity is

$$h = \frac{v^2}{2g} = \frac{11.2^2}{2 \times 32.16} = 1.95 \text{ feet}$$

Adding 10 per cent. for frictional losses in the sluice gate, $h = 2.15$ feet. Now, according to Art. **11**, not more than 50 per cent. of the kinetic energy of the stream due to the head h can be utilized; hence, the loss of head at this point is at least $\frac{1}{2}h$, or 1.08 feet.

The distance from the middle of the stream to the center of gravity of the buckets, which may be taken as 1 foot, is a further loss. The water discharges from the buckets before reaching the lower level, which is a third source of loss. The magnitude of this loss depends on the form of the buckets, but it varies from $.12\,D$ to $.2\,D$. Taking this as $.15\,D$, the loss of head is $.15 \times 16 = 2.4$ feet.

Finally, there is usually a clearance between the whee the water in the tailrace, and this constitutes a fourth lo head. Assuming this clearance to be 6 inches, or .5 foot, the total loss of head is 1.08 feet + 1 foot + 2.4 feet + .5 foot = 4.98 feet, say 5 feet. The effective head is, therefore, 20 feet − 5 feet = 15 feet, and the

$$\text{efficiency} = \frac{\text{net head}}{\text{total head}} = \frac{15 \text{ feet}}{20 \text{ feet}} = .75 = 75 \text{ per cent.}$$

The efficiency of the overshot waterwheel ranges from 70 to 90 per cent. in well-constructed wheels. When the supply of water is small, as during a drought, the buckets are only partly filled; hence, the loss from the water leaving the buckets too early is reduced, and the efficiency of the wheel is increased.

13. Breast Wheels.—The breast wheel is used where

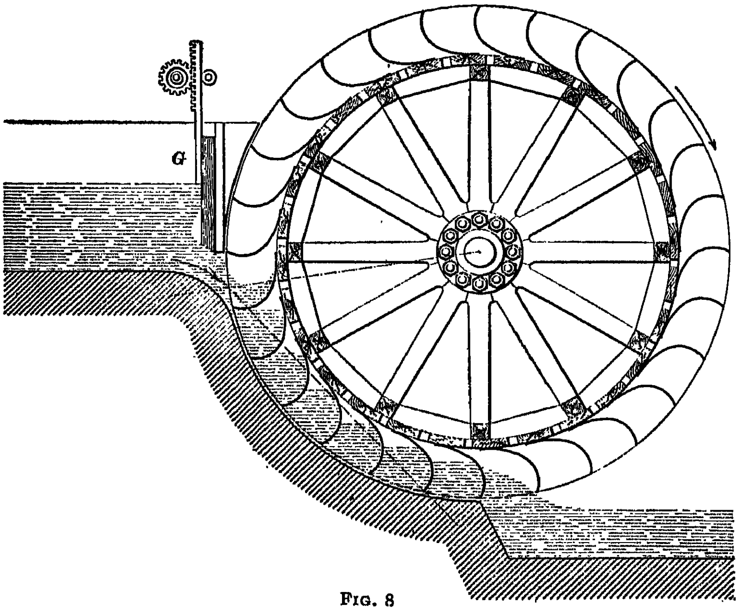


FIG. 8

the fall is too low for an overshot wheel. As shown in Fig. 8, the water flows through an opening in a reservoir or sluice and enters the wheel somewhat below a horizontal

plane through the center of the wheel. The size of the opening is regulated by the gate *G*. In breast wheels, the water acts more largely by its impulse than in overshot wheels, but generally the greater part of the action is due to the weight of the water. The efficiency of a breast wheel ranges from 50 to 70 per cent., the smaller value applying to the smaller sizes.

14. Undershot Wheels.—The **undershot wheel** is used for falls of 6 feet or less. The least efficient form has straight radial floats, as shown in Fig. 9, that are acted on

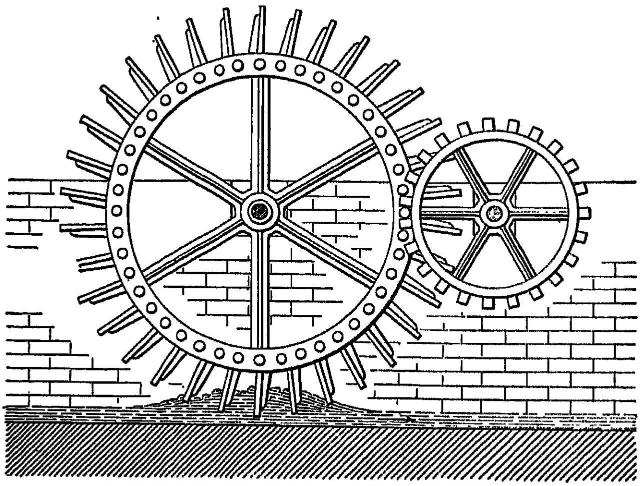


FIG. 9

directly by the current of a swiftly flowing stream. In this case the water acts only by impulse, and the efficiency is seldom greater than 25 per cent. The dimensions of these wheels may vary from 12 feet to 24 feet in diameter and they may have from twenty-four to forty-eight floats. The depth of the floats for best effect should be at least three times the depth of the stream. The velocity of the circumference of the wheel should be about one-half the velocity of the water in the stream; the depth of the stream should be from 4 to 6 inches, and the depth of the floats 12 to 20 inches. There should be as little clearance as possible between the floats and the bottom and sides of the race.

A formula for the horsepower that may be developed by the use of an undershot wheel, such as has been described, is easily derived.

By Art. **11**, the work of the stream on the series of floats is

$$W = \frac{Gu}{g}(v - u)$$

where v = velocity of water in race, in feet per second;
u = velocity of circumference of wheel, in feet per second.

If Q denotes the cubic feet of water flowing per second, then, from the experiments of Smeaton and Bossut, about .61 Q is the quantity striking the floats under usual conditions; hence, $G = 62.5 \times .61\ Q$, and

$$\text{H. P.} = \frac{W}{550} = \frac{62.5 \times .61\ Qu(v-u)}{550 \times 32.16}$$
$$= .00216\ Qu(v-u) \qquad (1)$$

For a **paddle wheel** suspended in an unconfined current, the horsepower may be computed from the formula

$$\text{H. P.} = .00282\ A\,v\,u\,(v-u) \qquad (2)$$

where v = velocity of current, in feet per second;
u = velocity of circumference of wheel, in feet per second;
A = area of immersed portion of the float, in square feet.

MACHINES UTILIZING THE PRESSURE OF WATER

15. Losses of Efficiency in Water Motors.—Machines in which water pressure is employed as the motive force are much used, especially in modern shops. There are hydraulic jacks, cranes, and elevators for hoisting, hydraulic presses for baling, flanging, forging, and many other purposes; and in the machine shop, hydraulic punches, shears, riveters, rail benders, etc. There are also engines and pumps driven by high-pressure water instead of steam. In all these machines, the water is introduced into a closed vessel or cylinder and acts on a movable piston. The theoretical work done is the

product of the weight G of the water used and the head h corresponding to the pressure; that is, work = Gh.

The sources of loss of efficiency are as follows: (1) Friction of the water in mains and passages; (2) losses due to shock when the water changes direction; (3) waste of water when the same quantity is used for light loads as for heavy loads; (4) friction of the mechanism.

16. Water-Pressure Engines.—An oscillating cylinder engine driven by water is shown in Fig. 10. This form is much used in Europe for domestic purposes and for small manufacturing, where only a small amount of power is

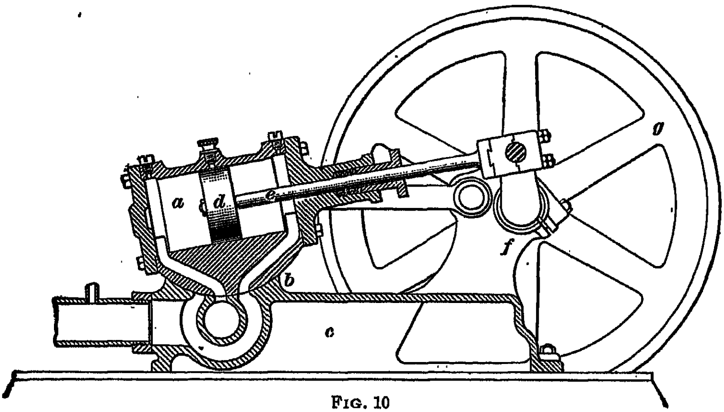


FIG. 10

desired. It consists of an oscillating cylinder a, which turns about the center of the cylindrical valve face b in the bed c as the engine runs. The piston d is a solid piston and may be made water-tight with leather packing. The rod e is guided by a long stuffingbox in the end of the cylinder cover. The bearings f, one on each side of the rod e, carry the crankshaft, on one end of which the flywheel g is fastened.

In this engine, the cylinder moves while the valve remains stationary. There are trunnions on each side of the cylinder with their centers at the center of the arc forming the valve face and provided with screws for holding the cylinder to the valve face. There are three ports in the valve; the middle one admits the water first to one end and then to the other

end of the cylinder. The exhaust or outlet ports are connected together around the inlet port and discharge the water into a waste pipe.

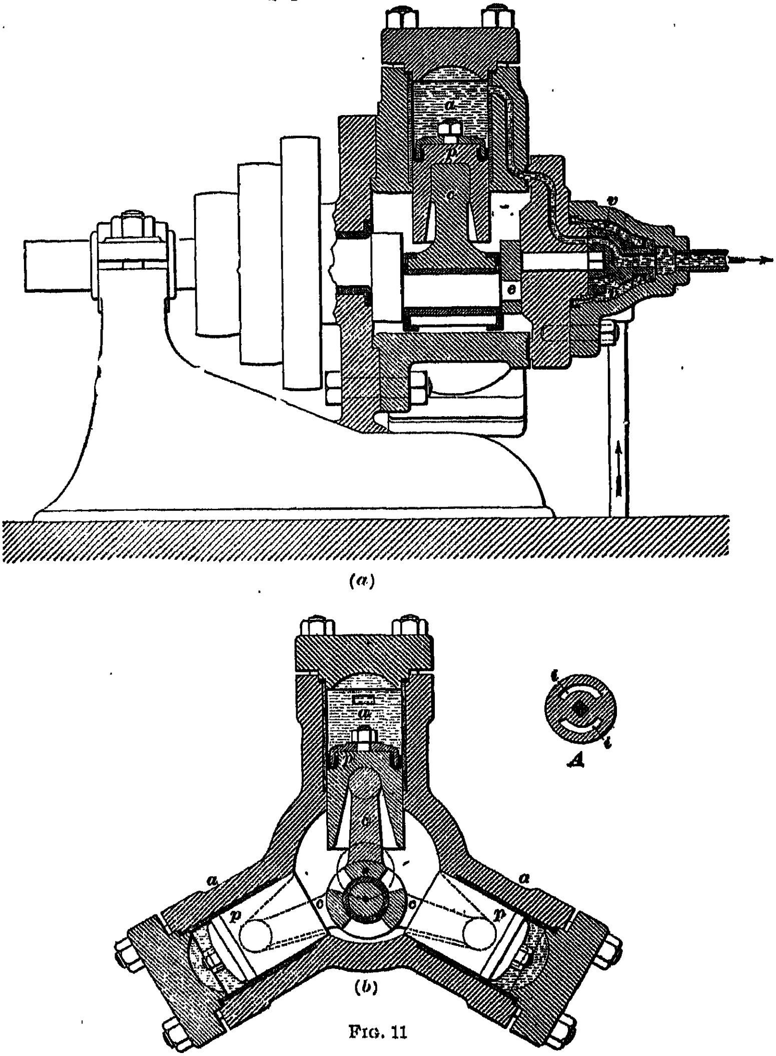


FIG. 11

17. A type of engine that has proved satisfactory in the use of water pressure for producing rotary motion is the

Brotherhood three-cylinder engine, two sectional views of which are shown in Fig. 11 (*a*) and (*b*), the one being taken parallel, and the other at right angles, to the shaft. It consists of three cylinders *a* in which work single-acting pistons *p*. These pistons are all connected to a single crankpin by means of the rods *c*. The water is admitted to the cylinders one after the other by the circular valve *v*, which also controls the exhaust. This valve has a lignum-vitæ seating and is rotated by the eccentric-pin *e* on the end of the crankpin. A view of the face of the valve, showing the ports *i*, *i*, is given at *A*. These ports pass over the passages leading to the cylinders, alternately admitting and exhausting the water. The arrangement of the three cylinders at angles of 120° secures a constant and nearly uniform turning force on the crankpin and makes it possible to start the engine in any position.

When the load is constant and the engine is designed for the load, a hydraulic-pressure engine is an efficient machine. As ordinarily constructed, however, the efficiency with variable loads cannot be high, owing to the fact that the cylinders must be filled with water at each stroke. Throttling the water in its passage to the cylinders reduces the pressure in them, since a part of the energy due to the pressure of the water in the supply pipe is expended in overcoming resistance to flow through the partly closed valves. This energy does no useful work and is therefore lost. A number of hydraulic pressure engines have been designed in which the stroke of the piston is varied to correspond with the work to be done. In this way, the water used is proportional to the work, while the pressure is kept constant, and the efficiency, consequently, is more nearly constant for all loads.

18. Hydraulic Flanging Press.—A press for flanging boiler heads, ship plates, etc. is shown in Fig. 12. The plate *p* is placed on a table *k* that is carried by two small plungers or rams working inside the small cylinders *h*, *h*, so that the table can be lowered when required, as shown by the dotted lines. A heavy adjustable crosshead carries a ring-shaped die *g*, made in the form in which the plate is to

be pressed. A large ram *b* works inside the cylinder *a* and carries a crosshead *c*, to which the annular die *d* is attached by means of the small columns *e*. The plate to be flanged is placed on the table *k* and pressed firmly against the die *g* by admitting water under pressure from a pump or accumulator to the cylinders *h*, *h*; then the die *d* is raised by the

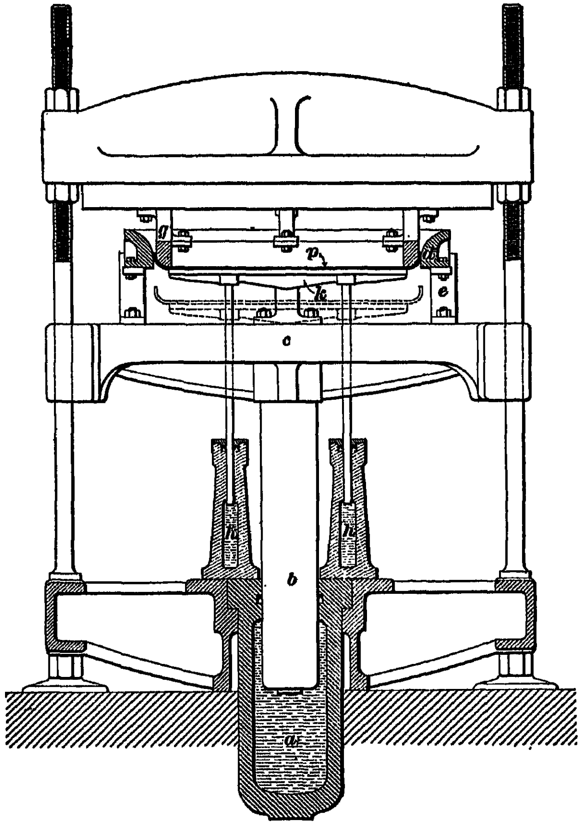


FIG. 12

ram *b* and the plate pressed around *g*, as shown. By this method, plates can be rapidly and accurately flanged, or pressed into any desired shape.

19. Hydraulic Riveter.—Fig. 13 shows a stationary hydraulic riveter of a form much used in boiler work. It

consists of a heavy frame G that carries the ram for pressing down the rivet. To this frame is bolted the stake V, which carries the die n' against which the rivet is held while being subjected to pressure. The rivet is headed by being compressed between the movable die n and the fixed die n'; m is a ring-shaped die that holds the plates to be joined against n' while the rivet is being headed. Water for operating the machine is brought in through the pipe r and is admitted through the valve v to the cylinder A by means of the

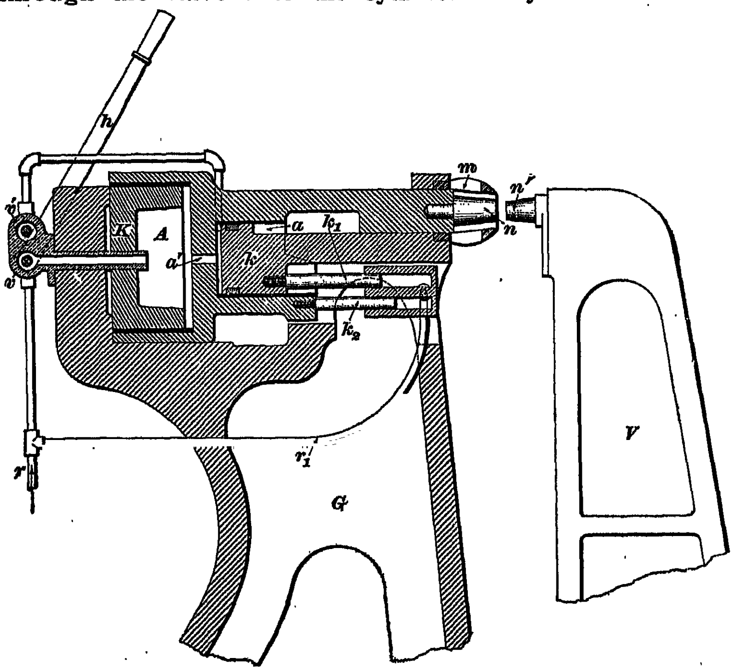


FIG. 13

lever h; K is a stationary piston, and, when water is admitted to A, the cylinder that carries the die n moves toward the rivet. The cylinder A carries a smaller cylinder a, in which works the piston k to which the ring die m is attached, and when water is admitted to A it also passes into a through the small hole a' and thus forces the die m against the plate. After the rivet head has been formed, the lever h is moved so

as to close the valve v and open v'. The water is then discharged from A and a, and the piston k and cylinder A is forced back by means of the two small plungers k_1 and k_2, which are constantly acted on by the water brought in through the branch pipe r_1.

20. General Formula for Pressure Machines.—The principal loss in slow-moving hydraulic machinery is that due to the friction of the packing required to keep the water from leaking past the ram or piston. This friction varies considerably with different methods of packing and the condition of the packing. For hemp packing in good condition, the loss from friction may be taken at from 3 to 8 per cent. of the total pressure. When the packing is new and very tight, the loss from friction will be greatly increased and may be as high as 25 per cent. under specially unfavorable conditions. Table I gives the results of experiments on the friction of leather packing on rams of different sizes.

TABLE I

Diameter of Ram Inches	Friction Per Cent.	Diameter of Ram Inches	Friction Per Cent.
2	2.00	12	.33
3	1.33	13	.30
4	1.00	14	.28
5	.80	15	.26
6	.66	16	.25
7	.57	17	.23
8	.50	18	.22
9	.44	19	.21
10	.40	20	.20
11	.38		

21. To find the net pressure exerted by the ram or plunger of a hydraulic press:

Let d = diameter, in inches, of a hydraulic piston or ram;
G = weight, in pounds, of the ram and attachments that must be lifted by the water;

p = pressure of the water, in pounds per square inch;
f = percentage of friction;
P = net pressure exerted by the ram.

Then, $$P = .7854 \times d^2 \times p \times \left(1 - \frac{f}{100}\right) - G$$

EXAMPLE.—What pressure will be exerted by the ram of a hydraulic press if its diameter is 10 inches, weight 1,500 pounds, pressure of water 550 pounds per square inch, and the friction .4 per cent.?

SOLUTION.—Applying the formula,
$P = .7854 \times 10^2 \times 550 \times (1 - .004) - 1{,}500 = 41{,}524$ lb. Ans.

22. To find the pressure per square inch required to exert a given net pressure, when the diameter and weight of the ram and the percentage of friction are given, solve for p in the formula of Art. **21.**

Then, $$p = \frac{P + G}{.7854\, d^2 \times \left(1 - \frac{f}{100}\right)}$$

EXAMPLE.—What must be the pressure per square inch on the horizontal ram of a hydraulic riveting machine if the diameter of the ram is 12 inches, the pressure required to head the rivet 80,000 pounds, and the friction $1\frac{1}{4}$ per cent.?

SOLUTION.—Since the ram is horizontal, its weight does not act against the pressure. Then, from the formula, and neglecting the factor G,

$$p = \frac{80{,}000}{.7854 \times 12^2 \times (1 - .0125)} = 716 \text{ lb. per sq. in.} \quad \text{Ans.}$$

23. To find the diameter of the piston or ram required to exert a given net pressure, solve for d in the formula of Art. **21.**

$$d = \sqrt{\frac{P + G}{.7854 \times p \times \left(1 - \frac{f}{100}\right)}}$$

EXAMPLE.—What must be the diameter of the ram of a hydraulic crane intended to lift a weight of 40,000 pounds, the weight of the moving parts of the crane being 12,750 pounds, the pressure 750 pounds per square inch, and the friction 2 per cent.?

SOLUTION.—Applying the formula,

$$d = \sqrt{\frac{40{,}000 + 12{,}750}{.7854 \times 750 \times (1 - .02)}} = 9.56 \text{ in.} \quad \text{Ans.}$$

24. Accumulators.—It is not often that a natural supply of high-pressure water is available; and, therefore, when it is not available, power-driven pumps are used to produce the extra pressure needed. Sometimes the water is used direct from the pumps; but when machines are working intermittently, some arrangement is needed by means of which the work of the pumps may be stored. The device most used for this purpose is the **accumulator.**

Fig. 14 shows an accumulator as usually constructed. There is a hydraulic cylinder *B* in which works a ram *C*. At its upper end, this ram carries a head *D*, from which a heavy weight *E*, *E* is suspended. The weight may consist of an annular cylinder filled with some heavy material, such as iron ore or scrap iron, or it may be made up of rings of cast iron. Water from the pumps enters the cylinder through the pipe *A*, and the hydraulic machines draw their supply through the pipe *F*. When the pressure of the water is great enough, the ram rises and lifts the weight. If the pumps supply more water than is used by the machines, the weight is lifted until it strikes the lever *G*. When *G* is raised, it operates the rods *r*, *r* and bell-crank lever *l*, and so closes the valve that admits steam to the pumps, thus stopping them until enough water is used to allow the weight to fall, when the steam valve again opens and sets the pumps in operation.

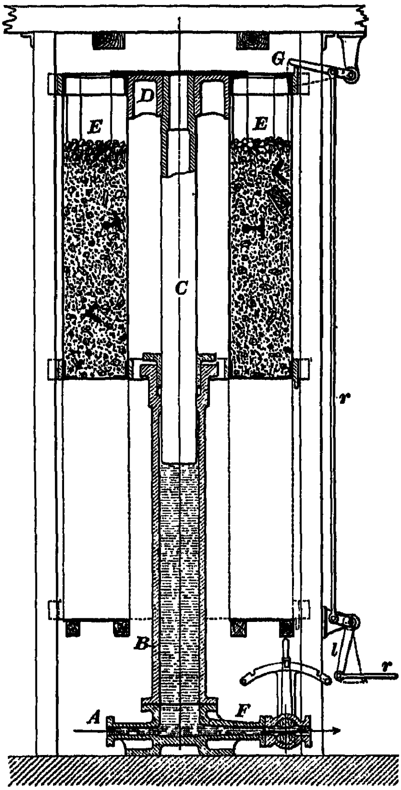


FIG. 14

25. Energy Stored in an Accumulator.—

Let p = pressure of water, in pounds per square inch;

G = total weight of ram with load;

d = diameter of ram, in inches;

l = stroke of accumulator, in feet;

Q = volume of water used, in cubic feet;

m = number of minutes in which Q is used;

H. P. = horsepower.

Neglecting the friction, which is small, the total weight of the ram and its load equals the total pressure on the ram, or

$$G = .7854\, d^2 p \qquad (1)$$

From which, by solving for the pressure,

$$p = \frac{G}{.7854\, d^2} \qquad (2)$$

Again, solving formula **1** for d,

$$d = \sqrt{\frac{G}{.7854\, p}} \qquad (3)$$

To raise the ram and load through the stroke l requires the performance of $G\,l$ foot-pounds of work; hence, the stored energy is

$$E = G\,l, \text{ or } E = .7854\, d^2 p\, l \qquad (4)$$

The volume of water in the cylinder when the ram is at its highest point is $\frac{.7854\, d^2}{144} \times l$ cubic feet; hence, the energy stored per cubic foot of water is

$$.7854\, d^2 p\, l \div \frac{.7854\, d^2 l}{144} = 144\, p \text{ foot-pounds.}$$

The number of cubic feet required per minute for each horsepower is therefore $\frac{33{,}000}{144 p} = \frac{229.2}{p}$. Then the number of cubic feet for any horsepower for m minutes is found by the formula

$$Q = \text{H. P.}\, \frac{229.2\, m}{p} \qquad (5)$$

EXAMPLE 1.—An accumulator is required to supply machines having a total of 45 horsepower for an interval of 15 seconds; the pressure is 600 pounds per square inch. If the ram is 12 inches in diameter, how far will the accumulator fall?

SOLUTION.—For 1 H. P. per min. $\frac{229.2}{p}$ cu. ft. is required; hence, for 45 H. P. for $\frac{1}{4}$ min., the volume required is found, by substituting in formula 5, to be

$$Q = 45 \times \frac{229.2 \times \frac{1}{4}}{600} = 4.3 \text{ cu. ft.}$$

The area of the ram is $.7854 \times 1^2 = .7854$ sq. ft.; hence, to deliver 4.3 cu. ft., the ram must fall $4.3 \div .7854 = 5.47$ ft. Ans.

EXAMPLE 2.—In example 1, what energy can be stored in the accumulator if its stroke is 12 feet?

SOLUTION.—Applying formula 4,

$$E = .7854 \times 12^2 \times 600 \times 12 = 814{,}300 \text{ ft.-lb., nearly.} \quad \text{Ans.}$$

EXAMPLE 3.—A 10-inch accumulator ram weighs 6,340 pounds; with what weight must it be loaded in order that the pressure in the cylinder shall be 2,000 pounds per square inch?

SOLUTION.—Applying formula 1,

$$G = .7854\, d^2 p = .7854 \times 10^2 \times 2{,}000 = 157{,}080 \text{ lb.}$$

Since the ram weighs 6,340 lb., the load must be

$$157{,}080 \text{ lb.} - 6{,}340 \text{ lb.} = 150{,}740 \text{ lb.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. What pressure will be exerted by the ram of a hydraulic press if it is 12 inches in diameter, the weight 2,000 pounds, the pressure of water 600 pounds per square inch, and the friction .33 per cent.? Ans. 65,635 lb.

2. An 8-inch accumulator ram weighs 4,320 pounds; with what weight must it be loaded in order that the pressure in the cylinder may be 1,800 pounds per square inch? Ans. 86,158 lb.

3. What is the energy that can be stored in an accumulator if the diameter of the ram is 10 inches, the stroke is 8 feet, and the pressure is 500 pounds per square inch? Ans. 314,160 ft.-lb.

MOTORS IN WHICH THE VELOCITY OF WATER IS UTILIZED

IMPULSE WHEELS

26. An **impulse wheel,** or **hurdy-gurdy,** is a water-wheel that has a number of vanes or buckets, against which a jet of water is made to impinge in such a way that the velocity and direction of motion of the water are changed in its passage over the moving vanes. This causes the water

to press against the vanes, and the kinetic energy of the jet is changed to work according to the principles stated in Art. **11**.

The simplest form of impulse wheel consists of a wheel provided with a series of flat radial vanes around its circumference, similar to the paddle wheel of a steamboat. A wheel of this kind can never have a high efficiency, since the water must leave the vanes with an absolute velocity nearly equal to the relative velocity with which it strikes. Experiments have shown a maximum efficiency of a little more than 40 per cent. for this kind of wheel.

27. The Pelton Waterwheel.—The **Pelton waterwheel**, shown in Fig. 15, is an impulse wheel that is used

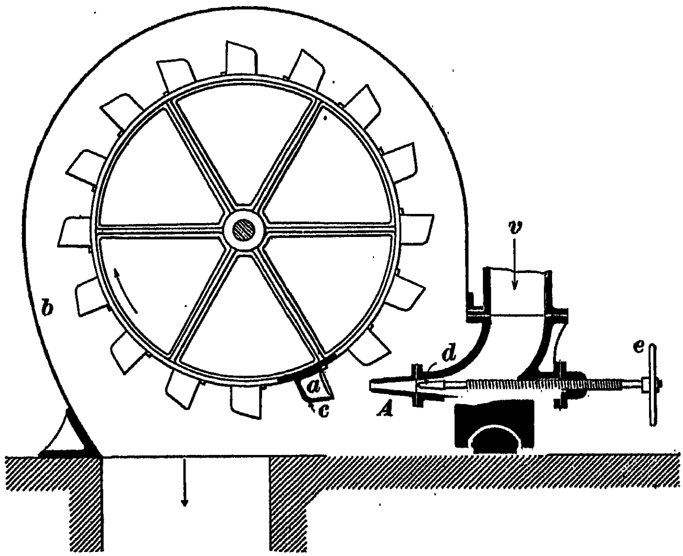


Fig. 15

for very high heads and comparatively small volumes of water. The wheel is covered by the casing *b*. The water enters at *v*, and the flow is regulated by the valve *d*, operated by the hand wheel *e*. The jet from the nozzle *A* impinges on the raised center *a* of the cups *c*, is deflected to both sides, and finally leaves the cups in a direction tangent to their

outer edges. In this way, the direction of the motion of the jet is changed nearly 180°; and when the velocity of the cup is equal to one-half the velocity of the jet, the theoretical efficiency of the wheel is 100 per cent. Experiments have shown that the actual efficiency is sometimes nearly 90 per cent. and that the best efficiency is obtained when the actual velocity of the cups corresponds nearly to the theoretical velocity.

The loss of efficiency is due to the friction of the water in passing through the cups and the energy that is lost in the absolute velocity of the water when it leaves them.

28. Fig. 16 shows two sections of the cups, and the common method of fastening them to the rim of a cast-iron wheel. The inclination of the outer edges *a* is such that the water, as it leaves them, flows clear of the wheel; this is done so that the water will offer no resistance to the motion of the wheel after leaving the cups. The faces of the cups are also inclined to the radius of the wheel, as shown, in order to give the water a slight tendency to flow away from the center of the wheel as it reacts from the cups. The outer edges of the cups are made sharp, so as to offer as little resistance to the water as possible, and the inside surface is sometimes finished for the purpose of reducing the loss by friction.

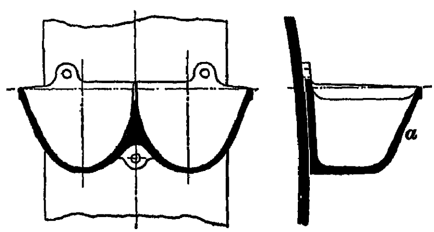


FIG. 16

A wheel that acts on the same principle as the Pelton wheel is the **Leffel cascade wheel.** The Pelton and Leffel cascade wheels are seldom used for heads of less than 50 feet, but are applicable to falls of any greater height. A number of wheels are in use under heads of more than 2,000 feet. They are specially applicable to mountain streams with high heads and small quantities of water, conditions that exist in the mining districts of Western America.

2(. **Calculations for Impulse Wheels.**—The circumferential velocity of an impulse wheel, that is, the actual velocity of the cups, depends on the head, and hence on the velocity of the jet. With a properly designed nozzle, the velocity of the jet will be nearly that due to the pressure head in the end of the pipe, and the best efficiency is obtained when the velocity of the cups is about one-half the velocity of the jet.

The number of revolutions, with a given velocity at the circumference, varies inversely as the diameter of the wheel; it is therefore possible to make the number of revolutions correspond to the speed of the machinery to be driven, within certain limits. In accordance with this principle, wheels are often designed so as to run at a speed that enables them to be connected directly to the shafts of dynamos, centrifugal pumps, or similar machinery without the use of belts or gearing.

EXAMPLE 1.—Required, the diameter of an impulse wheel that is to be directly connected to the shaft of a dynamo; the pressure head is 275 feet, the dynamo is required to make 850 revolutions per minute, and the coefficient of velocity of the jet is .98.

SOLUTION.—The velocity of the jet is $.98\sqrt{2gh} = .98 \times 8.02\sqrt{275} = 130.34$ ft. per sec. The circumferential velocity of the wheel, therefore, should be $130.34 \div 2 = 65.17$ ft. per sec., or (65.17×60) ft. per min., and the diameter required for 850 rev. per min. is

$$d = \frac{65.17 \times 60}{850 \times 3.1416} = 1.464 \text{ ft., say 18 in. Ans.}$$

Let e = efficiency of the wheel;
G = weight of water used per second;
h = head, in feet.

The work done per second is eGh and the horsepower is

$$\text{H. P.} = \frac{eGh}{550}$$

EXAMPLE 2.—In example 1, suppose that the jet has a diameter of $1\frac{1}{4}$ inches and the efficiency is .85; what is the horsepower developed?

SOLUTION.—The weight $G = \frac{.7854 \times (1\frac{1}{4})^2}{144} \times 130.34 \times 62.5$; hence, by substituting in the above formula,

$$\text{H. P.} = \frac{.85 \times .7854 \times (1\frac{1}{4})^2 \times 130.34 \times 62.5 \times 275}{144 \times 550} = 29.5. \text{ Ans.}$$

REACTION WHEELS

30. Barker's Mill.—In the simple reaction wheel, commonly known as **Barker's mill,** and shown in Fig. 17, the pressure that produces the motion is caused by the reaction of a jet of water that issues from an orifice under a head.

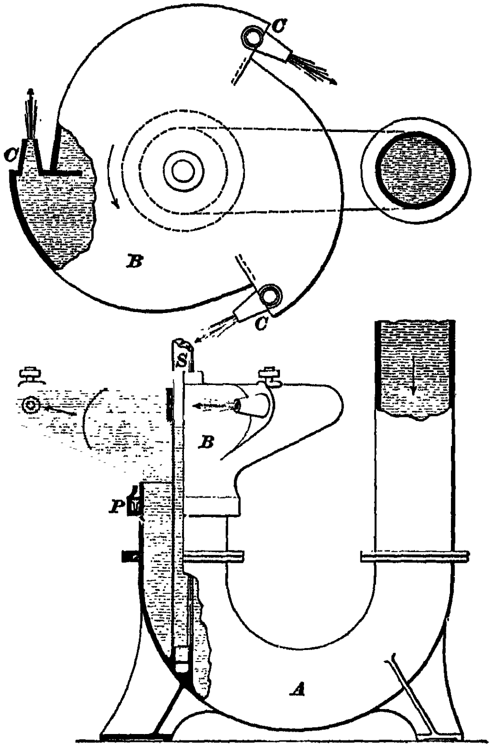


FIG. 17

Water is brought to the wheel through the curved pipe *A*, which opens into the revolving head *B*. From *B*, the water flows through the nozzles *C* and the pressure caused by the reaction of the issuing jets causes the head *B* to revolve. The head *B* is keyed to a shaft *S*, from which the power is taken, and a cup-leather packing *P* is provided to prevent leakage through the joint between *B* and the pipe *A*.

The efficiency of the simple reaction wheel can be but little more than 50 per cent., even under very favorable conditions, and for that reason it is not now used as a motor. A familiar example of the simple reaction wheel is the revolving lawn sprinkler.

31. Efficiency of Reaction Wheels.—In Art. **8**, it was shown that the reaction of the jet on the vessel from which it issues is $\frac{Gv}{g}$ pounds, provided that the vessel is at rest. If, however, the orifice has a speed of u feet per second, while the speed of the jet relative to the orifice is v feet per second, the reaction is $\frac{G}{g}(v-u)$ pounds, according to Art. **10**, and the work done per second is $\frac{G}{g}u(v-u)$ foot-pounds. The absolute velocity of the issuing water is $(v-u)$ feet per second, and the work wasted per second is therefore the kinetic energy $\frac{G}{2g}(v-u)^2$. Hence,

$$\begin{aligned}
\text{total work} &= \text{useful work} + \text{lost work} \\
&= \frac{G}{g}u(v-u) + \frac{G}{2g}(v-u)^2 \\
&= 2\frac{G}{2g}uv - 2\frac{G}{2g}u^2 + \frac{G}{2g}v^2 - 2\frac{G}{2g}vu + \frac{G}{2g}u^2 \\
&= \frac{G}{2g}(v^2-u^2)
\end{aligned}$$

$$\begin{aligned}
\text{Efficiency} &= \frac{\text{useful work}}{\text{total work}} \\
&= \frac{\frac{G}{g}u(v-u)}{\frac{G}{2g}(v^2-u^2)} \\
&= \frac{2u(v-u)}{(v+u)(v-u)} \\
&= \frac{2u}{v+u}
\end{aligned}$$

This efficiency becomes theoretically 100 per cent. when $u = v$, so that the water drops without velocity, but with

such a high value of u, the friction of the water through the nozzles and orifices is relatively large. In general, the efficiency is little more than 50 per cent. under the most favorable conditions.

TURBINES

32. Classification of Turbines.—The name **turbine** is given to a waterwheel that utilizes the energy of water by causing it to flow through curved vanes or buckets on which the water acts either by impulse alone, or by both impulse and pressure. Turbines may be divided into two chief classes:

1. **Impulse turbines,** in which the whole available energy of the water is converted into kinetic energy before it acts on the moving parts of the turbine, the water striking the turbine blades in the form of jets.

2. **Reaction turbines,** in which part of the energy of the water is in the form of kinetic energy and part is in the form of pressure energy.

In an impulse turbine, the water flows freely into the wheel from the guide buckets in the form of a jet, whose velocity is produced by the head of water on the buckets, and it passes over the wheel vanes without filling the space between them; the passages through the wheel are always open to the air, and consequently the pressure in the space between the wheel vanes is always nearly equal to the atmospheric pressure; the acting force is almost entirely the pressure due to the impulse of the jets issuing from the guides.

In a reaction turbine, the passages between the wheel vanes are always completely filled, so that the flow is said to be continuous. The pressure and the velocity of the water as it enters the wheel may, under different conditions, be equal to, greater, or less than the pressure and the velocity due to the head on the wheel; and the forces that act on the wheel vanes are: (1) a certain amount of static pressure; (2) the pressure caused by the change in direction of the moving water; and (3) a pressure due to the reaction of the

water as it issues from the vanes of the wheel. In most cases, the greatest of these forces is the pressure caused by the change in direction of the moving water in its passage through the wheel. If a reaction turbine is working open to the air, and the flow from the guides is restricted so that the passages between the wheel vanes are only partly filled, it becomes an impulse turbine; hence, the same wheel, under different conditions, may work either as a reaction turbine or as an impulse turbine.

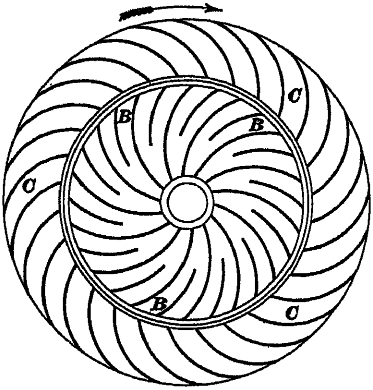


FIG. 18

33. Types of Turbines. Fig. 18 shows a cross-section of an **outward-flow turbine,** also called a **Fourneyron turbine,** from its inventor. The water is brought in at the center, passes outwards between the curved guide vanes B, B to the wheel vanes C, C and is discharged at the circumference of the wheel. The flow of water is regulated by a cylindrical gate that can be raised or lowered in an annular space between the wheel and guides.

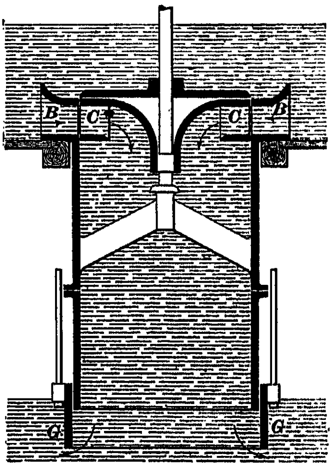


FIG. 19

34. In Fig. 19 is shown a vertical section, and in Fig. 20 a cross-section, through the vanes, of an **inward-flow,** or **Francis, turbine.** Here the water enters the guides B from the outside, passes inwards to the wheel vanes C, and is discharged near the center of the wheel. These wheels are often placed some distance above

the level of the tail-water, as shown, and discharge into an air-tight tube, commonly called a *draft tube*. The wheel is thus in a position that makes it possible to inspect and repair it easily, while at the same time it utilizes the total fall.

The supply of water in the wheel shown in the figure is regulated by a gate G at the outlet of the draft tube.

Outward-flow and inward-flow turbines are also called **radial turbines**, since the general direction in which the water moves through the wheel is radial.

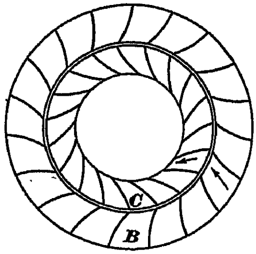


FIG. 20

35. In Fig. 21 is shown a **downward-flow**, or **Jonval, turbine**. Here the general direction of the water is always parallel to the shaft A, or axis; hence, wheels of this class are also known as *parallel-flow* and *axial turbines*.

The water usually enters the

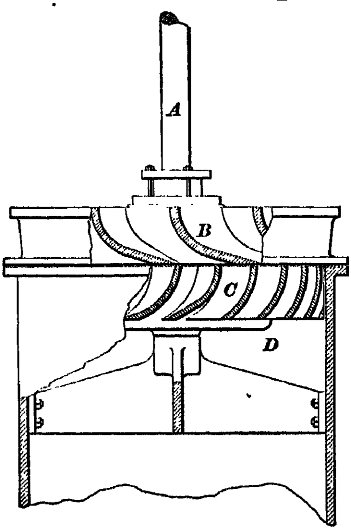


FIG. 21

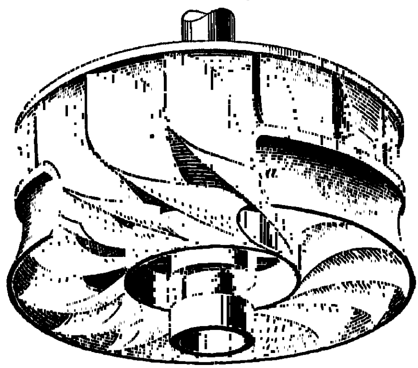


FIG. 22

guides B from above and is discharged downwards through the wheel C into a draft tube D, as shown in the figure. The discharge may also take place into the air or tail-water without the use of a draft tube.

36. Many American turbines are made with the wheel vanes so curved that the water enters the wheel in a radial

direction, like an inward-flow turbine, and is discharged in a downward, or axial, direction. These are called *mixed-flow turbines.*

Fig. 22 shows the wheel of a **Risdon mixed-flow turbine** with the double curvature of the vanes. This wheel is cast in one piece. The band *a* serves the double purpose of strengthening the wheel and of making the proper form for the passage of the water through the lower part of the wheel, confining it on all sides.

PUMPS

RECIPROCATING PUMPS

37. Pumps as Hydraulic Machines.—In the hydraulic machines and motors so far described, the water has acted as the motive force, that is, it has done work on the moving parts. Another class of hydraulic machines will now be considered, in which the moving parts do work on the water, either in raising it from a lower to a higher level or in forcing it through pipes. These machines are called **pumps.**

38. Lifting and Force Pumps.—The ordinary **lifting pump** is shown in Fig. 23, and the **force pump** in Fig. 24. The action of the lifting pump is as follows: Suppose the piston to be at the bottom of its stroke; then, as it is raised by the rod S it tends to leave a vacuum in the space below it and the atmospheric pressure on the surface of the water in the well forces the water up the pipe P and through the valve V. When the piston has reached the top of its stroke, the space below it is filled with water. When the piston starts to descend, the weight of the water in the cylinder forces the valve V shut, and the enclosed water passes up through the valves u, u. On the next upward stroke, the valves u, u close and the water above the piston is forced into the pipe P' through the valve c. At each upward stroke, therefore, a quantity of water equal in bulk

to the volume swept through by the piston is raised from the well.

The action of the force pump is easily seen from Fig. 24. During the upward stroke of the piston, the water rises in the pipe P, passes through the valve V, and fills the cylinder. When the piston starts to return, the pressure exerted on the water by the piston causes the valve V' to open, and the valve V to close, so that the water is forced from the cylinder into the pipe P'. With this pump, the water may be forced to almost any height.

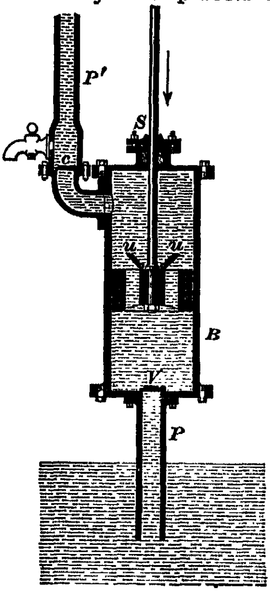


FIG. 23

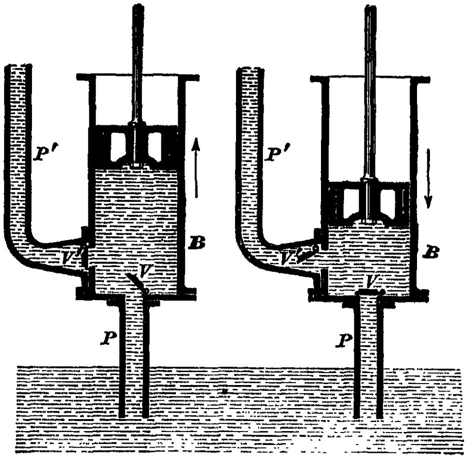


FIG. 24

39. Direct-Acting Steam Pump.—In the steam pump shown in Fig. 25, the motive force is steam pressure acting on the piston g. This piston, the rod r, and the plunger p in the water cylinder, form a single rigid piece that reciprocates, that is, moves to and fro, as steam is turned alternately into the two ends of the steam cylinder. The plunger p slides in a partition f cast with the cylinder.

Suppose the piston and plunger to move toward the right. The displacement of the plunger to the right increases the volume to the left of the partition f and decreases that to the right of f by the same amount. In consequence, water will flow through the suction pipe c into the chamber k; and

because of the partial vacuum to the left of f, will lift the valves s' and fill the space left vacant by the motion of the plunger. At the same time, the water in the right end will be forced through the valves v' out into the delivery pipe h. During the stroke to the left, the action is reversed; water flows into the right end of the cylinder through valves s and is forced out of the left end through the valves v. The hand-hole d is provided for examining the valves.

This pump is *double-acting*, as it forces water during both strokes. The pumps shown in Figs. 23 and 24 are *single-*

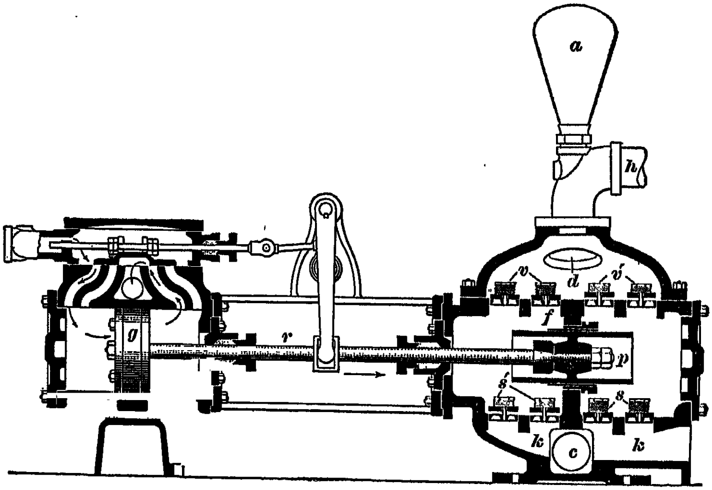


FIG. 25

acting, because they pump water only when the piston moves in one direction. Lift pumps are always single-acting, but force pumps are sometimes double-acting.

40. Air Chambers.—In even the double-acting pumps, there is an interruption of the flow at the end of the stroke, when the piston changes its direction of motion. This has the effect of bringing the column of water in the suction and discharge pipes to rest at the end of each stroke, and this column of water must be set in motion again as the next stroke is made. If the pipes are long, the force required to stop and start the water will be very great, and there will be

a severe shock at the end of every stroke that will absorb power and subject the pump and pipes to great pressures.

This difficulty is removed and the flow through the pipes is made more continuous and steady by the use of an **air chamber.** An air chamber is a vessel containing air, and is attached either to the pump just outside of the discharge valves or to the discharge pipe near the pump, as shown at *a*, Fig. 25.

The water, after being drawn in through the valves *s* is forced by the plunger *p* through the valves *v'* into the discharge pipe *h*; but part of it flows into the air chamber *a* and compresses the air therein. When the plunger reaches the end of its stroke and no more water is being forced into the discharge pipe, the compressed air in the air chamber expands and forces the extra water out through the discharge pipe. In this way, the air chamber acts as a reservoir that receives its supply during the motion of the plunger, and gives it out again when the plunger comes to a pause. The air in the air chamber acts as a spring or cushion that absorbs some of the force of each stroke of the plunger and gives it out while the plunger is at rest at the end of the stroke. The pump and pipe are thus relieved of shocks and a nearly constant rate of flow from the discharge pipe is insured.

41. Displacement and Discharge.—The **displacement** of a pump for a single stroke is the volume of water that would be displaced, that is, the volume swept through by the piston or plunger, during that stroke. The **theoretical discharge** of a pump is equal to the displacement. The **actual discharge** is generally less than the displacement, owing to leakage past the valves and piston, and also to the return of water through the valves while they are closing.

The difference between the displacement and the actual discharge, expressed as a percentage of the displacement, is called the **slip** of a pump. When the column of water in the suction and discharge pipes of a pump is long and the lift moderate, the energy imparted by the piston during the

discharge stroke may be sufficient to keep the column in motion during all or a part of the return stroke. Under these conditions, the actual discharge will be greater than the displacement, in which case the slip is said to be *negative*.

42. Let Q' = displacement or theoretical discharge, in cubic feet per minute;
Q = actual discharge, in cubic feet per minute;
N = actual discharge, in gallons per minute;
d = diameter of piston or plunger, in inches;
l = stroke of piston, in feet;
n = number of discharging strokes per minute;
s = the slip $= \frac{Q' - Q}{Q'}$.

The volume swept through by the piston in one stroke is $\frac{.7854\, d^2}{144} \times l$ cubic feet; hence,

$$Q' = \frac{.7854\, d^2\, l\, n}{144} = .005454\, d^2\, l\, n \qquad (1)$$

As $s = \frac{Q' - Q}{Q'} = 1 - \frac{Q}{Q'}$, $Q = Q'(1 - s)$; hence,

$$Q = .005454\, d^2\, l\, n\,(1 - s) \qquad (2)$$

Since there are 7.48 gallons per cubic foot, $N = 7.48\, Q$, and hence,

$$N = .0408\, d^2\, l\, n\,(1 - s) \qquad (3)$$

The diameter of the piston for a given discharge, in cubic feet per minute, is found by solving formula **2** for d; this gives

$$d = 13.54\sqrt{\frac{Q}{l\, n\,(1 - s)}} \qquad (4)$$

If the discharge is taken in gallons per minute, solve formula **3** for d; then,

$$d = 4.95\sqrt{\frac{N}{l\, n\,(1 - s)}} \qquad (5)$$

The slip varies from 0 to 40 per cent., depending on the tightness of valves, piston, etc.

EXAMPLE 1.—A single-acting plunger pump with a plunger 8 inches in diameter and 36 inches stroke discharges 33.5 cubic feet of water per minute when making thirty-five discharging strokes; what is the slip?

SOLUTION.—Using formula **1**, and substituting the given valves, $Q' = .005454 \times 8^2 \times 3 \times 35 = 36.65$ cu. ft. per min. The slip, therefore, is

$$\frac{36.65 - 33.5}{36.65} = .086 = 8.6 \text{ per cent., nearly. Ans.}$$

EXAMPLE 2.—A pump making thirty discharging strokes per minute is required to discharge 450 gallons per minute; the length of stroke is 36 inches. Assuming the slip to be .25, compute the diameter that must be given the plunger.

SOLUTION.—From formula **5**, by substituting,

$$d = 4.95\sqrt{\frac{450}{3 \times 30 \times (1 - .25)}} = 12.78 \text{ in., say 13 in. Ans.}$$

43. Work Done by a Pump.—The theoretical work done by a pump may be computed in either of two ways. In addition to the symbols given in Art. **42**, let p be the pressure on the piston, in pounds per square inch, and h be the height, in feet, through which the water is lifted. The area of the piston is $.7854\, d^2$ square inches; hence, the total pressure on it is $.7854\, d^2 p$ pounds. In one stroke, the resistance is overcome through a distance of l feet, and the work done per stroke is therefore $.7854\, d^2 p\, l$ foot-pounds. In 1 minute, the work done is $.7854\, d^2 p\, l\, n$ foot-pounds, and therefore the theoretical horsepower is

$$\text{H. P.} = \frac{.7854\, d^2 p\, l\, n}{33{,}000} = .0000238\, d^2 p\, l\, n \qquad (1)$$

But in 1 minute the pump raises G pounds of water through a height of h feet, requiring therefore the work $G h$ foot-pounds. If there is no slip, $G = 62.5\, Q'$, since a cubic foot of water weighs 62.5 pounds, and

$$\text{H. P.} = \frac{62.5\, Q'\, h}{33{,}000} = .001894\, Q'\, h \qquad (2)$$

That the two expressions for the horsepower are identical is readily shown; for, if $.434\, h$ is substituted for p in the first, and for Q' the value given in formula **1** of Art. **42**, in the second, the result is

$$\frac{.7854\, d^2 l\, n}{33{,}000} \times .434\, h = \frac{.7854\, d^2 l\, n}{33{,}000} \times \frac{62.5}{144} h$$

and as $.434 = \frac{62.5}{144}$, the expressions are identical.

If the discharge is given in gallons instead of cubic feet, formula **2** becomes

$$\text{H. P.} = .000253\,N h \qquad \textbf{(3)}$$

The actual work is always greater than the useful work, and, in practice, from 20 to 50 per cent. should be added to the results obtained from formulas **1**, **2**, and **3**. Work is required to overcome the friction of the piston or plunger in the cylinder or stuffingbox, and considerable work is also required to overcome the friction of the water in its passage through the pipes and the valves and passages of the pump. Some work must also be expended in giving the water the velocity it has when it leaves the discharge pipe.

According to the principles of hydraulics and the flow of water through pipes, it is evident that the power required to overcome the frictional resistance of the water will be reduced by making the pipes large and direct and the passages through the valves and pump of ample size and as direct as possible, so as to avoid loss from sudden change of direction of flow.

EXAMPLE.—What is the theoretical horsepower of a pump raising 375 gallons of water per minute a height of 120 feet?

SOLUTION.—Applying formula 3 and substituting,

$$\text{H. P.} = .000253 \times 375 \times 120 = 11.385. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. A single-acting pump with a plunger 10 inches in diameter and 40 inches stroke discharges 60 cubic feet of water per minute when making thirty-six discharging strokes per minute; what is the slip?
Ans. 8.32 per cent.

2. A pump making thirty-two discharging strokes per minute is required to discharge 300 gallons per minute; the length of stroke is 30 inches. Assuming the slip to be .20, compute the diameter that must be given the plunger. Ans. 10.72, say 11 in.

3. A pump raises 900 gallons of water per minute to a height of 175 feet; what theoretical horsepower is required? Ans. 39.85 H. P.

CENTRIFUGAL PUMPS

44. Reciprocating pumps may be considered as reversed pressure motors. A pressure motor, if driven from some external source, becomes a pump; the feedpipe of the motor becomes the suction pipe of the pump, and the exhaust pipe becomes the delivery pipe.

Likewise, the **centrifugal pump** may be considered as a reversed velocity motor. In the motor, water gives up kinetic energy, due to its velocity, and from this energy is

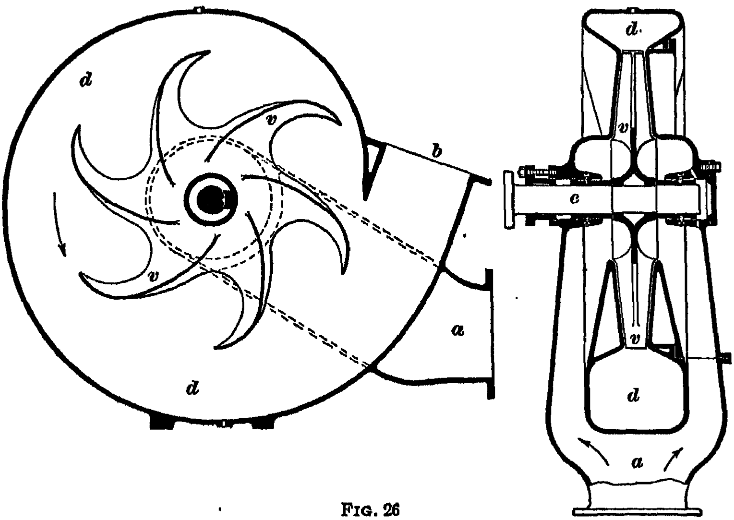


FIG. 26

obtained the work required to drive the moving parts. In the centrifugal pump, the action is reversed; the water enters with little velocity, and at atmospheric pressure work is done on it by the vanes of the *impeller* or wheel of the pump, and it leaves the pump with a higher velocity and higher pressure. By virtue of this increased velocity and pressure, the water is enabled to rise to the upper level. As the water rises, its kinetic and pressure energies are destroyed, but its potential energy is correspondingly increased.

In Fig. 26 are shown two sections of a centrifugal pump, taken at right angles to each other. The water enters

through a suction pipe *a* and is discharged at *b*. On the shaft *e* is keyed the wheel that carries the vanes *v*, *v*. By the rapid rotation of the wheel, the water is given a whirling motion as it passes outwards toward the ends of the vanes, and it is discharged into the passage *d* with considerable velocity and with a pressure in excess of atmospheric pressure.

Centrifugal pumps of this type are most efficient when working under low heads, say up to 40 feet. For low heads and large quantities of water, they give excellent results and are especially useful when the water contains grit or other impurities that would destroy the pistons and packing or prevent the closing of the valves of other pumps. Since there are no valves or other restricted passages, centrifugal pumps have been largely used in dredging machines for pumping water containing large quantities of mud, sand, and gravel; in fact, anything can be pumped that will be carried through the pump and pipes by a current of water. Recently, centrifugal pumps have been built for use with high heads, in some cases raising water several hundred feet. Such pumps are called *high-pressure centrifugal pumps.*

THE HYDRAULIC RAM

45. The **hydraulic ram** is a machine in which the pressure produced by suddenly stopping a column of moving water is used to raise a part of that water to a point above the level of the source of supply. Fig. 27 shows a section of a hydraulic ram. The pipe *a* that connects the ram with the source of supply is called the **drive pipe.** A valve *b* that closes the end of the pipe *a* has a stem that slides freely through a sleeve *c*. The sleeve *c* is provided with a regulating screw by means of which the stroke of the valve *b* may be regulated. An air chamber *f* is attached to the pipe *a* over an opening closed by a valve *d*, which opens from the pipe toward the air chamber. The action of the ram is as follows: Starting with the valve *b* open as shown, water will flow from the reservoir through the pipe *a* and out past *b* through

the opening at *c*. When the velocity of this water becomes sufficiently rapid, the current flowing past *b* will exert an upward pressure great enough to raise the valve and close it. The current is thus suddenly stopped, and the inertia of the column of water in the pipe *a* produces sufficient pressure to open the valve *d* and force some of the water into the air chamber *f*. The energy of the moving mass of water is thus absorbed in compressing the air in *f*, and the column is brought to rest. The stoppage of the column in *a* is so sudden that there is a slight recoil, which closes the valve *d* and causes a reduction in the pressure sufficient to open *b* again, when the process is repeated. The pressure of the com-

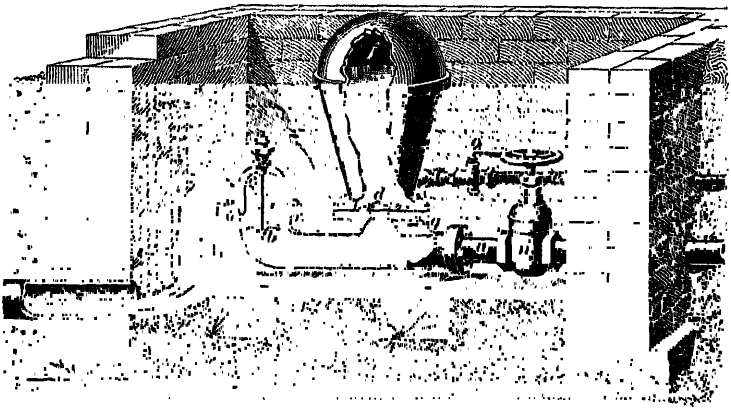


FIG. 27

pressed air in *f* forces a nearly constant stream out through the discharge pipe *e*. The air in the air chamber is gradually absorbed by the water, and therefore, in order to replace it, a **snifting valve** *g* is provided. When the recoil occurs, a small amount of air is drawn in through *g*, and this air is forced through *d* into the air chamber with the next charge of water. The valve *n* and the cock *o* regulate the flow through the inlet and discharge pipes, respectively.

The principal use of these rams is for the purpose of supplying buildings, water tanks, etc. from a source some distance below them. Tests have shown that when they are well made and adjusted their efficiency is about 50 per cent.

Rams may be used when the fall from the supply is no more than 18 inches, but the proportion of the water discharged varies almost directly as the ratio between the fall to the ram and the height to which the water must be raised. With moderate lengths of discharge pipe, the proportion of the water that can be raised is given as follows: One-seventh of the water can be raised to a level above the ram five times as high as the fall from the supply to the ram; one-fourteenth of the water can be raised to a level above the ram ten times as high as the fall to the ram; and so for other ratios between the fall and the height of discharge.

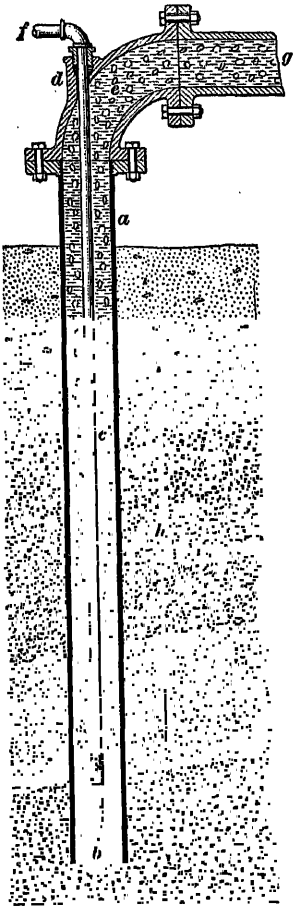


FIG. 28

AIR-LIFT PUMP

46. Another method of raising water from lower to higher levels is by the use of the **air-lift pump**, as shown in Fig. 28. This device consists of two pipes. The outer one *a*, or the casing, is suspended in the water to be pumped, so that the lower end *b* is at a distance *h* below the surface of the water. The inner pipe *c*, which is much smaller than the outer pipe, passes through a stuffingbox *d* in the elbow *e*, and extends downwards inside the casing. It is attached to an air compressor by means ot the pipe *f*, and its lower end is perforated with a number of small holes, as shown.

When the casing *a* is lowered into the water, the water rises to the same level inside the casing as outside. If air under pressure is then forced into the pipe *c*, it will escape

through the holes at the lower end of this pipe, and will form bubbles in the water inside the casing *a*. As a result, the casing will soon contain a mixture of water and air bubbles instead of the solid column of water it originally held.

The weight of the mixture of air and water being less, per cubic foot, than that of water alone, the column inside the casing will weigh less than a column of the same height outside the casing. The mixture of air and water will therefore be forced upwards inside the casing. If this casing were extremely long, the mixture would rise until the pressure it exerted at the lower end *b* of the casing was just equal to the pressure due to the head *h* of the water at that point. As usually built, however, an outlet *g* is provided at a height less than that to which the mixture would otherwise rise in order to balance pressures. As a result, the mixture of air and water is discharged at *g* in a continuous flow, and the velocity of discharge is equal to that corresponding to the additional height or head through which it is capable of rising.

The force that operates the air-lift pump, then, is the difference between the pressures inside and outside the casing at the bottom. To obtain the best results, the casing should extend below the water level a distance about $1\frac{1}{2}$ times as great as the height above the water level to which the water is to be pumped.

ELEMENTARY CHEMISTRY

ELEMENTARY SUBSTANCES

KINDS AND PROPERTIES OF SUBSTANCES

1. Definition of Chemistry.—Chemistry is the science that treats of the composition of substances and the changes that they undergo. Chemistry is used to determine the composition of coal, iron, feedwater, and other substances.

2. Substances.—A substance is anything that occupies space and has weight, such as iron, coke, sulphur, water, and air. There are three different classes of substances, namely, solid, liquid, and gaseous.

A *solid substance* is one that holds its shape under ordinary conditions, as, for example, cast iron, wood, coke, and stone.

A *liquid substance* is one that has no shape of its own, but takes the shape of the vessel that holds it, as, for example, water, oil, and molasses.

A *gaseous substance* is one that has no particular shape or volume and that must be kept in closed vessels to prevent it from escaping, as, for example, illuminating gas, hydrogen, oxygen, and carbon dioxide. Air is a gas, or a gaseous substance.

3. Properties of Substances.—All substances have certain characteristics, or properties, and these are of two kinds, namely, general properties and special properties.

General properties are those that all substances have; for example, all substances have weight, so weight is a general

property. All substances have volume; therefore, volume is a general property.

Special properties are those that are possessed by some substances, but not by all. Iron is hard, but lead is soft; therefore, hardness and softness are special properties. Glass is brittle, but clay is plastic; therefore, brittleness and plasticity are special properties.

4. **Physical and Chemical Changes.**—A change in the form of a substance without forming a new substance is called a physical change; thus, if a lump of lead is melted, or a piece of cast iron is broken up and ground into a fine powder, or water is changed to steam by boiling, the change is only a physical change. Any change that destroys a given substance and produces a new one is a chemical change; thus, if a match is struck and allowed to burn, a chemical change takes place, because the wood in the match is burned and the ash that remains after burning is an entirely new and different substance. The study of chemistry deals with chemical changes.

5. **Elements and Compounds.**—An element is a substance that cannot be resolved into anything simpler; for example, pure gold is an element, because it contains nothing but gold. On the other hand, limestone is not an element, because it is made up of calcium, carbon, and oxygen. A substance composed of a combination of two or more elements is called a compound. To form a compound the elements must combine or unite with one another; for if they simply mix without combining, they form a mixture. If iron filings and sulphur are shaken together at ordinary temperatures, they will form a mixture; but if heat is applied, they will combine and form a substance that will be different from either of them. This substance is a compound.

6. So far as is known at the present time, there are about eighty elements. These are not compounds; that is, they are not combinations of other substances and cannot be broken up into two or more other elements. Many elements are very rare and uncommon. The names of the more common

elements such as might be found in the composition of various forms of iron, steel, and fuels, are given in Table I, together with other information concerning them.

7. Atoms and Molecules.—Every substance is made up of very small parts or particles called atoms. The atom is the smallest part of an element that has been obtained chemically. The smallest part of a compound is called a molecule,

TABLE I

ELEMENTS, SYMBOLS, AND ATOMIC WEIGHTS

Name	Symbol	Atomic Weight $O=16$	Name	Symbol	Atomic Weight $O=16$
Aluminum	*Al*	26.97	Manganese	*Mn*	54.93
Antimony	*Sb*	121.77	Mercury	*Hg*	200.61
Arsenic	*As*	74.96	Molybdenum	*Mo*	96.0
Barium	*Ba*	137.37	Nickel	*Ni*	58.69
Bismuth	*Bi*	209.00	Nitrogen	*N*	14.01
Bromine	*Br*	79.92	Oxygen	*O*	16.00
Cadmium	*Cd*	112.41	Phosphorus	*P*	31.03
Calcium	*Ca*	40.07	Platinum	*Pt*	195.23
Carbon	*C*	12.00	Potassium	*K*	39.10
Chlorine	*Cl*	35.46	Silicon	*Si*	28.06
Chromium	*Cr*	52.01	Silver	*Ag*	107.88
Cobalt	*Co*	58.94	Sodium	*Na*	23.00
Copper	*Cu*	63.57	Strontium	*Sr*	87.63
Fluorine	*F*	19.00	Sulphur	*S*	32.06
Gold	*Au*	197.2	Tin	*Sn*	118.70
Hydrogen	*H*	1.008	Titanium	*Ti*	48.1
Iodine	*I*	126.93	Tungsten	*W*	184.0
Iron	*Fe*	55.84	Uranium	*U*	238.17
Lead	*Pb*	207.20	Vanadium	*V*	50.96
Magnesium	*Mg*	24.32	Zinc	*Zn*	65.38

which is composed of two or more atoms. Neither the atom nor the molecule is large enough to be seen, even under the most powerful magnifying glass, and they are too small to be weighed alone; however, the weight of an atom of one substance compared to the weight of an atom of some other substance can be found. These comparative weights are

given in Table I and are called the *atomic weights* of the various elements. They are the weights of the atoms as compared with the weight of an atom of oxygen, which is taken as 16. The *molecular weight* is the sum of the atomic weights of the atoms that form the molecule. Any given chemical compound always contains the same elements combined in the same proportions; this is the law of definite proportions. It means, for example, that the compound known as water is always made up in the proportion of two atoms of hydrogen to one atom of oxygen. The atomic weights given in Table I are exact; but in ordinary calculations it is more convenient, and quite accurate enough, to use an approximate value. For example, in ordinary calculations, the atomic weight of chlorine would be taken as 35.5 instead of 35.46; that of hydrogen would be taken as 1 instead of 1.008; that of nitrogen as 14 instead of 14.01; that of iron as 56 instead of 55.84; and so on for the other elements given.

MELTING POINT AND HEAT MEASUREMENT

8. Melting Point.—A great many solid substances change to liquid substances when they are heated. The temperature at which this change takes place is called the melting point, or *fusing point*, and it varies according to the nature of the substance. For example, beeswax melts in hot water, and tin or lead melts quickly in a flame; but an intense heat is required to melt iron. The approximate temperatures at which various metals melt are given in Table II, together with their specific gravities and hardness numbers.

The *specific gravity* is the ratio of the weight of a substance to the weight of an equal volume of water. For example, the table shows zinc to have a specific gravity of 7. This means that zinc weighs seven times as much as an equal volume of water.

The *hardness number* indicates the relative hardness of each substance as compared with the hardness of the diamond. It will be observed that few substances approach the diamond in hardness.

9. Heat Measurement.—The thermometer, which is used to measure temperature, shows only how hot or how cold a body is, without giving any idea as to how much heat there is in the body. To measure heat it is necessary to have some

TABLE II

PHYSICAL CONSTANTS OF METALS

Metal	Melting Point Degrees Fahrenheit	Specific Gravity Water = 1	Hardness Number Diamond = 10
Aluminum	1,218	2.6	2.9
Antimony	1,166	6.7	3.0
Bismuth	520	9.8	3.5
Cadmium	610	8.7	2.0
Cast iron, gray*	2,230	7.1	
Cast iron, white†	2,075	7.6	
Chromium	2,939	7.0	9.0
Cobalt	2,696	8.7	5.5
Copper	1,981	8.7	3.0
Gold	1,945	19.3	2.5
Iron	2,786	7.9	4.5
Lead	621	11.4	1.5
Magnesium	1,204	1.7	2.0
Manganese	2,246	8.0	5.0
Nickel	2,646	8.5	3.5
Platinum	3,159	21.5	4.3
Potassium	144	.87	.5
Silver	1,760	10.5	2.7
Sodium	208	.97	.4
Steel, soft‡	2,740	7.7	
Tin	450	7.3	1.8
Zinc	786	7.0	2.5

*Contains 3.5 per cent. of carbon, 1.75 per cent. of silicon, and .5 per cent. of phosphorus.
†Contains 4 per cent. of carbon.
‡Contains .1 per cent. of carbon and .3 per cent. of manganese.

unit of measurement, just as the inch is used as a unit in measuring length or the pound is employed as a unit in measuring weight. The common unit for measuring heat is called the *British thermal unit*, which is the amount of heat required to raise the temperature of 1 pound of pure water from 62° F. to 63° F. It is abbreviated *B. t. u.*

Another unit of heat is the *great calorie*, which is the amount of heat required to raise the temperature of 1 kilogram of water 1 degree centigrade from an initial temperature at or near 4° C. Still another unit is the *pound caloric*, which is the amount of heat required to raise 1 pound of water 1 degree centigrade in temperature. A great calorie is equal to 3.968 British thermal units, and 1 British thermal unit is equal to .252 calorie. One pound calorie is equal to $\frac{9}{5}$ British thermal unit and to .4536 great calorie.

CHEMICAL TERMS

10. Chemical Symbols.—To avoid writing the full names of the different elements and to aid in expressing the composition of substances briefly, symbols are used. The symbol is usually the initial letter of the name of the element, although sometimes it is an abbreviation of the Latin name. The symbols for the different elements mentioned in Table I are given in the second column. For example, the symbol for aluminum is *Al*; but for iron it is *Fe*, because the Latin name *ferrum* is abbreviated; and for mercury it is *Hg*, the abbreviation of the Latin name *hydrargyrum*.

11. Chemical Formulas.—A chemical formula is a combination of figures and chemical symbols that shows the elements contained in a compound and their relative amounts. For example, *NaCl* is the chemical formula for common salt. The two symbols that are used in it are *Na* and *Cl*, which are the symbols for sodium and chlorine. From the formula, then, it is known that salt contains sodium and chlorine, there being 1 atom of each in a molecule of salt. The approximate atomic weight of sodium is 23 and of chlorine is 35.5; therefore, each molecule of salt contains 23 parts of sodium, by weight, to 35.5 parts of chlorine. As a lump or any quantity of salt is simply a collection of molecules, the ratio of sodium to chlorine is the same in the large bulk, or 23 to 35.5. In 58.5 pounds of salt, therefore, there are 23 pounds of sodium and 35.5 pounds of chlorine.

12. The formula for pure water is H_2O. This expression indicates, first of all, that water is composed of hydrogen and oxygen, because H is the symbol for hydrogen and O for oxygen. The small figure 2 written to the right of and below the H means that there are 2 atoms of hydrogen combined with 1 atom of oxygen to form 1 molecule of water. Table I shows that the atomic weight of hydrogen is 1 and of oxygen is 16; but as there are 2 atoms of hydrogen to 1 atom of oxygen, the ratio of hydrogen to oxygen in water is 2 to 16, by weight. In 18 pounds of water, therefore, there are 2 pounds of hydrogen and 16 pounds of oxygen.

Again, consider the compound Fe_2O_3, which is an oxide of iron. In this case, 2 atoms of iron unite with 3 atoms of oxygen. The atomic weight of iron is 56 and of oxygen 16, so the relative weights of these elements in the oxide are $2\times56=112$ and $3\times16=48$; that is, in 160 pounds of the oxide Fe_2O_3 there are 112 pounds of iron and 48 pounds of oxygen.

Sometimes a number is placed before a formula to indicate the number of molecules considered; thus, in the expression $3NaCl$, the figure 3 indicates that 3 molecules of salt are considered. This figure multiplies each of the elements contained in the compound; thus, $3NaCl$ means that there are 3 atoms of sodium, Na, and 3 atoms of chlorine, Cl, in combination, forming 3 molecules of salt.

13. Chemical Equations.—When two substances unite chemically so as to form other substances or compounds, the process is called a *reaction*, and can be expressed simply by a chemical equation. To illustrate, suppose that 2 atoms of hydrogen and 1 atom of oxygen are brought together and ignited. They will unite and 1 molecule of water will be formed. The reaction can be expressed very simply as follows:

$$\underset{\substack{\text{hydrogen}\\ \text{2 atoms}}}{H_2} + \underset{\substack{\text{oxygen}\\ \text{1 atom}}}{O} = \underset{\substack{\text{water}\\ \text{1 molecule}}}{H_2O}$$

This expression is called a chemical equation. In any equation, the part to the left of the equality sign is always equal

to the part on the right. This is true in the foregoing case; for on the left there are 2 atoms of hydrogen and 1 atom of oxygen, and in the molecule of water on the right there are 2 atoms of hydrogen and 1 atom of oxygen. In the union of the hydrogen and oxygen, therefore, nothing has been lost; every atom has been accounted for. This is a general law; that is, no substance can be absolutely destroyed. If a current of electricity is passed through water, the water is *decomposed*, or broken up into two gases, hydrogen and oxygen, and the water itself disappears. But the amounts of hydrogen and oxygen obtained are exactly the same as had been combined in the form of water before the decomposition. The same thing is true of any other substance. Hence, in every chemical equation, every atom appearing on the left of the equality sign must also be accounted for on the right of the sign.

14. As further examples of chemical equations, the following may be given:

$$\underset{\text{sulphuric acid}}{H_2SO_4} + \underset{\text{sodium hydrate}}{2NaOH} = \underset{\text{sodium sulphate}}{Na_2SO_4} + \underset{\text{water}}{2H_2O}$$

This equation indicates that when sulphuric acid acts on sodium hydrate, these two substances are changed and in their place sodium sulphate and water are formed. On the left side of the equation, the sulphuric acid contains 2 atoms of hydrogen, H_2, and the 2 molecules of sodium hydrate, $NaOH$, also contain 2 atoms of hydrogen, so that there are 4 atoms of hydrogen to be accounted for on the right. As 2 molecules of water, H_2O, were formed by the reaction and each molecule contains 2 atoms of hydrogen, H_2, the total amount is 4 atoms, so that all the hydrogen is accounted for. The SO_4 in the sulphuric acid appears unaltered in the sodium sulphate on the right, and the 2 atoms of oxygen in the hydrate appear in the 2 molecules of water. The sulphur, S, and the oxygen, O, are therefore accounted for and check up correctly. Finally, the 2 atoms of sodium, Na, in the 2 molecules of hydrate appear as 2 atoms of sodium, Na_2, in the sulphate; this accounts for the sodium. Thus, in the reaction, every atom

put in reappears in the new substances formed, without gain or loss.

15. Acids.—To the chemist, the sourness of an acid is but an accidental property, as many substances that are not sour to the taste are classified as acids. An acid may be defined as a substance containing hydrogen, which hydrogen may be replaced by a metal, the resulting compound being a salt. Most acids are sour; they are also active chemical agents; most of them are characterized by their property of changing the color of a solution of *litmus*, a blue dye, to red. Narrow strips of paper saturated with a solution of litmus are commonly used for testing acids. These strips are called *litmus paper*. Some of the common acids are hydrochloric acid, HCl, nitric acid, HNO_3, and sulphuric acid, H_2SO_4.

16. Bases and Alkalies.—A *base* is a substance containing a metal, or group of elements equivalent to a metal, and hydrogen and oxygen. An *alkali* is a base of especially active character, soluble in water, and easily recognized by the soapy taste and feel it imparts to water. Bases have the ability to restore the blue color to a solution of litmus that has been reddened by an acid. The principal bases are calcium hydrate, $Ca(OH)_2$, sodium hydrate, $NaOH$, and potassium hydrate, KOH, the latter two being alkalies. Hydrates are also called *hydroxides* as they are formed by the combination of metallic oxides with water. This is represented by the following equations:

$$\underset{\text{sodium oxide}}{Na_2O} + \underset{\text{water}}{H_2O} = \underset{\text{sodium hydrate or sodium hydroxide}}{2NaOH}$$

$$\underset{\text{calcium oxide}}{CaO} + \underset{\text{water}}{H_2O} = \underset{\text{calcium hydrate or calcium hydroxide}}{Ca(OH)_2}$$

17. Neutralization.—When an acid and a base react with one another in the right proportion, they lose their properties and a new compound, having different properties, and water are formed. This process is called *neutralization;* or, the

base is said to *neutralize* the acid. The new compound formed belongs to the class of *salts*, which are neither acids nor bases. The process of neutralization is represented by the following equation:

HCl	$+$	$NaOH$	$=$	$NaCl$	$+$	H_2O
hydrochloric acid		sodium hydrate		sodium chloride		water

METALS AND NON-METALS

CLASSIFICATION OF ELEMENTS

18. The various elements are divided into two classes, known as the *metals* and the *non-metals*. The latter are sometimes called the *metalloids*. The number of metals exceeds that of non-metals. Sometimes it is very difficult to tell definitely whether an element should be classed as a metal or as a non-metal. The metallic properties of most metals, particularly of those commonly met with, are so plain that there is no doubt about them. In the case of the more common non-metals, it is usually just as easy to determine that they are not metals. The elements which are doubtful are generally those which are very uncommon, and with these, fortunately, the foundryman does not have to deal. In the following articles are given the properties of a number of elements with which the foundryman does have to deal and of which he should have some knowledge.

NON-METALS OR METALLOIDS

GENERAL REMARKS

19. Before considering the most important metals, some of the most important non-metals will be discussed; these are: hydrogen, oxygen, nitrogen, chlorine, sulphur, phosphorus, carbon, and silicon. The atomic weight given with the

description of each element is the approximate value, which is the more convenient to use and is satisfactory for all ordinary calculations. The more exact value will be found in Table I.

HYDROGEN

Symbol H. Atomic weight 1.

20. Occurrence.—Hydrogen occurs to some extent in the free condition and issues from the earth in small quantities in some localities. It is, for example, a constituent of the gases that escape from some oil wells. It occurs chiefly, however, in combination with oxygen, as water, of which it forms 11.11 per cent. It occurs also in most animal and vegetable substances. In these products of life, it is combined with carbon and oxygen or with carbon, oxygen, and nitrogen. Hydrogen also occurs in petroleum, coal, natural gas, etc.

21. Properties.—Hydrogen is a colorless, odorless, and tasteless gas. It is the lightest element known, being 14.43 times lighter than air and 11,000 times lighter than water. Its molecular weight, therefore, is smaller than that of any other known substance. Hydrogen weighs .0053 pound per cubic foot. It burns with a colorless flame, combining with oxygen to form water. It is soluble to a very slight extent in water, 100 volumes of which dissolve but $1\frac{1}{2}$ volumes of hydrogen. At a very low temperature, hydrogen can be condensed to a liquid with the aid of pressure.

OXYGEN

Symbol O. Atomic weight 16.

22. Occurrence.—Oxygen occurs abundantly in nature, both in the free state and in combination with other elements. It occurs uncombined in the atmosphere, of which it constitutes about one-fifth. In the combined form, it constitutes eight-ninths, by weight, of water, and nearly one-half of the earth's solid crust.

23. Properties.—Oxygen is an odorless, colorless, and tasteless gas. It is somewhat heavier than air, the ratio being about 1.105 for oxygen to 1 for air. It is only slightly soluble in water, 100 volumes of water dissolving about 3 volumes of oxygen. It is capable of entering into combination with nearly all the elements; but in the state in which it is usually obtained, heat is necessary to accomplish this union.

24. Combustion.—In the ordinary sense of the word, combustion is burning, and is the result of the union of oxygen with some other element producing light and heat. It does not take place at ordinary temperatures; the substances must first be heated above its kindling temperature before it will burn. The kindling temperature of iron or carbon, for instance, is much higher than that of sulphur or phosphorus. The products of combustion are oxides of the substances which burn; thus, carbon in burning forms carbon dioxide, CO_2.

COMPOUNDS OF HYDROGEN AND OXYGEN

25. Introduction.—There are two compounds of hydrogen and oxygen, namely, *water*, H_2O, and *hydrogen peroxide*, H_2O_2. Water is one of the most important substances in nature, and although hydrogen peroxide is of considerable interest to the chemist and of some importance in the arts, it is not of sufficient importance to justify more than a reference to it in this Section.

26. Water.—Every one is familiar with the occurrence of water in its three physical forms: in a solid form as ice, in a liquid state as water, and in a gaseous state as steam. Between the temperatures of 32° F. and 212° F., water is a tasteless, colorless, odorless, and clear liquid. Its most characteristic property is its great solvent power, there being comparatively few substances that it does not dissolve in large or in small quantities. When cooled to 32° F., it solidifies into ice; when heated to 212° F., it is changed to steam. It unites with most oxides, forming either acids or bases.

27. Chemically pure water is usually obtained by freeing natural water from the small quantity of foreign substance that it contains; and as most of these bodies are in the state of solution, the water is commonly purified by distillation. By means of an electric current, water can be decomposed or separated into the elements hydrogen and oxygen, the volume of the hydrogen produced being twice that of the oxygen, and its weight one-eighth that of the oxygen. When hydrogen burns, it unites with the oxygen of the air, forming water. If 2 volumes of hydrogen are mixed with 1 volume of oxygen, and the mixture is ignited—best by means of an electric spark—a violent explosion takes place; and if the original gases have been measured at a temperature above 212° F., and the resulting water, as steam, is measured at the same temperature, it will be found that the 3 volumes of mixed gases yield 2 volumes of steam.

28. Oxidation and Reduction.—It has been seen that oxygen is an exceedingly active element, combining with many other substances to form oxides. Such addition of oxygen is called oxidation. Copper, for example, if heated in oxygen, is converted into the black copper oxide, CuO. If this copper oxide is heated in hydrogen, the hydrogen will combine with the oxygen of the copper oxide and set free, or reduce, the metal. Thus, $CuO+H_2=Cu+H_2O$. Any process by which oxygen is removed from a compound is called reduction, and the substance used to effect this reduction is called a *reducing agent.*

29. Oxides.—Compounds of oxygen and metals are called oxides. These occur in great quantities in nature, a large number of the common ores being oxides. For instance, the most important iron ore is hematite, Fe_2O_3. To convert oxides into the respective metals, a process of reduction is necessary, carbon in the form of coke or coal being mostly used as the reducing agent with the help of heat. In any process of reduction there must also be oxidation, the reducing agent being oxidized. Thus, in the example of the reduction of copper oxide by the reducing agent hydrogen, the hydrogen itself is oxidized.

CHLORINE

Symbol Cl. Atomic weight 35.5.

30. Occurrence.—Chlorine is not known to exist free in nature; but, as chlorides, it is widely distributed in combination with metals. The most important of these is *sodium chloride*, *NaCl*, or common salt, which not only occurs in enormous quantities dissolved in sea-water, and the water of salt springs, but also in immense beds of rock salt.

31. Properties.—Chlorine is a yellowish-green gas with an irritating odor; when breathed in only minute quantities it causes coughing, and in larger quantities it causes serious inflammation of the lungs and air passages. It is nearly two and one-half times as heavy as air. At a temperature of $-40°$ F., or at a pressure of four atmospheres, it condenses to a dark-yellow liquid. It is very soluble in water, 1 volume of water dissolving between 2 and 3 volumes of the gas, forming a solution that has practically the same properties as the gas itself.

32. In its chemical properties, chlorine is an exceedingly active substance; even at an ordinary temperature it combines with many elements and acts on many compounds. Many metals combine with chlorine with the production of light, and all metals are capable of combining with it, forming chlorides. Chlorine also unites readily with many non-metals. The attraction of chlorine to hydrogen is specially strong, the two gases exploding violently when mixed together and exposed to sunlight. Owing to its slight attraction to oxygen, it does not burn in the air at any temperature. Chlorine is largely used as a bleaching and disinfecting agent.

33. Hydrochloric acid, *HCl*, also known as *muriatic acid*, is the only known compound of hydrogen and chlorine. It is a colorless gas with a sharp, pungent odor, and gives off strong fumes in the air. It cannot be breathed and puts out a flame. By cold and pressure, it may be condensed to a clear, colorless, liquid. Hydrochloric acid is remarkably soluble in

water, 1 volume of which dissolves 450 volumes of the gas at 59° F., forming a strongly acid liquid. This solution of the gas in water is what is ordinarily called hydrochloric acid.

Hydrochloric acid is a strong acid; it readily turns blue litmus red, and attacks many metals with the formation of the corresponding chloride and the liberation of hydrogen. With zinc the reaction is

$$\underset{\text{zinc}}{Zn} + \underset{\substack{\text{hydrochloric}\\\text{acid}}}{2HCl} = \underset{\substack{\text{zinc}\\\text{chloride}}}{ZnCl_2} + \underset{\text{hydrogen}}{H_2}$$

SULPHUR

Symbol S. Atomic weight 32.

34. Occurrence.—Sulphur occurs free in nature, principally in the vicinity of active or extinct volcanoes. It is separated from the accompanying rock by fusion. Besides occurring in the free state, it is found in certain mineral springs in the form of hydrogen sulphide and is otherwise widely distributed in nature in combination with various metals, as sulphides and sulphates.

35. Properties.—Sulphur is capable of existing in three distinct forms. The most common form of sulphur, the one that is found in the natural state, and to which the others tend to change, is a lemon-yellow brittle solid. It is insoluble in water and soluble in carbon bisulphide. Another form is also crystalline; but the crystals are different from the first. Still a third variety is plastic, having a consistency like gum. Each variety melts at 239° F., becoming a pale-yellow clear liquid; as the temperature is increased, it becomes dark in color and viscous, until, between 392° F. and 482° F., the vessel may be inverted and the sulphur will not run out of it. At about 662° F., it again becomes a liquid, which boils at 824° F. During cooling, these changes occur in reverse order. When heated to 500° F. in the air, sulphur takes fire and burns with a pale blue flame and gives off a strangling smoke. Sulphur readily combines with the metals, many of which take fire if

thrown in its vapor. It also combines with many of the non-metals.

36. Compounds.—Sulphur forms a very large number of compounds. *Hydrogen sulphide*, H_2S, is a colorless combustible gas having a very disagreeable odor, which resembles that of rotten eggs. *Sulphur dioxide*, SO_2, is a colorless incombustible gas, having the strangling effect of burning sulphur. *Sulphuric acid*, H_2SO_4, is a heavy, oily liquid that has no smell. It contains two atoms of hydrogen capable of being replaced by a metal. Sulphides are compounds of the metals with sulphur, such as lead sulphide or galena, PbS. Iron and steel contain very small quantities of sulphur in the form of iron sulphide, FeS, and manganese sulphide, MnS. Coal and coke always contain some sulphur too.

NITROGEN

Symbol N. Atomic weight 14.

37. Occurrence.—Nitrogen occurs in the free state in the air, of which it constitutes about four-fifths by bulk; it is found in native nitrates, combined with oxygen and metals, and is an important element in many bodies of plant or animal origin.

38. Properties.—Nitrogen is a colorless, odorless, and tasteless gas. It differs remarkably in its properties from oxygen, with which it is associated in the air. Oxygen is specially characterized by its great chemical activity; but nitrogen is one of the most inactive bodies, entering into direct combination with only a few elements. With oxygen itself, it combines only at very high temperatures. Nitrogen puts out the flame of burning bodies introduced into it.

39. Air.—The air, or the atmosphere, is mainly a mixture of nitrogen and oxygen, containing approximately 21 per cent. of oxygen and 79 per cent. of nitrogen by volume, or 23 per cent. of oxygen and 77 per cent. of nitrogen by weight. However, it does not consist wholly of these two elements,

but always contains small amounts of carbon dioxide, CO_2, vapor of water, and other gases.

40. Compounds of Nitrogen.—Although the element nitrogen is exceedingly inactive chemically, some of its compounds are very active substances. *Ammonia*, NH_3, is a gas having a pungent irritating odor. It can be liquefied rather easily. Ammonia is very soluble in water, and it is this solution of the gas that it is known as ammonia or *hartshorn*. The solution of ammonia acts like a base, the ammonia combining with the water to form *ammonium hydrate*.

$$\underset{\text{ammonia}}{NH_3} + \underset{\text{water}}{H_2O} = \underset{\substack{\text{ammonium}\\ \text{hydrate}}}{NH_4OH}$$

Nitric acid, HNO_3, is a liquid with a strong oxidizing action. It is sometimes known as *aqua fortis*.

PHOSPHORUS

Symbol P. Atomic weight 31.

41. Occurrence.—Phosphorus never occurs free in nature. It is present in bones in the form of calcium phosphate, $Ca_3(PO_4)_2$. Several minerals also contain phosphates of different metals.

42. Properties.—Phosphorus exists in several forms, the common one being a pale yellow, nearly transparent, wax-like solid that is very poisonous, takes fire quickly, and glows and emits a white smoke if exposed to the air. It burns in air if heated to 111° F., and therefore must be kept under water. *Red phosphorus*, another variety, is not a poison, and can be kept in the air. The molecules of most elements consist of 2 atoms, but the phosphorus molecule contains 4 atoms. Phosphorus is present in all iron ores as phosphates, and in cast iron as iron phosphide, Fe_3P.

CARBON

Symbol C. Atomic weight 12.

43. Occurrence.—Carbon occurs in nature in two crystalline forms, known as diamond and graphite. More or less impure carbon is found in coal and, combined with hydrogen, in petroleum. With oxygen and calcium it forms limestone. Few elements have so many forms as carbon; the transparent, colorless, hard, crystalline diamond; the opaque, soft, metallic-looking graphite; the dull and porous wood charcoal—all these are carbon.

44. Properties.—Carbon in the form of the diamond is crystalline, usually transparent, colorless, has great brilliancy, and is the hardest substance known. Its specific gravity is about 3.5. The form of carbon known as graphite is a lead-colored solid with a metallic luster. It is soft, and is used in the manufacture of lead pencils; it is used also as a lubricant, as a facing material in foundry work, and for other purposes. Various other sorts of amorphous carbon are known as charcoal, lampblack, coke, etc. These substances do not form crystals, but are exceedingly useful.

45. Carbon can unite directly with many elements; but in the case of all elements, except fluorine, the two must be brought together at high temperatures. The number of known compounds of carbon is far greater than of any other element. The elements most frequently entering into these compounds are hydrogen, oxygen, and nitrogen. Though the temperature necessary to bring about the combination varies, being highest for graphite and diamond, when any of the three varieties of carbon burns in sufficient oxygen the product is *carbon dioxide*, CO_2, which is a colorless gas with a slightly pungent odor and taste. This gas is often called *carbonic-acid gas*. It is about $1\frac{1}{2}$ times as heavy as air, can be liquefied by cold and pressure, and neither burns nor supports combustion. Water dissolves about its own volume of carbon dioxide and the resulting liquid contains *carbonic acid*, H_2CO_3. This acid, however,

cannot be obtained by itself in the pure state, as it readily breaks up into H_2O and CO_2; but its salts are of considerable importance.

46. *Carbon monoxide*, CO, is a colorless, odorless, poisonous gas. It is nearly as heavy as air, is hard to condense to a liquid, is very slightly soluble in water, and burns with a blue flame, forming carbon dioxide. It is a strong reducing agent.

47. Most of the ordinary forms of fuel consist of carbon with more or less hydrogen. If the burning is complete, the products are carbon dioxide and—if hydrogen is in the fuel—water. When the combustion is incomplete, carbon monoxide may also be formed. Because of its strong affinity, or attraction, for oxygen at high temperatures, carbon is an excellent reducing agent, and many metals are reduced from their oxides by means of one of the forms of carbon.

In such operations, the carbon not only burns with the oxygen of the air to furnish the necessary heat, but part of it removes the oxygen from the oxide of the metal, forming CO_2 and CO. In the reduction of iron ores to the metal in the blast furnace, the iron absorbs about 4 per cent. of carbon, which is present in the pig iron partly as free graphite and partly combined as *iron carbide*, Fe_3C.

SILICON

Symbol Si. Atomic weight 28.

48. Occurrence and Properties.—Silicon does not occur free in nature, but it is found most abundantly in combination with oxygen. In combination with oxygen as well as with aluminum, potassium, calcium, and other elements, it constitutes a large portion of all known rock formations. Silicon resembles carbon in many respects. It is a solid occurring in two forms. *Crystalline silicon* forms black crystals that will scratch glass. *Amorphous silicon* is a reddish-brown powder with a specific gravity of 2.35; it burns in the air, forming *silicon dioxide*, SiO_2.

49. *Silicon dioxide*, better known under the name of *silica*, occurs widely distributed in nature. Its purest natural form is a transparent and colorless variety of quartz known as *rock crystal*, which is recognized by its great hardness, scratching glass almost as readily as the diamond. *Sand*, of which the white varieties are nearly pure silica, appears to have been formed by the breaking up of silicious matter, and has very often a red or yellow color, owing to the presence of iron oxide. *Flint* consists essentially of silica colored with various impurities. Silicon dioxide, in the form in which it is usually obtained, is a white uncrystallized powder. Silicon is an important constituent of cast iron, and is also used in the foundry in the form of *ferro-silicon*, an alloy of iron and silicon of different proportions.

METALS

PROPERTIES OF METALS

50. As a rule, metals are opaque; that is, it is impossible to see through them. They also have a certain luster, or shine, known as metallic luster. Except mercury, which is a liquid at ordinary temperatures, all metals are solids. Chemically, the metals are capable of replacing the hydrogen of acids, forming salts. Combinations of metals with each other are called *alloys*. Generally, alloys are not definite chemical compounds, but are merely mixtures of different metals; only occasionally is an alloy a true chemical compound. The properties of metals and alloys are sometimes changed to a remarkable extent by the presence of minute quantities of other elements.

SODIUM

Symbol Na. Atomic weight 23.

51. **Occurrence and Properties.**—Sodium is abundantly and widely distributed in nature, but occurs only in combined forms. As *sodium chloride*, or common salt, it is found not only in sea-water, but also in enormous deposits of rock salt;

as *sodium nitrate* (Chile saltpeter) it occurs in South America. As *sodium silicate*, it forms a part of many minerals and crystalline rocks. Traces of sodium salts are always found in the dust floating in the air. Sodium is a shining, white, soft metal that melts at about 200° F. and that becomes gaseous at about 1,370° F., forming a colorless vapor. It quickly oxidizes in the air; hence it is preserved in petroleum. It burns with a yellow flame. It combines with water, forming sodium hydrate and hydrogen, the hydrogen burning instantaneously. It should therefore always be kept in oil, as oil has no action on it. Sodium forms a large number of compounds, the best known of which are: *sodium hydrate*, $NaOH$ (caustic soda), *sodium chloride*, $NaCl$ (common salt), *sodium sulphate*, Na_2SO_4 (Glauber's salt), *sodium nitrate*, $NaNO_3$ (Chile saltpeter), *sodium carbonate*, Na_2CO_3, and *sodium bicarbonate*, $NaHCO_3$.

POTASSIUM

Symbol K. Atomic weight 39.

52. Occurrence and Properties.—Compounds of potassium occur in nature very extensively. Potassium exists principally in the silicates, especially feldspar and mica. The largest source of supply, however, are the double salts of potassium and magnesium, known as *carnallite* and *kainite*, found in the mines of Stassfurt, Germany. Potassium is very similar to sodium and forms similar salts. It has a silver-white luster and is almost as soft as wax at ordinary temperatures. It melts at 140° F. and boils at about 1,330° F., turning then to a green vapor. It quickly oxidizes in moist air, its surface becoming covered with potassium hydrate. Like sodium, it has to be preserved under petroleum.

CALCIUM

Symbol Ca. Atomic weight 40.

53. Occurrence, Properties, and Compounds.—Calcium does not occur free in nature. Calcium carbonate and calcium sulphate occur in very large deposits. Calcium silicate is a

constituent of nearly all silicious minerals, and calcium phosphate occurs as apatite and as phosphorite. Calcium, as a metal, in its free state, is of no commercial interest or value; but a number of its compounds are of great importance. *Calcium oxide*, CaO, is known as burnt lime and is used, on account of its high refractory or heat-resisting power, in the construction of crucibles. *Calcium hydrate*, $Ca(OH)_2$, or slaked lime, is used in the manufacture of cement and for building purposes. *Calcium carbonate*, $CaCO_3$, constitutes limestone and marble. It cannot be dissolved in pure water, but water charged with carbon dioxide has a solvent action, the carbonic acid converting the calcium carbonate into calcium bicarbonate, $Ca(HCO_3)_2$. Limestone is used in the blast furnace and the cupola as a flux. *Calcium sulphate*, $CaSO_4+2H_2O$, is soluble in 400 parts of water and loses part of its water on being heated to 248° F.; the product, $2CaSO_4+H_2O$, is called burnt gypsum or plaster of Paris.

MAGNESIUM

Symbol Mg. Atomic weight 24.

54. Occurrence and Properties.—Magnesium occurs as magnesium carbonate in magnetite and, mixed with calcium carbonate, in many impure limestones; it occurs also as a silicate in asbestos, talc, and meerschaum. The metal itself is white; it does not tarnish in dry air, but oxidizes readily in moist air. It burns with a dazzling white light and on this account is of great value in flash-light photography. Like calcium carbonate, magnesium carbonate is insoluble in pure water, but it can be dissolved in water that contains carbon dioxide. Magnesium sulphate is readily soluble in water. All soluble magnesium salts have a bitter taste.

ALUMINUM

Symbol Al. Atomic weight 27.

55. Occurrence.—Next to oxygen and silicon, aluminum is the most abundant element in nature. It never occurs in the free state, but exists as the oxide Al_2O_3 in corundum and

emery. Mica, clay, feldspar, and many other minerals contain aluminum silicate.

56. Properties.—Aluminum is a silvery white metal and is the lightest of the common metals; it can be hammered or drawn into various shapes, and fuses at about 1,216° F. In the air it soon becomes coated with a thin layer of oxide, which protects the metal from further attack. It is little affected by dilute nitric or sulphuric acids; but dilute hydrochloric acid dissolves it readily. A dilute acid is one that is weakened by the addition of water. Aluminum is a powerful reducing agent, reducing many oxides with the production of heat. It enters into some important alloys, as, for example, aluminum bronze, which is an alloy of copper and about 10 per cent. of aluminum, and is characterized by being hard and of a golden color. In recent years, the so-called light-weight alloys have come into extensive use in the construction of aircraft and automobiles. They are alloys of aluminum with copper, zinc, or magnesium, which are either cast or forged. They have a specific gravity of only 2.5 to 3, combined with great strength and durability. Aluminum hydrate, $Al(OH)_3$, is a white, jelly-like mass, formed by adding ammonium hydrate to a solution of an aluminum salt.

IRON

Symbol Fe. Atomic weight 56.

57. Occurrence.—Iron is the most important and one of the most common metals. It does not occur in the metallic state on the earth's surface, the native iron so found probably coming almost entirely from meteors, or shooting stars. In combination, it is found in nearly all kinds of rock, clay, etc., its presence being generally indicated by the color of the rock, iron being the most common of natural mineral coloring ingredients. In most rocks, however, the amount of iron is not enough to be of any practical value. Iron ores are rather abundant, the most important of these being the oxides Fe_2O_3, or hematite; Fe_3O_4, or magnetite; hydrated oxide, $2Fe_2O_3$, $3H_2O$, or limonite; and a carbonate $FeCO_3$, or

siderite. A sulphide of iron, FeS_2, or pyrite, contains a large percentage of iron; but it is not used as an ore on account of the injurious effects of the sulphur on the metal.

58. Properties.—Few metals besides iron have their properties modified to such an extent by very small quantities of other elements. A large number of widely different kinds of iron, such as cast iron, wrought iron, and the various kinds of steel, are well known. Although the causes of the wide variation in properties are not thoroughly understood, in the main these variations are due to the presence in the metal of varying quantities of other elements. The most important of these elements are carbon and silicon, though sulphur, phosphorus, manganese, tungsten, vanadium, molybdenum, nickel, and chromium are important, as well as some rarer elements not so universally present in commercial grades of iron or steel. The different varieties of commercial iron may be divided into three classes—wrought iron, cast iron, and steel. The first is nearly pure iron; the second contains generally from 2 to 5 per cent. of carbon, with varying quantities of silicon and other elements; in composition, the third lies between the other two.

Wrought iron, when rolled or forged, has a dark, fibrous structure and is remarkably tough, though comparatively soft. It melts at about 2,912° F., but becomes a soft, wax-like mass at a considerably lower temperature, and is then capable of being welded. Cast iron melts at about 2,208° F., but does not soften before melting. It is too brittle to be rolled or forged, but is easily cast into all kinds of shapes and in this form is used in enormous quantities. Its fracture varies from a silver-white to dark gray, depending on the composition and rate of cooling. Steel is more easily melted than wrought iron, but not as easily as cast iron. It has greater strength than either, can be hardened and tempered, and its physical properties are changed to a remarkable degree by heat treatment. It can be rolled, forged, and welded, but is not so easily cast. Iron is attracted by a magnet, and can itself be magnetized. A solid mass of iron does not tarnish in dry air, but if heated it oxidizes readily, with the production of black scales of oxide,

Fe_3O_4; and when more strongly heated, or when heated in pure oxygen, it burns with the formation of the same scales. In pure water, iron does not lose its brilliancy, but if a trace of carbonic acid is present and air is permitted to reach it, the iron begins at once to oxidize or rust on its surface. Among the important compounds of iron may be mentioned ferric oxide, Fe_2O_3, which is not only an important ore of iron, but is also of considerable importance as a mineral paint.

NICKEL

Symbol Ni. Atomic weight 58.5.

59. Occurrence and Properties.—Nickel does not occur free in nature, except in small quantities in some meteorites, which are alloys of iron and nickel. Native compounds of nickel are also not very abundant. Among the important minerals containing nickel are *garnierite*, a silicate of nickel and magnesium, and *niccolite* (copper nickel, or kupfernickel), $NiAs$.

Nickel is almost as white as silver, is very tough, and has a high metallic luster. It dissolves sparingly in hydrochloric and sulphuric acids, but freely in nitric acid. It is permanent in the air, though slight tarnishing takes place. It is employed in nickel-plating metallic objects, and is a constituent of several alloys. *German silver* contains about 50 per cent. of copper, 25 per cent. of zinc, and 25 per cent, of nickel. Nickel coins contain 75 per cent. of copper and 25 per cent. of nickel. A small amount of nickel added to steel greatly increases its toughness, and much nickel is used for this purpose, especially in the manufacture of armor plate. Nickel is sometimes added to brass and bronze in small quantities to improve their properties and make sounder castings.

MANGANESE

Symbol Mn. Atomic weight 55.

60. Occurrence and Properties.—Manganese is found chiefly as *pyrolusite*, MnO_2, *braunite*, Mn_2O_3, and *rhodochrosite*, $MnCO_3$. Manganese sulphide, arsenide, and silicate are also known as minerals. The metal itself has not been applied to any very useful purpose in the arts, but with other metals forms some useful alloys.

Manganese is best prepared by reducing MnO_2 by means of aluminum powder. It is a grayish-white, hard, brittle metal. It is hard to melt, requiring a temperature of 2,237° F., and oxidizes readily in the air. It dissolves easily in dilute hydrochloric and sulphuric acids, Mn displacing H_2. It resembles iron in its tendency to combine with carbon at a high temperature to form a compound corresponding to cast iron; in this form the manganese is not oxidized by air.

Spiegeleisen and *ferromanganese* are alloys containing iron, manganese, and carbon, and are largely used in the production of steel.

61. *Manganese dioxide*, or *peroxide*, MnO_2, is the chief compound in which manganese is found in nature; it is also the source from which practically all manganese compounds are obtained. Its chief mineral form is pyrolusite, which forms steel-gray prismatic crystals having a specific gravity of 4.9; but it is also found uncrystallized, as *psilomelane*, and in the hydrated state as *wad*. In commerce, pyrolusite is known as black manganese and is largely imported, as well as mined, in the United States of America, for the use of the steelmaker, the manufacturer of bleaching powder, and the glass maker. It is also used as a source of oxygen, which it gives off when heated to redness without melting, leaving the red *oxide of manganese*, Mn_3O_4.

CHROMIUM

Symbol Cr. Atomic weight 52.

62. Occurrence and Properties.—Chromium is not found in the metallic state, but largely as chrome iron ore, having the formula $Fe(CrO_2)_2$, and occasionally as lead chromate, $PbCrO_4$, in the mineral crocoisite. Chromium is a light-gray, or tin-white, lustrous, crystalline metal. It is very hard, difficult to melt, and oxidizes very slowly when heated in the air; but when heated in oxygen, or in the oxyhydrogen flame, it burns to chromic oxide, Cr_2O_3. Chrome iron ore is now reduced in the electric furnace to *ferrochromium*, which is used extensively in the manufacture of chrome steels and stainless steel.

TUNGSTEN

Symbol W. Atomic weight 184.

63. Occurrence and Preparation.—Tungsten occurs in nature in a number of minerals. Its symbol, *W*, is derived from the mineral *wolframite*, which is tungstate of iron and manganese, and from which it was first obtained; it occurs in *scheelite*, which is calcium tungstate; in *stolzite*, which is lead tungstate; and in other minerals. Metallic tungsten is obtained by reducing tungsten trioxide, WO_3, with hydrogen or aluminum powder as an iron-gray metal. Tungsten is extremely hard, very difficult to melt, and is unaffected by either hydrochloric or sulphuric acid. It is a very heavy element, having a specific gravity of 18.8. Steel alloyed with 13 to 19 per cent. of tungsten and smaller amounts of other metals is known as *high-speed steel.* It is harder than ordinary steel and retains its hardness when hot, and is therefore used for lathe, planer, and other machine-shop tools that operate at high speeds. Tungsten is used for filaments of electric light bulbs on account of its high melting point.

VANADIUM

Symbol V. Atomic weight 51.

64. Occurrence and Properties.—Vanadium occurs in nature in the form of vanadates, which are similar to phosphates. It has been found in clay, coal, and some rare minerals. To prepare the metal, vanadium pentoxide, V_2O_5, is molded with carbon and paraffin into sticks and an electric current is passed through, which reduces the oxide to metallic sticks. Ferro-vanadium is now also produced in the electric furnace by reduction with aluminum, and is used in steel manufacture. Besides removing impurities from steel, vanadium increases its hardness and resistance to shock, .1 to .2 per cent. being enough to have a marked effect. Vanadium is a silver-white element with a metallic luster, and is not oxidized by the air.

COPPER

Symbol Cu. Atomic weight 63.

65. Occurrence and Properties.—Copper has been used by man from the earliest times. It is sometimes found native, immense masses of the metal being found in the region around Lake Superior. Copper also occurs rather abundantly as carbonate and oxide; but its most important source is the sulphide, being in this form usually associated with sulphides of iron and other metals. From these ores it is purified by smelting and by electrolysis. It is then known as *electrolytic copper*, which is purer than Lake copper, particularly because it is free from arsenic.

Copper is a lustrous metal, flesh red in color, and somewhat softer than iron. When rubbed, it gives off a peculiar odor. It may be hammered or drawn into wire, and possesses considerable strength. It melts at 1,981° F., and is slightly volatile at a white heat. In dry air, at ordinary temperatures, it is unaltered; but in the presence of moisture and carbon dioxide it becomes covered with a green layer of carbonate of copper called *verdigris*. Heated to redness in the air, scales of oxide

form on its surface. It dissolves readily in nitric acid. Weak acids, alkalies, and salt solutions act on it slowly; hence, as all its salts are poisonous, copper vessels should not be used in cooking.

Copper is an exceedingly useful metal. In the pure state, it is very largely used as a conductor of the electric current. Many of the most useful alloys contain copper, brass being an alloy of copper and zinc, and bronze an alloy of copper and tin.

ZINC

Symbol Zn. Atomic weight 65.

66. **Occurrence and Properties.**—Zinc is not found in the metallic state. The chief ore is the sulphide, ZnS, though considerable carbonate, oxide, and silicate are mined and reduced. Zinc is a bluish-white metal of crystalline structure. It is hard and brittle at ordinary temperatures, but between about 200° F. and 300° F. it becomes malleable and ductile; that is, it can be hammered and drawn out into different shapes. At this temperature it may be rolled into sheets that preserve to some extent their malleability on cooling. At 392° F., it is again hard and brittle. At 786° F., it melts; and at about 1,900° F., it boils, or, if air is present, takes fire and burns with a luminous, greenish flame, forming zinc oxide. It oxidizes readily when exposed to moist air and is easily attacked by acids.

Zinc is not only used in the form of sheet zinc, but it is used in some alloys, as, for example, in brass. It is extensively used also in making the so-called *galvanized iron*, which is iron covered with a coating of zinc by being dipped in melted zinc. Zinc oxide, ZnO, is used as a white paint, and zinc chloride, $ZnCl_2$, is used as a soldering flux.

LEAD

Symbol Pb. Atomic weight 207.

67. **Occurrence and Properties.**—Lead was one of the metals known to the ancients. It is not found as a metal. The principal ore is the sulphide, or *galena*, PbS; it also occurs as carbonate, as sulphate, and in other compounds.

Lead is a soft, bluish-white, brilliant metal; it is malleable and ductile, but possesses little strength, and melts at 621° F. It is slightly volatile at higher temperatures; that is, it gives off gases or fumes if heated above the boiling point. A freshly cut surface of lead tarnishes in ordinary air, but remains bright in perfectly dry air, and also in water entirely free from air. When melted in the air, it rapidly absorbs oxygen and becomes covered with a film of oxide, which, by the continuous action of heat and air, is transformed into a yellow powder, known as *litharge* and as *massicot*. Lead is only slightly attacked by sulphuric or hydrochloric acids at ordinary temperatures, but is readily attacked by nitric acid. In the presence of air and moisture, it is acted on by very feeble acids, such as acetic and carbonic acids. Lead forms several oxides, one of which, *red lead*, Pb_3O_4, is used as a paint. A carbonate of lead constitutes *white lead*, an exceedingly useful paint. The metal itself is used in the form of pipe, for storage battery plates, and in some alloys, especially solder, which is an alloy of lead and tin, and bearing metals.

TIN

Symbol Sn. Atomic weight 119.

68. **Occurrence and Properties.**—Tin in the metallic state is not found in nature. Tin ore occurs in only a few localities. The most important ore is the oxide SnO_2. Tin is a white metal, resembling silver; it is soft, malleable, and ductile, but possesses little strength. It crackles when a bar is bent, producing what is known as *the cry of tin*. It is easily fusible, melting at about 450° F. Tin does not lose its luster on being exposed to air at ordinary temperatures; but if

strongly heated it takes fire, forming a white powder of stannic oxide, SnO_2. It is attacked by acids. Tin is extensively used for plating iron. What is commonly known as tin is really sheet iron plated with that metal. It is a constituent of some important alloys, bronze being an alloy of tin and copper, and ordinary solder being an alloy of tin and lead.

ANTIMONY

Symbol Sb. Atomic weight 120.

69. Occurrence and Properties.—Antimony can be found in the metallic state, but more commonly it occurs as the sulphide Sb_2S_3. Antimony is a bluish-white, brittle metal. It melts at 1,166° F., and at a white heat may be distilled. The brittleness of antimony renders it useless in its natural state; but it is a constituent of a number of useful alloys. Type metal is an alloy of lead and antimony. Many bearing-metal alloys contain antimony.

BISMUTH

Symbol Bi. Atomic weight 208.

70. Occurrence and Properties.—Bismuth is found in the metallic state and also as an oxide, sulphide, carbonate, etc. The metallic bismuth of commerce is seldom pure, but mostly contains arsenic, iron, and traces of other metals. Bismuth is a hard, brittle metal. It is white, but has a slight reddish tinge. It melts at 518° F., and on solidifying expands about one thirty-second of its bulk. It remains unaltered in dry air, but tarnishes in moist air. At a red heat, it burns with a bluish-white flame, forming bismuth oxide. Bismuth is not used alone, but is a constituent of some very easily fused alloys and of some bearing metals.

HEAT

(PART 1)

MANIFESTATIONS AND MEASUREMENT OF HEAT

1. The Nature of Heat.—The sensations of warmth and cold are familiar to every one. If the hand is placed in water, a sensation is produced; and, according to the sensation, the water is pronounced cold, lukewarm, or hot. It is customary to ascribe the cause of the differences between these states of the water to something called **heat**; thus, when the water gives the sensation of warmth, it is said to have been heated, or to have had heat added to it; when it gives the opposite sensation, it is said to have been cooled, or to have had heat abstracted from it.

Regarding the nature of heat, there have been two theories. The older theory, now discarded, assumed that it was a substance, a sort of fluid without weight, which filled the spaces between the particles of a body, and that a body was hotter or colder according as it had more or less of the fluid stored in it.

The modern theory is that heat is a result of the rapid vibration of the molecules of a body. The application of heat to a body causes a more rapid vibration, while the withdrawal of heat causes a less rapid vibration; and it is to the rate of vibration that the sensation of hotness or coldness is due. As will be shown later, heat is a form of energy. For the present, it is sufficient to consider heat merely as something that can be recognized and measured.

2. Sensible Heat.—The heat that manifests itself to the senses is called **sensible heat,** because any change from a given state to a hotter or colder state is indicated at once by the sense of feeling or by the aid of an instrument called a **thermometer.** The more sensible heat a body possesses, the hotter it is; the more sensible heat that is taken away from it, the colder it is.

3. Temperature.—The term **temperature** is used to indicate how hot or cold a body is. When heat is added to a body and it becomes hotter, its temperature is said to rise; when it cools, its temperature is said to fall. According to the modern theory, the temperature of a body is a measure of the speed with which the molecules composing it are vibrating. A rise in temperature indicates an increase of molecular speed, and a fall in temperature indicates a decrease of molecular speed.

MEASUREMENT OF TEMPERATURE

4. Thermometers.—Owing to the imperfection of the senses, it is impossible to determine by their aid, with any degree of accuracy, the temperatures of different bodies; hence, for this purpose, the **thermometer** is used. In these instruments the effects of heat on bodies are made use of in obtaining the temperature, the most common method being to utilize the expansive effect of heat on liquids. Liquids are used for ordinary purposes instead of solids or gases, because in the first the expansion is too small, and in the second it is too great. Mercury and alcohol are the only liquids used—the former because it boils only at a very high temperature, and the latter because it does not solidify at ordinary temperatures.

5. Mercury Thermometers.—In Fig. 1 is shown a mercurial thermometer with two sets of graduations on it. The one on the left, marked *F*, is the **Fahrenheit graduation,** named after its orignator, and is the one commonly used in the United States and England; the one on the **right,**

marked *C*, is the **centigrade graduation**, generally used by scientists throughout the world because its graduations are better adapted for calculations. As will be seen, the instrument consists of a closed glass tube terminating in a bulb at the lower end. Before closing the upper end, the tube is partially filled with mercury, and the air above it is driven out by heating the mercury to near its boiling point. When the tube above the mercury is filled with mercurial vapor, it is sealed; on cooling, the vapor condenses and a vacuum results. The expansion or contraction of the mercury on the application or withdrawal of heat from the body with which the bulb is in contact causes the highest point of the mercury column to rise and fall, and since for equal changes of temperature the mercury rises or falls equal distances, this instrument, when properly made and graduated, indicates any change in temperature with great accuracy.

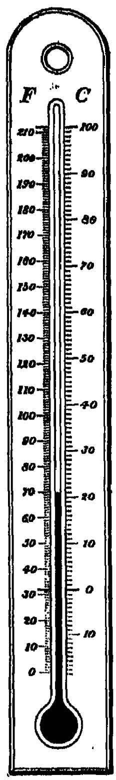


FIG. 1

6. Graduation of Thermometers.—The inside diameter of a good thermometer tube should be the same throughout its length. The graduation of the thermometer is accomplished as follows: It is first placed in melting ice and the point to which the mercury column falls is marked **freezing**. It is then placed in the steam rising from water boiling in an open vessel, and the point to which the mercury column rises is marked **boiling**.

There are now two fixed points—the freezing point and the boiling point. If it is desired to make a Fahrenheit thermometer, the distance between these two fixed points is divided into 180 equal parts, called degrees. The freezing point is marked 32°, and the boiling point 212°. Thirty-two parts are marked off from the freezing point downwards, and the last one is marked 0°, or zero. The graduations are carried above the boiling point and below the zero point as far as desired.

This thermometer was invented in 1714, and was the first to come into general use.

7. In graduating a centigrade thermometer, the freezing point is marked 0°, or zero, and the boiling point 100°; the distance between the freezing and boiling points is divided into 100 equal parts; these equal divisions are carried as far below the freezing point and above the boiling point as desired. The reason that Fahrenheit placed the zero point on his thermometer 32° below freezing was because that temperature was the lowest he could obtain, and he supposed that it was impossible to obtain a lower one. Where there can be any doubt as to the thermometer used, the first letter of the name is placed after the degree of temperature. For example, 183° F. means 183° above zero on the Fahrenheit instrument; 183° C. means 183° above zero on the centigrade instrument.

8. In Russia and a few other countries, another instrument is used, called the **Réaumur** thermometer; the freezing point is marked 0°, or zero, and the boiling point 80°, the space between these two points being divided into 80 equal parts; 183° R. would mean 183° on the Réaumur thermometer.

9. Of these three thermometers, the centigrade is used the most; but, since the Fahrenheit instrument is the one in general use in the United States, all temperatures given here will be understood to be in Fahrenheit degrees, unless otherwise stated. In order to distinguish the temperatures below the zero point from those above, the sign of subtraction is placed before the figures indicating the number of degrees below zero. Thus, −18° C. means a temperature of 18° below the zero point on the centigrade thermometer; −25.4° F. means 25.4° below zero on the Fahrenheit thermometer.

10. Absolute Temperature.—As was stated in *Pneumatics*, absolute zero, or −460° F., is the temperature at which all vibratory motion of the molecules ceases. It is

supposed that, at this temperature, and under a heavy pressure, so that the molecules would be brought close enough together, all gases would have solidified. The absolute zero on the centigrade scale is $-273\frac{1}{3}°$ C.

The **absolute temperature** is the temperature measured above the point of absolute zero. Hence, on the Fahrenheit scale, the absolute temperature T is $460° + t°$ when t = the ordinary temperature, and is above zero. If $t°$ is below zero, its value is negative, and the absolute temperature T is $460° + (-t°) = 460° - t°$.

Throughout the following pages, where temperatures are mentioned, t will denote the ordinary temperature indicated by the thermometer, and T the absolute temperature.

EXAMPLE.—What are the absolute temperatures corresponding to 212°, 32°, and −39.2°?

SOLUTION.—Since no scale is specified, the Fahrenheit is the one intended to be used.

$$T = 460° + 212° = 672°. \quad \text{Ans.}$$
$$T = 460° + 32° = 492°. \quad \text{Ans.}$$
$$T = 460° - 39.2° = 420.8°. \quad \text{Ans.}$$

The absolute temperature on the centigrade scale is $T = 273\frac{1}{3}° + t°$ when $t°$ is above zero, or $T = 273\frac{1}{3} - t°$ when $t°$ is below zero.

EXAMPLE.—What are the absolute temperatures corresponding to 100°, 4°, and −40° C.?

SOLUTION.— $T = 273\frac{1}{3}° + 100° = 373\frac{1}{3}°$ C. Ans.
$T = 273\frac{1}{3}° + 4° = 277\frac{1}{3}°$ C. Ans.
$T = 273\frac{1}{3}° - 40° = 233\frac{1}{3}°$ C. Ans.

11. Conversion of Centigrade and Fahrenheit Temperatures.—It is frequently necessary to change from the centigrade scale to the Fahrenheit scale, or the reverse. Since the number of degrees between the freezing point and boiling point on the centigrade scale is 100 and on the Fahrenheit 180, it is evident that if F equals the number of degrees Fahrenheit, and C equals the number of degrees centigrade,

$$F : C = 180 : 100, \text{ or } F = \tfrac{180}{100} C = \tfrac{9}{5} C$$

And, $$C = \tfrac{100}{180} F = \tfrac{5}{9} F$$

That is, the number of Fahrenheit degrees above freezing is $\frac{9}{5}$ of the number of centigrade degrees above the same point. But since, on the Fahrenheit scale, the freezing point is 32° above zero, 32 must be added to the number of degrees above the freezing point.

Let t_c = centigrade temperature;

and t_f = Fahrenheit temperature.

Then,
$$t_f = \tfrac{9}{5}\, t_c + 32 \qquad (1)$$
and
$$t_c = \tfrac{5}{9}\,(t_f - 32) \qquad (2)$$

Formulas 1 and 2 may be expressed as follows:

Multiply the temperature centigrade by $\frac{9}{5}$, and add 32°; the result will be the temperature Fahrenheit.

Subtract 32° from the temperature Fahrenheit, and multiply by $\frac{5}{9}$; the result will be the temperature centigrade.

EXAMPLE 1.—Change: (*a*) 100° C., (*b*) 4° C., and (*c*) −40° C. into Fahrenheit temperatures.

SOLUTION.—

(*a*) $t_f = \frac{9}{5} t_c + 32 = \frac{9}{5} \times 100 + 32 = 212°$ F. Ans.

(*b*) $t_f = \frac{9}{5} \times 4 + 32 = 39.2°$ F. Ans.

(*c*) $t_f = \frac{9}{5} \times -40 + 32 = -40°$ F. Ans.

EXAMPLE 2.—Change: (*a*) 60° F., (*b*) 32° F., and (*c*) −20° F. into their corresponding centigrade temperatures.

SOLUTION.—

(*a*) $t_c = \frac{5}{9}(t_f - 32) = \frac{5}{9}(60 - 32) = 15\frac{5}{9}°$ C. Ans.

(*b*) $t_c = \frac{5}{9}(32 - 32) = 0°$ C. Ans.

(*c*) $t_c = \frac{5}{9}(-20 - 32) = -28\frac{8}{9}°$ C. Ans.

12. Alcohol Thermometers.—Since mercury freezes at −39° F., which corresponds to about −39.5° C., it cannot be used to obtain temperatures below this point. For this purpose alcohol is used instead of mercury. This liquid has not been frozen until very recently, and then only at the extremely low temperature of −202.9° F. Since alcohol vaporizes at 173° F., the boiling point of water cannot be marked on the alcohol thermometer by heating it to that point. The freezing point is determined as for mercury. An alcohol and a mercurial thermometer are placed in a vessel containing hot water or other liquid, and the point to which the alcohol column rises is marked. Suppose that the

point to which the mercury column rises is marked 132°, then the distance on the alcohol thermometer between the point marked and the freezing point would be divided into $132 - 32 = 100$ equal parts, and each one of these parts would correspond to one degree on the mercurial thermometer. These equal divisions are then carried below the zero point as far as desired.

There are many other kinds of thermometers, some of which depend on the expansion and contraction of different metals and gases when heated and cooled. For temperatures above 675° F., the point at which mercury vaporizes, other means are employed to obtain the temperatures.

EXPANSION OF BODIES BY HEAT

13. The volume of any body—solid, liquid, or gaseous—is always changed if the temperature is changed, other conditions remaining the same; nearly all bodies expand when heated and contract when cooled. In solids, expansion may be considered in three ways, according to the conditions: (1) Expansion in one direction, as the elongation of an iron bar; this is called **linear expansion**; (2) **surface expansion**, which refers to an increase in area; (3) **cubic expansion**, which refers to a general increase in the whole volume.

14. In Fig. 2 is shown an apparatus for exhibiting the linear expansion of a solid body. A metal rod *a* is fixed at one end by a screw *b*, the other end passing freely through the eye *c*, held in the post, and pressing against the short arm of the indicator *f*. The rod is heated in the way shown, and its elongation causes the indicator to move along the arc *de*.

An illustration of surface expansion is afforded in machine shops, particularly in locomotive shops, where piston rods, crankpins, etc. are shrunk in and tires are shrunk on their centers. In shrinking on a tire, it is bored a little smaller than the wheel center and is then heated until it is expanded

enough to go over the wheel center. It is then cooled with cold water, when it contracts, tending to regain its original size, but is prevented by the wheel center, which is a trifle larger. The tire is thus caused to bind around the center

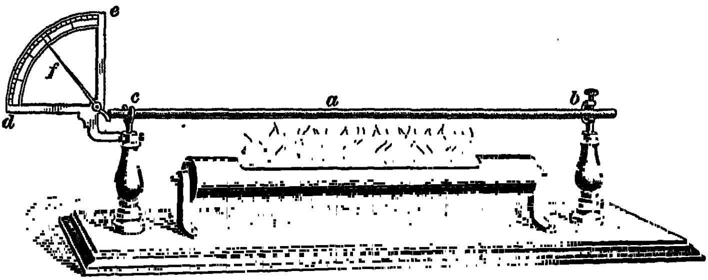


FIG. 2

with great force, and the excessive friction between the tire and the center prevents them from separating.

Cubic expansion may be illustrated by means of a *Gravesandes' ring*. This consists of a brass ball a, Fig. 3, which at ordinary temperatures passes freely through the ring m, of very nearly the same diameter. When the ball is heated, it expands so much that it will no longer pass through the ring.

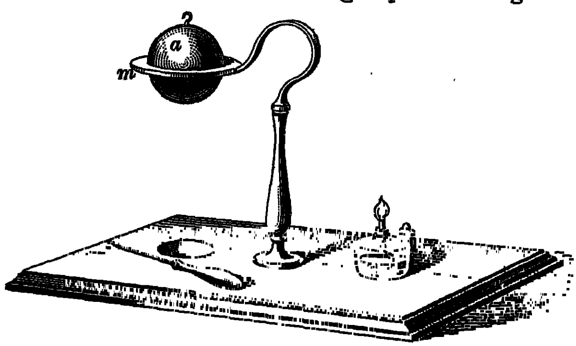


FIG. 3

The bars of a furnace must not be fitted tightly at their extremities, but must be free at one end; otherwise, in expanding, they will crack the masonry. In laying the rails on railways, a small space is left between the successive rails; for, if they touched, the force developed by the

expansion would cause them to curve. Long straight lines of steam piping must be fitted with expansion joints. If a glass vessel is heated or cooled too rapidly, it cracks, especially if it is thick; the reason for this is that, since glass is a poor conductor of heat, the sides become unequally heated and, consequently, unequally expanded. The expansion of liquids is shown in the mercurial and alcohol thermometers; the expansion of gases was discussed to some extent in *Pneumatics*.

15. Coefficient of Expansion.—Suppose that the temperature of the metal rod, shown in Fig. 2, was 32° F. before heating, and that the rod was exactly 10 feet long; that after

TABLE I

COEFFICIENTS OF EXPANSION

Name of Substance	Linear Expansion C_1	Surface Expansion C_2	Cubic Expansion C_3
Cast iron	.00000617	.00001234	.00001850
Copper	.00000955	.00001910	.00002864
Brass	.00001037	.00002074	.00003112
Silver	.00001060	.00002120	.00003180
Wrought iron	.00000686	.00001372	.00002058
Steel (untempered) . . .	.00000599	.00001198	.00001798
Steel (tempered)	.00000702	.00001404	.00002106
Zinc	.00001634	.00003268	.00004903
Tin	.00001230	.00002460	.00003690
Mercury	.00003334	.00006668	.00010010
Alcohol	.00019259	.00038518	.00057778
Gases			.00203252

the temperature had been raised 1°, or to 33°, the bar was 10 feet + $\frac{1}{1200}$ inch long. The linear expansion then was (10 feet + $\frac{1}{1200}$ inch) − 10 feet = $\frac{1}{1200}$ inch, and the ratio of this expansion to the original length of the bar was $\frac{1}{1200} : 10 \times 12 = \frac{1}{1200 \times 120} : 1 = .000006944$.

For every increase of temperature of 1°, the rod became longer by .000006944 of its length. This number .000006944,

which is equal to the expansion of the rod for 1° rise of temperature divided by the original length, is called the **coefficient of linear expansion.** If the temperature of the rod were increased 100° instead of 1°, the amount of elongation would be $.000006944 \times 100 = .0006944$ of its length, or $.0006944 \times 120 = .083328$ inch, or $\frac{1}{12}$ inch. Table I contains the coefficients of expansion, per degree Fahrenheit, for a number of solids, mercury, and alcohol, and the average cubic expansion of gases. No liquids are given, except mercury and alcohol, for the reason that the coefficient of expansion of a liquid is different at different temperatures.

16. Formulas for Expansion.—

Let L = length of any body;

l = amount of expansion or contraction due to heating or cooling the body;

A = area of any section of the body;

a = increase or decrease of area of the same section after heating or cooling the body;

V = volume of the body;

v = increase or decrease in volume due to heating or cooling the body;

C_1, C_2, and C_3 = coefficients taken from Table I;

t = difference of temperature between original temperature and temperature of body after it has been heated or cooled.

Then,

$$l = L\,C_1\,t \qquad (1)$$

$$a = A\,C_2\,t \qquad (2)$$

$$v = V\,C_3\,t \qquad (3)$$

EXAMPLE 1.—How much will a bar of untempered steel, 14 feet long, expand if its temperature is raised 80°?

SOLUTION.—Since only one dimension is given, that of length, linear expansion only can be considered. From Table I the coefficient of linear expansion per unit of length for a rise in temperature of 1° is found to be .00000599 for untempered steel. Hence, using formula **1**, $l = L\,C_1\,t = 14 \times .00000599 \times 80 = .0067088$ ft., or $.0067088 \times 12 = .0805056$ in. Ans.

This seems a very small amount, but in engineering constructions, where long pieces are rigidly connected, it must be taken into account. If the cross-section of the bar were 2 inches square, and the bar were fitted tightly between two supports, an expansion of the above amount would exert a pressure against the supports of about 58,000 pounds.

EXAMPLE 2.—An iron rod $1\frac{1}{4}$ inches in diameter and 100 feet long is used as a tie-rod in constructing a bridge. It was put in place and securely fastened to two rigid supports during a warm day in summer when the temperature in the sunlight was, say 110°. On a cold day in winter, when the thermometer registers zero, how much will the bar tend to shorten, owing to this change in temperature?

SOLUTION.—From formula 1,

$$l = .00000686 \times 100 \times 110 = .07546 \text{ ft.} = .90552 \text{ in. Ans.}$$

If this rod were rigidly secured, so that it could neither stretch nor shorten, it would exert a pull on the supports estimated at about 33,400 lb.

EXAMPLE 3.—The wheel center of a locomotive driver is turned to exactly 50 inches in diameter. If the steel tire is bored 49.94 inches in diameter, to what temperature must the tire be raised in order that it may be easily placed over the center? Assume that the diameter of the tire is increased until it is $\frac{1}{1000}$ inch larger than the center, and that the original temperature is 60°.

SOLUTION.—For this case, formula 2 may be used. The original diameter of the tire is 49.94 in., and it is to be increased to 50.001 in. The area of a circle 49.94 in. in diameter is 1,958.79 sq. in.; the area of a circle 50.001 in. in diameter is 1,963.58 sq. in. The difference between the two areas is 1,963.58 − 1,958.79 = 4.79 sq. in. = a in formula 2. Hence, since C_2 = .00001198, and A = 1,958.79, substitute these values in $a = A\,C_2\,t$, and $4.79 = 1{,}958.79 \times .00001198 \times t = .023466\,t$. Therefore,

$$t = \frac{4.79}{.023466} = 204.13°, \text{ and } 204.13° + 60° = 264.13°. \quad \text{Ans.}$$

NOTE.—Owing to the form of the equation here denoted by formula 2, and to the manner in which the coefficients C_2 were determined, this example may be more easily solved by means of formula 1. Thus, regard the diameter as a linear dimension and apply formula 1. Increase in diameter is l = 50.001 − 49.94 = .061 in. L = 49.94 and C_1 = .00000599. Substituting in $l = L\,C_1\,t$, $.061 = 49.94 \times .00000599 \times t$, or $t = \frac{.061}{49.94 \times .00000599} = 203.92°$, and 203.92° + 60° = 263.92°. Ans. The slight difference in the two results is immaterial, and was to have been expected.

EXAMPLE 4.—What is the decrease in volume of a copper cylinder 30 inches long and 22 inches in diameter if cooled from 212° to 0°?

SOLUTION.—The volume is $V = 22^2 \times .7854 \times 30$ = 11,404 cu. in. Apply formula 3, $v = V\,C_3\,t$. By substituting,

$$v = 11{,}404 \times .00002864 \times 212 = 69.24 \text{ cu. in. Ans.}$$

17. It will be found, on trial, that the three preceding formulas for calculating expansion will not work backwards; that is, if the length of a bar, after it has been heated, be found by formula **1**, Art. **16**, and an attempt be made to reduce the bar to its original length by again applying the same formula and substituting for t the same value as in the first case, the value obtained for l will be slightly different in the two cases. The difference, however, is so slight that it is neglected in practice. If, however, it is desired to obtain exactly the same result in both cases, the following more cumbersome formula must be used, in which t_1, t_2, l_1, and l_2 are, respectively, the original and final temperatures and the original and final lengths, and C_1 has the same value as in formula **1**, Art. **16**:

$$l_2 = \left[\frac{1 + C_1\,(t_2 - 32)}{1 + C_1\,(t_1 - 32)}\right] l_1$$

18. Expansion of Water.—Although, as stated before, the expansion of solids and liquids is nearly uniform throughout all ranges of temperature, water is a marked exception to the general rule. If water is cooled down from its boiling point, it continually contracts until it reaches 39.1° F., when it begins to expand, until it freezes at 32° F. On the other hand, if water at 32° F. is heated, it contracts until it reaches 39.1° F., when it commences to expand. Therefore, the density of water is greatest where this change occurs. The importance of this exception is seen in the fact that ice forms on the surface of water, since it is lighter than the warmer body of water lying at varying depths below it. Were it not for this fact all the large bodies of water would freeze solid, and the climate of the earth would thereby be seriously affected. The coefficient of expansion of water is such a changeable quantity (varying with the temperature) that a special table of coefficients is required.

EXAMPLES FOR PRACTICE

1. What are the absolute temperatures corresponding to: (*a*) 120° C., and (*b*) 120° F?

Ans. $\begin{cases} (a) & 393\tfrac{1}{3}° \text{ C.} \\ (b) & 580° \text{ F.} \end{cases}$

2. **Change −10° C. to the corresponding Fahrenheit reading.**

Ans. 14° F.

3. (*a*) How much will an iron tie-rod 60 feet long expand when the temperature is raised from 40° to 110°? (*b*) Calculate the expansion by the formula in Art. **17** also. (*c*) What is the difference of the two results?

Ans. (*a*) .345744 in.; (*b*) .345725 in.; (*c*) .000019 in.

4. To what temperature must a steel tire of 59.93 inches internal diameter be raised in order that its diameter may be 60.0015 inches? Original temperature is 71°. Ans. 270°

HEAT PROPAGATION

19. Heat is propagated through matter and space in three ways—by *conduction*, by *convection*, and by *radiation.*

20. Conduction.—The progress of heat from places of higher to places of lower temperature in the same body is called **conduction.** The rate at which heat is conducted

TABLE II

HEAT CONDUCTIVITY OF METALS

Metal	Conductivity	Metal	Conductivity
Silver	100.0	Iron	11.9
Copper	73.6	Steel	11.6
Gold	53.2	Lead	8.5
Aluminum	31.3	Platinum	8.4
Zinc	28.1	Bismuth	1.8
Tin	15.2	Mercury	1.3

varies greatly with different substances, the good conductors being those in which conduction is most rapid, and the bad conductors being those in which it is very slow. A non-conductor is a substance that will not conduct heat. No perfectly non-conducting substances are known, but a number of materials are such poor conductors of heat that they are ordinarily called non-conductors. The metals furnish the best conductors, and of these, silver stands first, and copper second. Fluids, both liquid and gaseous, are very

poor conductors of heat. Water, for example, can be made to boil at the top of a vessel while a cake of ice is suspended in the water within a few inches of the surface. If thermometers are placed at different depths, while water boils at the top, it is found that the conduction of heat downwards is very slight.

Representing the conductivity of silver by 100, Table II shows the conducting power of a number of the metals.

Organic substances conduct heat poorly. It is because of this fact that trees withstand great and sudden changes in the atmosphere without injury. The bark is a poorer conductor than the wood beneath it. Cotton, wool, straw, bran, etc. are all poor conductors. Rocks and earth are poorer conductors the less dense and homogeneous is the mass; hence the length of time required for the heat of the sun's rays to penetrate the earth. In Central Europe, the air near the ground has the highest temperature in the month of July, but at a depth of from 25 to 28 feet in the earth the time of highest temperature is in the month of December.

21. Convection.—The transfer of heat by the motion of the heated matter itself is called **convection.** It can take place only in liquids and gases. For example, as heat is applied to the bottom and sides of a vessel of water, the heaviness of the water nearest the source of heat is decreased; it rises, and the colder and heavier water above descends and takes its place. There is thus a constant circulation going on, and this tends to equalize the temperature of the whole mass by bringing the hotter parts of the water in contact with the colder.

22. Radiation.—The communication of heat from a hot body to a colder one across an intervening space is called **radiation.** The best example of radiated heat is that received from the sun, the distance intervening in this case being 93,000,000 miles. A person standing in front of a fire, but at some distance from it, feels a sensation of warmth that is not due to the temperature of the air, for, if a screen be interposed between him and the fire, the sensation

immediately ceases, which would not be the case if the surrounding air had a high temperature. Hence, bodies can send out rays that excite heat and penetrate the air without heating it. This is **radiant heat**, and it manifests itself in all directions around the body.

The intensity of heat radiation from a given source:

1. *Varies as the temperature of the source;*
2. *Varies inversely as the square of the distance from the source;*
3. *Grows less as the inclination of the rays to the surface grows less.*

The truth of all these laws has been established by careful experiment.

Radiant heat is transmitted in a vacuum as well as in air. This is demonstrated by the following experiment: In the top of a glass flask, a thermometer *t* is fixed in such a manner that its bulb occupies the center of the flask, Fig. 4. The neck of the flask is next carefully narrowed by means of a blowpipe; the flask is then attached to an air pump, and a vacuum is produced in the interior. This being accomplished, the tube is sealed at the narrow part. On immersing in hot water, or on bringing the flask near some hot charcoal, the mercury is seen to rise at once. It can rise only by reason of the radiation through the vacuum in the interior, for glass is such a poor conductor that the heat could not travel with sufficient rapidity through the sides of the flask and the stem of the thermometer to cause this almost instantaneous rise.

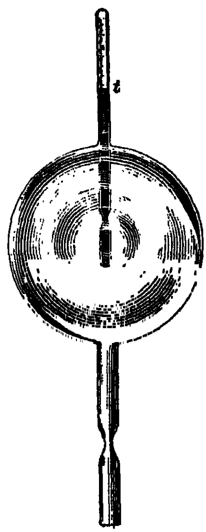


FIG. 4

23. The **radiating power of heated surfaces** also depends very greatly on the form, shape, and material of which they are composed. If a cubical vessel, filled with hot water, has one of its vertical sides coated with polished

silver, another with tarnished lead, a third with mica, and the fourth with lampblack, experiment has shown that the radiating powers will be represented relatively by the numbers 25, 45, 80, 100; hence, bright surfaces radiate less heat than dark ones having the same temperature.

In the same way, it is found that the heat-absorbing power of bodies varies in a similar manner. Lampblack reflects few of the heat rays that impinge on it; nearly all are absorbed, while, on the other hand, polished silver reflects the greater percentage of the rays and absorbs only about $2\frac{1}{2}$ per cent.

Some substances neither absorb nor reflect the heat rays to any extent, but transmit nearly all of them just as glass transmits light. For example, rock salt reflects less than 8 per cent. of the radiation it receives, absorbs almost none, and transmits 92 per cent.

It is apparent that there is a sort of exchange going on between heated bodies at all times, which tends toward an equalization of temperature. The hot bodies are always cooling, and the cold bodies are always tending toward a rise in temperature, so that heat is created only to be diffused and apparently lost. That it is not lost, however, will be shown in the subsequent pages.

24. Dynamical Theory of Heat.—The view now generally taken as to the mode in which heat is propagated is thus stated in Ganot's Physics: "A hot body is one whose molecules are in a state of vibration. The higher the temperature of a body, the more rapid are these vibrations, and a diminution in temperature is but a diminished rapidity of the vibrations of the molecules. The propagation of heat through a bar is due to a gradual communication of this vibratory motion from the heated part to the rest of the bar. A good conductor is one which readily takes up and transmits the vibratory motion from molecule to molecule, while a poor conductor is one which takes up and transmits the motion with difficulty. But even through the best of the conductors the propagation of this motion is comparatively slow. How then can be explained the instantaneous

perception of heat when a screen is removed from a fire, or when a cloud drifts from the face of the sun? In this case, the heat passes from one body to another without affecting the temperature of the medium which transmits it. In order to explain these phenomena, it is imagined that all space, the space between the planets and the stars, as well as the interstices in the hardest crystal and the heaviest metal—in short, matter of any kind—is permeated by a medium having the properties of matter of infinite tenuity, called **ether.** The molecules of a heated body, being in a state of intensely rapid vibration, communicate their motion to the ether around them, throwing it into a system of waves which travel through space and pass from one body to another with the velocity of light. When the undulations of the ether reach a given body, the motion is given up to the molecules of that body, which in their turn begin to vibrate; that is, the body becomes heated. This process of this motion through the ether is termed radiation, and what is called a ray of heat is merely one series of waves moving in a given direction."

MEASUREMENT OF HEAT

HEAT UNITS

25. The British Thermal Unit.—To measure heat, some standard unit is required. The unit commonly employed in English-speaking countries *is the amount of heat required to raise the temperature of 1 pound of water from 62° to 63° F.* This unit is called the **British thermal unit,** usually written B. T. U.

For accurate work, it is necessary to specify the particular degree on the thermometric scale, for it is found by experiment that the heat required to raise the temperature of a pound of water 1° is not the same for all parts of the scale. For ordinary calculations, however, this is not required.

26. The Calorie.—The amount of heat necessary to raise the temperature of 1 kilogram of water 1° C. is called

a calorie. One kilogram equals 2.2046 pounds and 1° C. $= \frac{9}{5} \times 1°$ F.; hence, a calorie is $2.2046 \times \frac{9}{5} = 3.9683$ B. T. U. The calorie is used in France, and in other countries in which the metric system of weights and measures has been adopted.

SPECIFIC HEAT

27. Definition of Specific Heat.—If equal weights of two substances are heated under exactly similar conditions to a certain temperature, it will take longer to heat one than to heat the other. If, at this higher temperature, they are plunged into water, in vessels containing equal quantities of water at the same temperature, the temperature of the water will be raised more by the substance that required the longer time to heat. For example, it requires more time to raise the temperature of 1 pound of iron from 70° to 300° than to raise the temperature of 1 pound of lead to the same point under the same conditions. If each is then cooled in a separate vessel containing, say, 5 pounds of water at 70°, the water in which the iron cools will be heated to a higher temperature than that in which the lead cools. This indicates that it takes more heat to raise the temperature of 1 pound of iron a certain number of degrees than it does to raise the temperature of 1 pound of lead the same number of degrees, and that the iron, at the same temperature as the lead, held more heat than did the lead.

The **specific heat** of a substance is the ratio between the amount of heat required to raise the temperature of the substance 1° and the amount of heat required to raise the temperature of the same weight of water 1°. Thus, if the specific heat of lead is .0314, the amount of heat required to raise a certain weight of lead 1° will raise the same weight of water only .0314 of 1°, or, what is the same thing, .0314 B. T. U. will raise the temperature of 1 pound of lead 1° F.

EXAMPLE.—The specific heat of copper is .0951; how many B. T. U. will it take to raise the temperature of 75 pounds 180°?

SOLUTION.—Since it takes .0951 B. T. U. to raise 1 lb. of copper 1°, it will take $75 \times 180 \times .0951$ B. T. U. to raise 75 lb. 180°. Hence, the heat required is $.0951 \times 75 \times 180 = 1{,}283.85$ B. T. U. Ans.

28. Heat Required for a Given Rise of Temperature.—The following formula gives the number of B. T. U. required to raise the temperature of a substance a given number of degrees, or the number of B. T. U. given up by a body in cooling a given number of degrees:

Let G = weight of body, in pounds;
s = specific heat of substance composing the body;
t_1 = original temperature of body;
t_2 = final temperature of body;
Q = number of B. T. U. required, or given up, in changing temperature of body from $t_1°$ to $t_2°$.

Then, $$Q = Gs(t_2 - t_1)$$

EXAMPLE.—A piece of wrought iron weighing 31.3 pounds and having a temperature of 900°, is cooled to a temperature of 60°; how many units of heat did it give up? The specific heat of wrought iron is .1138.

SOLUTION. Using the above formula, $Q = Gs(t_2 - t_1)$,
$31.3 \times .1138 \times (60 - 900) = -2,992$ B. T. U. Ans.

If a body is cooled from a temperature t_1 down to a temperature t_2, the value of Q as given by the above formula will be negative, the minus sign indicating that the heat is withdrawn.

29. In Table III are given the specific heats of a number of substances under constant pressure.

The reason that there are two values for the specific heat of gases is that less heat is required to raise the temperature of a gas when the volume is constant than when the pressure is constant and the volume varies. This point will be more fully discussed later.

30. Mixing Bodies of Unequal Temperatures.—If a certain quantity of water having a temperature of 40° is mixed with a like quantity having a temperature of 100°, the temperature after mixing will be $\frac{40 + 100}{2} = 70°$. But, if 5 pounds of copper having a temperature of 100° is immersed in 5 pounds of water having a temperature of 40°, the resulting temperature will not be 70°.

TABLE III

SPECIFIC HEATS

Solids

Substance	Specific Heat	Substance	Specific Heat
Copper	.0951	Cast iron	.1298
Gold	.0324	Lead	.0314
Wrought iron	.1138	Platinum	.0324
Steel (soft)	.1165	Silver	.0570
Steel (hard)	.1175	Tin	.0562
Zinc	.0956	Ice	.5040
Brass	.0939	Sulphur	.2026
Glass	.1937	Charcoal	.2410
Aluminum	.2143	Nickel	.1089

Liquids

Water	1.0000	Lead (melted)	.0402
Alcohol	.6200	Sulphur (melted)	.2340
Mercury	.0333	Tin (melted)	.0637
Benzine	.4500	Sulphuric acid	.3350
Glycerine	.5550	Oil of turpentine	.4260

Gases

	Specific Heat	
	Constant Pressure	Constant Volume
Air	.23751	.16902
Oxygen	.21751	.15507
Nitrogen	.24380	.17273
Hydrogen	3.40900	2.41226
Superheated steam	.48050	.34600
Carbon monoxide	.24790	.17580
Carbon dioxide	.21700	.15350

When different substances having different specific heats and different temperatures are mixed or are brought into close contact, the resulting temperature may be found by the following formula, provided that there is no change of state in any substance, as when ice melts, etc.:

$$t = \frac{G_1 s_1 t_1 + G_2 s_2 t_2 + G_3 s_3 t_3 + etc.}{G_1 s_1 + G_2 s_2 + G_3 s_3 + etc.}$$

in which t = final temperature of mixture;
G_1, s_1, and t_1 = weight, specific heat, and temperature, respectively, of one body;
G_2, s_2, and t_2 = same for second body;
G_3, s_3, and t_3 = same for a third body, etc.

Apply this formula to the case of the copper immersed in water. The specific heat of water is 1 and the specific heat of copper from Table III is .0951; then the final temperature is found from the equation

$$t = \frac{5 \times 1 \times 40 + 5 \times .0951 \times 100}{5 \times 1 + 5 \times .0951} = 45.21°\text{, nearly}$$

EXAMPLE 1.—If 21 pounds of water at a temperature of 52° is mixed with 40 pounds of water at a temperature of 160°, what is the temperature of the mixture?

SOLUTION.—Since the specific heat of water is 1, it may be left out in applying the formula, and the temperature is found to be

$$t = \frac{21 \times 52 + 40 \times 160}{21 + 40} = 122.82°. \quad \text{Ans.}$$

EXAMPLE 2.—A copper vessel weighing 2 pounds is partly filled with water having a temperature of 80° and weighing 7.8 pounds. A piece of wrought iron weighing $3\frac{1}{4}$ pounds and having a temperature of 780° is dropped into this water. What is the final temperature?

SOLUTION.—Substituting the values given in the formula, and remembering that the original temperatures of the copper vessel and the water that it contains are the same, then

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 80 + 3.25 \times .1138 \times 780}{2 \times .0951 + 7.8 + 3.25 \times .1138} = 110.97°\text{, nearly.}$$

Ans.

EXAMPLE 3.—A wrought-iron ball weighing 1 pound is placed in a furnace; when it has attained the temperature of the furnace, it is taken out and placed in a copper vessel weighing $\frac{1}{2}$ pound and containing exactly 2 pounds of water at a temperature of 75°. Assuming that no water escapes as steam, and that the temperature of the ball, water, and vessel after mixing is 156°, what is the temperature of the furnace?

SOLUTION.—Substituting the values given in preceding formula,

$$156 = \frac{1 \times .1138 \times t_1 + 2 \times 75 + .5 \times .0951 \times 75}{1 \times .1138 + 2 + .5 \times .0951}$$

or,
$$156 = \frac{.1138\, t_1 + 153.566}{2.16135}$$

and clearing of fractions, $156 \times 2.16135 = .1138\, t_1 + 153.566$; hence, $.1138\, t_1 = 183.604$,

or,
$$t_1 = \frac{183.604}{.1138} = 1{,}613.4^\circ. \quad \text{Ans.}$$

31. Calculating Specific Heat.—By means of the formula in Art. **30**, the specific heat of a substance may be obtained. Thus, in the formula,

$$t = \frac{G_1 s_1 t_1 + G_2 s_2 t_2 + G_3 s_3 t_3 + etc.}{G_1 s_1 + G_2 s_2 + G_3 s_3,\ etc.}$$

suppose that the specific heat s_3 is required, and that all the other quantities, including t, are known. Solving the above equation for s_3, $t\,(G_1 s_1 + G_2 s_2 + etc.) + t\, G_3 s_3 = G_1 s_1 t_1 + G_2 s_2 t_2 + G_3 s_3 t_3 + etc.$, or $t\, G_3 s_3 - t_3 G_3 s_3 = G_1 s_1 t_1 - G_1 s_1 t + G_2 s_2 t_2 - G_2 s_2 t + etc.$,

or,
$$s_3 = \frac{G_1 s_1 (t_1 - t) + G_2 s_2 (t_2 - t) + etc.}{G_3 (t - t_3)}$$

EXAMPLE.—A silver vessel weighing 13 ounces is suspended by a string; 1 pound 4 ounces of water having a temperature of 120° is poured into it, and in this is placed a piece of metal weighing 14 ounces and having a temperature of 100°. If the temperature of the vessel is 72°, and the final temperature is 117°, what is the specific heat of the piece of metal?

SOLUTION.—Using the formula, and letting G_1, s_1, and t_1 represent, respectively, the weight, specific heat, and temperature of the silver vessel G_2, s_2, and t_2 the same quantities for the water, and G_3, s_3, and t_3 those for the piece of metal,

$$s_3 = \frac{G_1 s_1 (t_1 - t) + G_2 s_2 (t_2 - t)}{G_3 (t - t_3)}$$

$$= \frac{13 \times .057\,(72 - 117) + 20 \times 1\,(120 - 117)}{14\,(117 - 100)} = \frac{-33.345 + 60}{238} = .112$$

Ans.

All weights must be reduced to either pounds or ounces before substituting.

EXAMPLES FOR PRACTICE

1. How many units of heat are required to raise the temperature of 10 ounces of platinum from 80° to 2,000°? Ans. 38.88 B. T. U.

2. In order to determine the specific heat of a certain alloy, a piece weighing 12½ ounces was heated to a temperature of 320°, and was then immersed in 2 pounds 6 ounces of water contained in a lead vessel weighing 4 pounds 7 ounces. The temperature of the water and of the vessel being 70°, what was the specific heat of the alloy if the final temperature was 79°? Ans. .1202

3. In order to determine the temperature of a chimney, a silver bar weighing 20 ounces is placed in it until it has attained the same temperature. It is then immersed in 1 pound of water contained in a brass vessel weighing 10 ounces. The temperature of the vessel and water being 65°, and the final temperature 98½°, what is the temperature of the chimney? Ans. 596°

4. An iron casting weighing 3 tons is cooled from 2,100° to 100°; how many units of heat does it give up? Ans. 1,557,600 B. T. U.

LATENT HEAT

32. Doctor Black's Experiment.—Heretofore, the phenomena relating to sensible heat only have been considered; heat that is not sensible, that is, heat the existence of which is not revealed by the senses or by a thermometer, will now be considered. If a quantity of pounded ice at a temperature of 32° be put in a vessel and held over the flame of a spirit lamp, heat passes rapidly into the ice and melts it; but a thermometer resting in this mixture of ice and water shows no tendency to rise; it will remain at 32° until all the ice has been melted. Where has the heat gone that was supplied to the ice? This question was first investigated by Doctor Black, of Edinburgh, in 1760, and is easily explained by the modern dynamical theory of heat.

Doctor Black took 1 pound of water and 1 pound of ice, both having a temperature of 32°, and placed them in two vessels suspended in a chamber that was kept at as nearly a uniform temperature as possible. At the end of ½ hour the temperature of the water was 39.2°, but the ice did not reach that temperature until 10½ hours had passed, being melted,

of course, in the meantime. Doctor Black reasonably assumed that the ice received the same quantity of heat that the water did in each $\frac{1}{2}$ hour, because it was placed in exactly the same position with respect to the surrounding air; that is to say, it received $39.2 - 32 = 7.2$ units of heat every $\frac{1}{2}$ hour, or 14.4 units every hour, and $14.4 \times 10\frac{1}{2} = 151.2$ units in $10\frac{1}{2}$ hours. Hence, it took $151.2 - 7.2 = 144$ units of heat to change the 1 pound of ice at 32° into water at 32°. This value will be used hereafter whenever the occasion arises for using it.

If 1 pound of water having a temperature of 212° be mixed with 1 pound of water having a temperature of 32°, the temperature of the mixture will be $\frac{212 + 32}{2} = 122°$, the boiling water giving up 90° and the cold water receiving 90°, thus bringing both to a common temperature. If 1 pound of ice at a temperature of 32° be mixed with 1 pound of water at a temperature of 212°, the temperature of the mixture will be only 50° instead of 122, as in the previous case. Here, the water has given up $212 - 50 = 162$ units of heat in order to bring both bodies to a common temperature. Since the temperature of the ice was raised from 32° to 50°, it follows that $50 - 32 = 18$ units of heat were used to raise the temperature of the ice after it had been melted into water, and that $162 - 18 = 144$ units of heat were necessary to convert the ice at 32° into water of the same temperature.

33. Latent Heat of Fusion.—The extra amount of heat that is necessary to convert a solid into a liquid of the same temperature without raising the temperature of the solid is called the **latent heat of fusion,** and the temperature at which this change of state in the body takes place is called the **melting point,** or **temperature of fusion.** All solids probably have a latent heat of fusion, the word *probably* being used because some solids have never been melted, except at such high temperatures that accurate measurements are not possible.

The value of the latent heat varies greatly for different substances, being 144 units for ice, while for frozen mercury its value is only 5.09; that is, to change 1 pound of frozen mercury at its temperature of fusion ($-39°$ F.) into liquid mercury of the same temperature requires only 5.09 units of heat.

It is reasonable to suppose that if 144 units of heat are required to convert 1 pound of ice at 32° into water at 32°, then the same number of heat units will be given up when water at 32° is changed into ice at 32°. Experiment has verified this supposition.

34. Latent Heat of Vaporization.—If water be heated to its boiling point of 212° under a constant pressure of 14.696 pounds per square inch, it has been found, by experiment, that about 965.8 units of heat are required to change 1 pound into steam at 212°. This extra number of units of heat necessary to convert a liquid into vapor of the same temperature and pressure is called the **latent heat of vaporization**, and the temperature at which this change of state takes place is called the **temperature of vaporization.**

35. Nature of Latent Heat.—According to the modern theory of heat, the extra quantity of heat necessary for a change of state of a body is used in forcing the molecules of a body farther apart, and in overcoming the force of cohesion. This latent heat is not lost, but performs work in giving additional energy to the molecules of a body, and it always reappears when the body resumes its former state. Thus, for instance, 1 pound of steam under a pressure of one atmosphere contains about $965.8 + 180 = 1,145.8$ units of heat more than does 1 pound of water at 32°. Hence, if 1 pound of steam at 212° be mixed with $\frac{965.8}{180} = 5.37$ pounds of water at 32°, the temperature of the mixture will be exactly 212°, or the boiling point of water; in other words, the steam raised 5.37 pounds of water from the freezing point to the boiling point without lowering its own temperature by merely changing from steam into water. If 1 pound

of water at a temperature of 32° be changed into ice of the same temperature, 144 units of heat will be given up during this change of state.

36. In Table IV are given the temperatures of fusion and of vaporization, and the latent heats of fusion and vaporization for the cases in which they have been determined with sufficient accuracy:

TABLE IV

HEATS OF FUSION AND VAPORIZATION

Substance	Temperature of Fusion Degrees F.	Temperature of Vaporization Degrees F.	Latent Heat of Fusion B. T. U.	Latent Heat of Vaporization B. T. U.
Water	32	212	144.00	965.8
Mercury	− 39	675	5.09	
Sulphur	237.2	831	16.86	
Tin	446		25.65	
Lead	626		9.67	
Zinc	786	1,900	50.63	
Alcohol	− 202.9	173		372
Oil of turpentine	14	313		124
Linseed oil		600		
Aluminum	1,220		51.4	
Copper	2,000			
Cast iron	2,192			
Wrought iron	2,912			
Steel	2,520			
Platinum	3,200			

EXAMPLE 1.—How many units of heat will be required to change 12 pounds of ice at a temperature of −20° C. into steam of 212° F.?

SOLUTION.—By formula **1**, Art. **11**, $t_f = (\frac{9}{5} \times -20) + 32 = -4°$ F. This is equivalent to $32° + 4° = 36°$ F. below the freezing point. In Table III, the specific heat of ice was given as .504; hence, it will take $12 \times 36 \times .504 = 217.73$ B. T. U. to raise the temperature of 12 lb. of ice from −4° to 32°. To convert this ice into water of 32° will require $144 \times 12 = 1,728$ B. T. U. To raise this water from 32° to a temperature

of 212° will require $12 \times 180 = 2{,}160$ B. T. U. To convert it into steam of 212° will require $965.8 \times 12 = 11{,}589$ B. T. U. The total number of units of heat required is therefore

$217.73 + 1{,}728 + 2{,}160 + 11{,}589 = 15{,}695$ B. T. U. Ans.

EXAMPLE 2.—How many units of heat will it take to evaporate 25 pounds of alcohol from a temperature of 70°?

SOLUTION.—The temperature of vaporization of alcohol is 173°, and the specific heat is .62; the increase in temperature from 70° will be $173° - 70° = 103°$. The number of units of heat required will be $25 \times 103 \times .62 = 1{,}596.5$ heat units. The latent heat of vaporization is 372; hence, $1{,}596.5 + 25 \times 372 = 10{,}896.5$ B. T. U. will be required. Ans.

37. Freezing Mixtures.—A solid may be changed into a liquid, not only by melting it, but also by dissolving it, as salt or sugar is dissolved in water. Since the particles of the solid body must be torn asunder, in opposition to the forces that hold them together, it is reasonable to suppose that a certain amount of heat will be required to do this. That such is a fact may be easily proved by any one having a thermometer. Put a thermometer in a vessel of water and leave it there until it indicates the temperature of the water, then put in some salt or sugar, and stir so as to make it dissolve more quickly. It will be found that the mercury has fallen several degrees. In fact, if any solid is dissolved in a liquid that does not act chemically on it, the temperature of the mixture will be lower than if the solid did not dissolve. It is this principle that is taken advantage of in the so-called freezing mixtures. A mixture of one part of nitrate of ammonia and one part of water will reduce the temperature from 50° to 4°, a fall of 46°. The effects are still more striking when both bodies are solids, one of which is already at the freezing point. Thus, in a mixture of two parts of snow, or finely pounded ice, and one part of common salt, the temperature is lowered from 32° to −5°, a fall of 37°, while in a mixture of four parts of potash and three parts of snow or pounded ice the temperature is lowered from 32° to − 51°, a fall of 83°.

38. Latent Heat in Nature.—Latent heat plays an important part in the formation and melting of ice and snow

in nature. It takes a long time and severe cold to freeze the water of a river to any depth, even though the thermometer goes far below the freezing point. This is because 144 units of heat must be given up by every pound of water, after being brought to the freezing point, before the ice can form. If it were not for this fact, the rivers, lakes, and other bodies of water would be frozen solid as soon as the water reached the freezing point, and would be melted as soon as the temperature rose above that point. In the spring all the snow on the hills would be melted during a warm day, and great floods would be the consequence. As it is, 144 units of heat must be supplied to every pound of snow at 32° to convert it into water at 32°, and considerable time must elapse before the whole of this large quantity of heat can be supplied.

EXAMPLES FOR PRACTICE

1. If 1 pound of steam at 212° and 7 pounds of ice at 32° are mixed, what will be the resulting temperature? Ans. 49.23°

2. How many units of heat are required to melt 10 pounds of mercury at −39° and raise it to a temperature of 0°? Ans. 63.9 B. T. U.

3. How many pounds of oil of turpentine at 60° can be vaporized by the heat from 1 pound of coal, if the coal gives out 13,400 B. T. U. during combustion? Ans. 57.8 lb.

4. How many pounds of water at 32° can be vaporized by the heat from the combustion of 1 pound of coal of the quality used in example 3? Ans. 11.7 lb.

5. How many pounds of coal of the same quality as in example 3 are required to raise 100 pounds of wrought iron from 85° to its melting point? Ans. 2.4 lb.

SOURCES OF HEAT

39. Heat is derived from the following sources: *Physical sources*, which are, the radiation of heat from the sun, terrestrial heat, change of state in bodies, and electricity; *chemical sources*, or molecular combinations, more especially combustion; *mechanical sources*, comprising friction, percussion, and pressure

40. Physical Sources.—The greatest of all the sources of heat is the sun. Most scientists are of the opinion that all the heat received or given up by the earth has, or has had, its source in the sun; but it is not required to discuss this theory fully here. It is the heat radiated from the sun and received by the earth that causes the changes of seasons, and that causes the water in the rivers, lakes, and seas to evaporate and form the clouds, to be again precipitated as rain or snow. Without this heat, no living thing—animal or vegetable—could exist.

The earth possesses a heat peculiar to itself, called *terrestrial heat*. When a descent is made below the surface, the temperature is found to gradually increase. This is not caused by the heat radiated from the sun, for the material comprising the earth is such a poor conductor that the heat of the sun's rays penetrates only a very short distance below the surface. The explanation usually given for this phenomenon is that the interior of the earth is in a molten condition. The terrestrial heat exerts but a slight effect, not raising the temperature of the surface more than $\frac{1}{20}°$.

If a liquid be poured on a finely divided solid, as a sponge, flour, starch, roots, etc., the temperature will be increased from 1° to 10° according to conditions. This phenomenon may be called *heat produced by capillarity*.

The heat produced by a change of state has already been described; it is the heat given off when a body is converted from a gas or liquid to a liquid or solid.

Extremely high temperatures may be produced by the electric current. By means of it, quicklime, firebrick, osmium, porcelain, and several other substances, which until very recently have resisted every attempt to melt them, may be made to run like water.

41. Chemical Sources.—Whenever substances that act chemically on one another are brought together and allowed to combine, heat is evolved. When heat is produced by oxygen uniting with carbon or some other substance, and is accompanied by light, it is called *combustion*.

42. Mechanical Sources.—The friction between any two bodies rubbed together produces heat. Rubbing one hand briskly against the other will soon make the hands too warm for comfort. The friction between a journal and its bearing causes heat; the heat causes the journal and bearing to expand, the journal expanding more rapidly because it is smaller and is heated more quickly; the expansion causes a greater pressure on the bearing, thus producing more friction and heat. If the bearing is not properly oiled, the heat will become so intense in a short time that the soft metal in the bearing will melt.

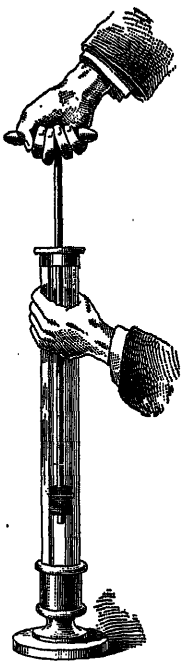


FIG. 5

When meteors or so-called shooting stars strike the earth's atmosphere their velocity is so great (sometimes as high as 150 miles a second) that the friction of the atmosphere causes them to take fire almost instantly. Friction is always accompanied by the production of heat.

Heat is also generated by percussion. The repeated blows of a hammer on a piece of iron, lead, or other metal, will soon make it quite hot.

The generation of heat by compression was mentioned in connection with gases. It is a matter of common experience that when a gas is compressed its temperature rises. The cylinder of an air compressor is too warm to rest the hand on, and a bicycle pump can be made quite warm by vigorous use. The same thing is also true of solids and liquids, but the results are not so marked.

The production of heat by the compression of gases is easily shown by means of the pneumatic syringe shown in Fig. 5, which consists of a glass tube with thick sides, hermetically closed with a leather piston. At the bottom is a small cavity in which a piece of cotton, moistened with ether or carbon disulphide, is placed. The tube is filled with air

and the piston is suddenly plunged downwards. The compression of the air generates so much heat that the cotton is ignited and can be seen to burn when the piston is suddenly withdrawn. The ignition of the cotton in this experiment indicates a temperature of at least 570°, since it will not ignite at a lower temperature under these conditions.

THERMODYNAMICS

FUNDAMENTAL RELATIONS OF HEAT AND WORK

43. Production of Heat by Work.—In Art. **42**, it was shown that heat may be produced mechanically by friction, percussion, or the compression of gases. In every case in which heat is thus produced mechanically, there is work done. In the case of the journal and bearing, the frictional resistance at the contact surface is overcome and work is done against this resistance; in compressing a gas, there is work done in moving the piston; when a bar of iron is heated by hammering, work is done on the bar by the impact of the hammer.

Furthermore, the quantity of heat produced depends on the amount of work done. If a journal is rough and unlubricated, the friction between journal and bearing is greater than when it is smooth and lubricated; as a consequence, more work is done in overcoming friction in a given time and it is a matter of experience that more heat is produced; that is, the journal heats in a shorter time. In compressing a gas, the further the compression is carried the more work there is done and the more the gas is heated. Likewise, the more work expended in hammering a bar of iron, the hotter it becomes; that is, the more heat there is generated. The statements of this paragraph lead therefore to the following important principle:

The performance of mechanical work results in the production of heat, and the quantity of heat thus produced depends, in some way, on the amount of work done.

44. Performance of Mechanical Work by Heat.
Heat may be produced by the expenditure of work, and, conversely, work may be produced by the expenditure of heat. As an example, let the metal rod *a*, Fig. 6, be heated; it will lengthen and raise the mass *b* a distance equal to the increase of length; here heat has been expended, and as a result work has been done in raising the block. The expansion of gases furnishes instances of the production of work by heat. In Fig. 7, air is confined in the cylinder *a* by the piston *b*. Let the confined air be heated; then, if its pressure remains the same, which will be the case if the piston is free to move, the volume of the confined air must be increased, for the temperature is increased and the volume must increase with the absolute temperature when the pressure remains constant. But if the air expands, the piston must rise, and consequently work is done. The more heat expended, the more the air expands and the more work there is done.

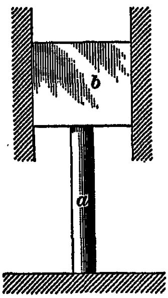


FIG. 6

Perhaps the most conclusive proof that heat produces work is furnished by Hirn's experiments on the steam engine. Hirn measured the heat taken into the engine cylinder per stroke, from the boiler, and the heat rejected per stroke to condenser. In every case, the heat given up to the condenser was less by a considerable amount than that received from the boiler, proving that some of the heat received was expended in the performance of the work required to drive the engine.

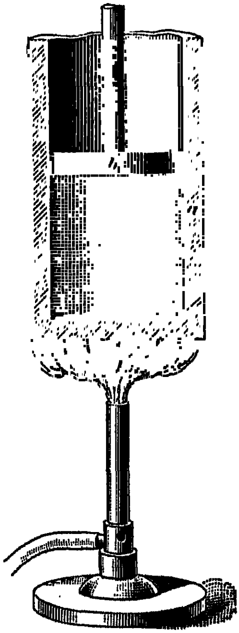


FIG. 7

45. Mechanical Theory of Heat.—The fact that heat is produced by the expenditure of work, and vice versa,

leads to the conclusion that heat is simply a form of energy. When work is done and reappears as heat, it has simply been transformed into energy of another form. To illustrate this statement, take the case of work done on a gas in compressing it. If the piston, Fig. 7, is pushed downwards, the air below is heated and its temperature rises. Work has been done on the air, and as a result heat has been generated. Before the piston was moved, the confined air had a definite temperature; the molecules were vibrating to and fro with an average speed that may be denoted by v_1, and it is the continual striking of the molecules against the piston that gives rise to what is called the pressure on the containing walls and piston.

Let the piston move downwards and compress the air. The molecules, being now closer together, strike the containing walls with greater frequency, thus causing the increased pressure. It is found that the temperature is also increased; this results from the fact that the molecules are now moving with an average speed v_2, which is greater than the original speed v_1. According to the principles of mechanics, a body of weight G having a speed v possesses the kinetic energy $\frac{G v^2}{2g}$. Considering the molecules of the gas as small bodies, the kinetic energy of a molecule moving with the speed v_1 is $\frac{G v_1^2}{2g}$ when G denotes the weight of the molecule. The sum of the kinetic energies of all the molecules of the confined air is the kinetic energy of the weight of air. After compression, the molecules have the velocity v_2, and therefore the kinetic energy is $\frac{G v_2^2}{2g}$, which is greater than the original kinetic energy $\frac{G v_1^2}{2g}$, hence, the total kinetic energy of the contained air has been increased. Also, there may be a change in the potential energy of the air, but this point will be considered later.

When work is done on a body, that work is stored up in the body as kinetic energy and the increase of energy is

equal to the work done. For example, the net work done in starting a railway train from rest and getting it up to speed is stored in the train and is given back when the train comes to rest. In the same way, the work done in compressing the air is expended in increasing the total energy of the air; that is, in making the molecules move at a higher rate of speed.

Consider next the reverse operation. Suppose the air under the piston to be heated by a flame while the piston is held stationary. The temperature of the air rises and the pressure is correspondingly increased. Now, remove the flame and let the piston be released so that it is free to move. The pressure of the confined air being greater than the pressure of the atmosphere above the piston, there is a net upward force that causes the piston to rise. As the piston rises, the confined air expands and its temperature falls. The fall of temperature indicates that the molecules of the air move with less speed, and this in turn indicates that the air as it expands is losing kinetic energy. According to the law of conservation of energy, the energy thus lost by the air must reappear somewhere; it cannot be destroyed. Where it reappears in this case is easily seen; the energy lost by the air is precisely that required to do the work of raising the piston.

From the foregoing, it is apparent that *the heat in a body is simply the stock of energy the body possesses;* this energy may be kinetic energy, due to the motions of the molecules composing the body, or potential energy, due to the relative positions of the molecules and their distances from each other; or it may be partly kinetic and partly potential. To heat a body is to increase its stock of energy; and, conversely, to cool a body is to decrease its stock of energy.

46. **The Mechanical Equivalent of Heat.**—As already explained, heat is measurable, and quantities of heat are expressed in terms of a unit called the British thermal unit. Work and energy are expressed in foot-pounds. Since heat is merely a form of energy, there should be some numerical

relation between the two units, that is, a B. T. U. should be equal to some definite number of foot-pounds, or the reverse.

This relation has been determined by many experimenters. Doctor Joule, of Manchester, England, found as the result of many experiments that the heat required to raise the temperature of 1 pound of water 1° F. could, if expended in work, raise a weight of 772 pounds a distance of 1 foot; that is, 1 B. T. U. = 772 foot-pounds. Later experiments of Professor Rowland, of Baltimore, Maryland, show that Joule's figure is too low, and that 778 is the correct value. This number, 778, is called the *mechanical equivalent of heat*, and sometimes *Joule's equivalent;* it is denoted by the letter J.

47. The First Law of Thermodynamics.—The formal statement of the relation between heat and work constitutes the first law of thermodynamics.

Law.—*Heat and mechanical work are mutually convertible. A unit of heat requires for its production, or produces by its disappearance, J units of work.*

Let Q = heat produced or given up, in B. T. U.;
W = work, in foot-pounds, produced by Q or required to produce Q;
J = Joule's equivalent.

Then the first law of thermodynamics is expressed by the formula,

$$JQ = W, \text{ or } Q = \frac{W}{J}$$

EXAMPLE 1.—The combustion of 1 pound of coal results in the generation of 13,700 B. T. U.; if all this heat could be transformed into work, what horsepower could be obtained by the burning of 300 pounds of coal per hour?

SOLUTION.—The B. T. U. liberated per minute is $\frac{300}{60} \times 13{,}700 = 68{,}500$. Since 1 B. T. U. = 778 ft.-lb., the work per minute is $68{,}500 \times 778 = 53{,}293{,}000$ ft.-lb. Hence, since 1 H. P. is equal to 33,000 ft.-lb. per min., the horsepower is $53{,}293{,}000 \div 33{,}000 = 1{,}614.9$ H. P. Ans.

EXAMPLE 2.—A journal 4 inches in diameter bears a load of 10,000 pounds and makes 80 revolutions per minute. The coefficient of friction is .02. How much heat is produced per hour?

SOLUTION.—The frictional resistance is $10,000 \times .02 = 200$ lb. A point on the circumference travels $3.1416 \times \frac{4}{12} \times 80 \times 60 = 5,026.56$ ft. per hr. The work done against friction is, therefore, $W = 5,026.56 \times 200 = 1,005,312$ ft.-lb., and from the above formula the heat produced is

$$Q = \frac{W}{J} = \frac{1,005,312}{778} = 1,292.2 \text{ B. T. U.} \quad \text{Ans.}$$

48. Three Effects of Heat.—When a quantity of heat is imparted to a body, it, in general, performs three kinds of work:

1. The temperature of the body is raised—that is, the sensible heat is increased; in consequence, the molecules are caused to move at greater speeds and the kinetic energy is increased. The work required to raise the temperature, or what is the same thing, to increase the kinetic energy, is called the **vibration work.**

2. Usually, the heated body expands and, on the whole, the molecules are farther apart than they were before the body was heated. Since the molecules attract each other, work must be expended in moving them farther from each other. After being thus separated, the molecules, when they again approach each other, possess a certain capacity for doing work; hence, the expanded body has a certain potential energy that is due merely to the separation of the molecules. The work required thus to increase the potential energy is called the **disgregation work.**

3. The body, in expanding, must overcome an external pressure through some definite distance and work is thus done against the external pressure. For example, the expansion of the air in the cylinder shown in Fig. 7 causes the piston to rise. Above the piston is the pressure of the atmosphere, and the piston, in rising, does work against this pressure. To the work expended in overcoming external pressure, the name **external work** is given.

49. Equation of Energy.—According to the first law of thermodynamics, the heat absorbed must be precisely equal to the total work done; hence, the heat, in B. T. U. $\times$ 778 = vibration work + disgregation work + external work.

Let K = vibration work, in foot-pounds;
D = disgregation work, in foot-pounds;
W = external work, in foot-pounds;
Q = heat imparted, in B. T. U.

Then, $$JQ = K + D + W \quad (1)$$

K is the increase of the kinetic energy of the body and D is the increase of the potential energy; hence, the sum $K + D$ is the total change of energy. If E_1 denotes the energy possessed by the body originally, and E_2 the energy after the heat is imparted, both expressed in foot-pounds, then $K + D = E_2 - E_1$, and, substituting in formula **1**,

$$JQ = E_2 - E_1 + W \quad (2)$$

that is, the heat imparted to a body is equal to the increase in the energy of the body plus the external work.

50. Formula **1**, Art. **49**, may now be applied under various circumstances.

1. *Heating a Solid Body.*—A solid body expands but little; the disgregation and external works are therefore small and may be neglected when the vibration work is being considered. When heating a solid, as a piece of iron, it is assumed, therefore, that all the heat is used in raising the temperature.

2. *Melting a Solid.*—During the melting of a solid body, there is no change of temperature; hence, the vibration work K becomes zero. Usually, there is little change in the volume during the melting, and the external work is therefore small. Nearly all the heat is expended in disgregation work, which in this case consists in changing from the molecular state of a solid to that of a liquid; that is, in tearing the molecules from their fixed positions relative to each other in the solid and giving them the freedom that molecules have in the liquid state. If the very small external work be neglected, the disgregation work is the equivalent of the latent heat of fusion (see Art. **33**).

3. *Heating a Liquid.*—In heating a liquid, the conditions are the same as in the heating of a solid.

4. *Vaporization of a Liquid.*—There is no change of temperature during vaporization; therefore, $K = 0$, and $JQ = D + W$. Since the volume of the vapor is many times greater than that of the liquid, there is considerable external work, though the larger part of the heat Q is expended in disgregation work.

5. *Heating a Gas.*—In the case of a gas, the molecules are so far apart, considering their size, that their attraction for each other is almost inappreciable, and practically no work is required to separate them farther; hence, the disgregation work is taken as zero, and formula **1**, Art. **49**, becomes

$$JQ = K + W$$

The relative magnitudes of K and W depend on the conditions under which the gas is heated. Suppose that the air in the cylinder, Fig. 7, is being heated. The piston may be held in the original position, in which event the external work W is zero and all the heat is expended in raising the temperature. Or the piston may be raised at such a rate that the decrease of temperature due to the expansion of the air just offsets the increase due to the heating; in this case, the temperature remains constant, $K = 0$, and all the heat is expended in doing external work. The work W may even be negative; thus, imagine the piston to be pushed down while the air is being heated so that work is done by the external pressure instead of against that pressure. As work done by the air against the external pressure has been considered as positive, the work done on the air by the external pressure must be considered as negative. In this case, therefore, $JQ = K - W$, or $JQ + W = K$. The work W assists the heat Q in doing the vibration work K.

51. Abstraction of Heat.—When heat is abstracted or taken from a body, the three works K, D, and W are, in general, negative and the equation of energy takes the form

$$-JQ = (-K) + (-D) + (-W)$$

or

$$-JQ = -K - D - W$$

The negative signs indicate, respectively, that the heat is abstracted instead of added, that the temperature falls, that

the molecules approach each other, and that the body thus gives up potential energy, and that work is done by the external prèssure on the body as the latter contracts.

Under special conditions, one of the quantities may be positive. Take, for example, the case of freezing water; the temperature remains constant; hence, $K = 0$, and, as is well known, water in freezing expands and the work W is therefore positive. The energy equation for this case is therefore

$$-JQ = -D + W$$

As another example, suppose that the piston in Fig. 7 is pushed downwards, so as to compress the air confined below it, and suppose further that the lower part of the cylinder is surrounded by a stream of cold water, which abstracts heat from the air. If the air is compressed very slowly, its temperature will fall and K will be negative; but if the air is compressed quickly, the temperature will rise, notwithstanding the abstraction of heat by the cold water, and K will therefore be positive. In the latter case,

$$-JQ = K - W$$

A thorough understanding of formula **1**, Art. **49**, the energy equation, is very necessary as a preparation for the work that is to follow. Arts. **49**, **50**, and **51** should be thoroughly studied, and the nature of the processes described completely understood.

HEAT

(PART 2)

THERMODYNAMICS OF GASES

FUNDAMENTAL RELATIONS OF GASES

1. General Equation of Gases.—As explained in *Pneumatics*, the relation between the pressure, volume, and temperature of 1 pound of a gas is expressed by the equation

$$p\,V = R\,T$$

and for a weight of G pounds,

$$p\,V = G\,R\,T$$

in which p = absolute pressure, in pounds per square inch;
V = volume, in cubic feet;
R = a constant;
T = absolute temperature;
G = weight, in pounds.

In the formulas given in the following pages, it is convenient to express pressures in pounds per square foot, rather than in pounds per square inch. Therefore, let P equal pressure in pounds per square foot. Then $P = 144\,p$, and

$$P\,V = 144\,p\,V = 144\,R\,T$$

For air, R was given, in *Pneumatics*, as .37; hence, $144\,R = 144 \times .37 = 53.28$, and

$$P\,V = 53.28\,T$$

The symbol R may also be used to denote 53.28, with the understanding that the pressure is to be taken in pounds per square foot.

The other form of the general equation, given in *Pneumatics*, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, will frequently be used in this and succeeding Sections.

2. Specific Heats of a Gas.—From the definition in *Heat*, Part 1, the specific heat of a gas is numerically equal to the quotient obtained by dividing the British thermal units (B. T. U.) imparted to a pound of gas by the rise in temperature; that is, if Q is the B. T. U. imparted; s, specific heat; and t_1 and t_2, the initial and final temperatures, respectively; then

$$s = \frac{Q}{t_2 - t_1}$$

For gases, $JQ = K + W$ (see *Heat*, Part 1); hence,

$$Q = \frac{K + W}{J}, \text{ and } s = \frac{K + W}{J(t_2 - t_1)}$$

in which Q = heat in B. T. U.;
K = vibration work, in foot-pounds;
W = external work, in foot-pounds;
t_1 and t_2 = initial and final temperatures, in degrees Fahrenheit;
J = mechanical equivalent of heat, or 778 foot-pounds per B. T. U.

The specific heat of a gas may have any value, depending on the conditions under which the heat is imparted; for, as has been shown, in *Heat*, Part 1, K and W may be varied at pleasure; either may be made negative; and for a given rise in temperature the sum $K + W$ may be made small or large.

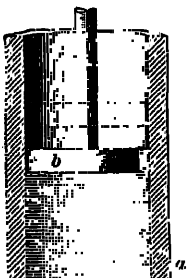


FIG. 1

There are two specific heats of special importance: that at *constant pressure* and that at *constant volume*. Suppose that the cylinder shown at a, Fig. 1, contains air, and that the piston b is free to move. The pressure of the confined air is due only to the weight of the piston and to the pressure of the atmosphere on the piston, and evidently this pressure is the same for all

positions of the piston. As heat is added to the air, the expansion causes the piston to rise to some higher position, as that shown by the dotted lines, and external work is done. The amount of this external work may now be calculated. Suppose the weight of the confined air to be 1 pound, and let V_1 denote the volume before heating and V_2 the volume after heating. The constant pressure is P pounds per square foot. If A is the area in square feet, PA is the total pressure of the piston on the air; and if h is the distance the piston rises, in feet, the external work done is

$$W = P A h \text{ foot-pounds}$$

But $A h$ is the volume swept through by the piston and is, therefore, the increase in the volume of the air; hence,

$$A h = V_2 - V_1$$

and

$$W = P(V_2 - V_1) \qquad (1)$$

From the general equation, then,

$$P V_1 = R T_1$$

and

$$P V_2 = R T_2$$

where T_1 and T_2 denote, respectively, the initial and final absolute temperatures. Subtracting the first from the second,

$$P V_2 - P V_1 = R T_2 - R T_1, \text{ or } P(V_2 - V_1) = R(T_2 - T_1)$$

which is also equal to W; that is, the work done by 1 pound of air when heated at constant pressure is R times the rise in temperature.

Let c_p denote the specific heat at constant pressure; then,

$$c_p = \frac{K}{J(t_2 - t_1)} + \frac{R}{J} \qquad (2)$$

$t_2 - t_1$ being the rise in temperature.

For the derivation of formula **2**, see Appendix I at the end of this Section. Values of c_p for various gases are given in *Heat*, Part 1.

Suppose that the piston in Fig. 1 is held in one position, so that the air cannot expand, but must retain the same volume, and that heat is added. Under this condition, there is no external work done, that is, $W = 0$. Let the specific heat at constant volume be denoted by c_v; then,

$$c_v = \frac{K + W}{J(t_2 - t_1)} = \frac{K}{J(t_2 - t_1)} \qquad (3)$$

3. Relation Between Specific Heats at Constant Pressure and Constant Volume.—Comparing the expressions for c_p and c_v in Art. **2**, it appears that c_p is the larger. That this must be the case is evident, for in heating at constant pressure part of the heat is used in doing external work, and therefore more heat is required for the same rise in temperature.

For a given rise in temperature $(t_2 - t_1)$, the change of energy K, which depends only on the change of temperature, has a definite value; hence, $\frac{K}{J(t_2 - t_1)}$, in formula **3**, Art. **2**, is the same as in formula **2**, and subtracting formula **3** from formula **2**,

$$c_p - c_v = \left[\frac{K}{J(t_2 - t_1)} + \frac{R}{J}\right] - \frac{K}{J(t_2 - t_1)} = \frac{R}{J}$$

For air, Regnault found the value of c_p to be .23751; hence, $.23751 - c_v = \frac{53.28}{778}$, and

$$c_v = .23751 - \frac{53.28}{778} = \frac{.23751 \times 778 - 53.28}{778} = .16902$$

The ratio $\frac{c_p}{c_v}$ is frequently used, and is denoted by k.

For air, $k = \frac{.23751}{.16902} = 1.4052$, say 1.405. The constant k has been determined experimentally. The value thus found agrees closely with the value given above.

4. Change of Energy of a Gas.—From *Heat*, Part 1, the change of energy in the gas, sometimes called the change of **intrinsic energy**, is, in general,

$$E_2 - E_1 = K + D$$

in which E_1 and E_2 = initial and final energies of the gas, in foot-pounds;
K = vibration work, in foot-pounds;
D = disgregation work, in foot-pounds.

For a gas, however, the disgregation work is practically zero, and

$$E_2 - E_1 = K$$

But, from formula **3**, Art. **2**, $K = Jc_v(t_2 - t_1)$ for 1 pound of gas; hence, for G pounds,

$$E_2 - E_1 = Jc_v G(t_2 - t_1) = Jc_v G(T_2 - T_1) \qquad (1)$$

Experiments have shown that when a gas expands without doing external work, the temperature remains practically unchanged. To be exact, there is a very slight change of temperature, owing to the small amount of heat required to do the disgregation work; but for a gas, as already stated, the disgregation work may be neglected.

EXAMPLE 1.—Six pounds of air is heated at constant volume from 60° F. to 83° F.; what is the increase of energy?

SOLUTION.— $E_2 - E_1 = 778 \times .16902 \times 6 \times (83 - 60) = 18{,}147$ ft.-lb., nearly. Ans.

A second formula for the change of energy is

$$E_2 - E_1 = \frac{P_2 V_2 - P_1 V_1}{k - 1} \qquad (2)$$

If the pressures are expressed in pounds per square inch,

$$E_2 - E_1 = \frac{144(p_2 V_2 - p_1 V_1)}{k - 1} \qquad (3)$$

For derivation of formula **2**, see Appendix II. In most cases, formula **2** is more convenient than formula **1**, as the weight of the air need not be known.

EXAMPLE 2.—Air confined in a cylinder has a volume of 15 cubic feet and a pressure of 60 pounds per square inch, absolute; the air is heated and expands to a volume of 22 cubic feet, and the pressure is 55 pounds per square inch. What is the change of energy of the air?

SOLUTION.— $E_2 - E_1 = \dfrac{144(55 \times 22 - 60 \times 15)}{1.405 - 1} = 110{,}222$ ft.-lb. Ans.

EXPANSION OF GASES

5. A gas may pass from one state to another in a number of ways; in practice, however, the expansion of a gas takes place according to one of a few well-defined laws. The most common forms of expansion are the following:

1. *Expansion at constant pressure.*
2. *Isothermal expansion,* during which the temperature remains constant.

3. *Adiabatic expansion*, in which the gas expands without receiving heat from or giving up heat to any external body.

4. *Expansion according to the law* $P V^n = c =$ a constant, in which the gas expands in such a way that the pressure always varies inversely as the *n*th power of the volume.

In connection with the expansion of a gas from some initial state to a second state, according to any given law, it is desired to know the following: (1) The external work done; (2) the change of energy in the gas; (3) the heat added to or abstracted from the gas; (4) the relations between pressure and volume, pressure and temperature, and volume and temperature, respectively.

It is desirable to take up, in order, the four expansions just noted and derive expressions for external work, heat added, etc. The derivation of some of the formulas cannot be accomplished except by the use of higher mathematics; in these cases the formulas must be taken for granted.

EXPANSION AT CONSTANT PRESSURE

6. The expansion of a gas when the pressure remains constant may be represented, graphically, in the way shown in Fig. 2. From any point O, the line OP is drawn vertically and the line OV horizontally. From O, a length OA_1 is laid off to represent, to some scale, the initial volume of the gas, and from A_1 the line A_1A is drawn parallel to OP, and of such a length as to represent, to some scale, the initial pressure of the gas. The point A is said to represent the initial state of the gas, as regards pressure and volume; its distance from OP represents the volume and its distance from OV represents the pressure. As the gas changes its state, the point representing the state must move; thus, if the volume increases, it must move away

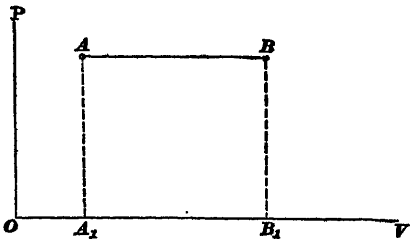


FIG. 2

from OP so that its perpendicular distance from OP continually represents the volume; while, if the pressure increases, it must move away from OV, and vice versa.

In the case under consideration, the volume increases but the pressure remains the same; hence, the point must move away from OP, but it must remain at the same distance from OV; that is, it must move along AB parallel to OV. The point B is located by making OB_1 equal to the final volume, and drawing B_1B perpendicular to OV.

As shown by formula **1**, Art. **2**, the external work is the product of the pressure and the increase in volume; that is,

$$W = P(V_2 - V_1) \qquad (1)$$

in which V_1 = initial volume;
V_2 = final volume.

In Fig. 2, the width A_1B_1 represents the increase of volume, $V_2 - V_1$, and the height $A_1A = B_1B$, the constant pressure P; hence the area of the rectangle A_1ABB_1 represents the external work.

The change of energy is given by either formula **1** or formula **2**, Art. **4**. These formulas are of general application and hold good for all expansions or changes of state of a perfect gas.

The heat added per pound of gas during the expansion is evidently the product of the specific heat and the rise in temperature, that is, $c_p(T_2 - T_1)$. For G pounds,

$$Q = Gc_p(T_2 - T_1) \qquad (2)$$

Another expression for the heat added is as follows:

$$JQ = E_2 - E_1 + W = \frac{k}{k-1}(PV_2 - PV_1) \qquad (3)$$

This equation has the advantage that the weight G need not be known; for its derivation, see Appendix III.

To obtain the relation between the volume and the temperature, the general equation $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, from *Pneumatics*, is used. Since $P_1 = P_2$ (the pressure being constant), $\frac{V_1}{T_1} = \frac{V_2}{T_2}$; that is, the volume varies directly as the absolute temperature.

EXAMPLE.—Air having a volume of 5 cubic feet, a pressure of 60 pounds per square inch, absolute, and a temperature of 40° F., expands at constant pressure until the volume is 8 cubic feet. Compute: (*a*) the final temperature; (*b*) the external work; (*c*) the change in energy; (*d*) the heat imparted during the expansion.

SOLUTION.—(*a*) $T_1 = 460 + 40 = 500$, $V_1 = 5$, and $V_2 = 8$.

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$, hence $\frac{5}{500} = \frac{8}{T_2}$, or $T_2 = \frac{8 \times 500}{5} = 800$, and $t_2 = 800 - 460 = 340°$ F. Ans.

(*b*) $P = 144p = 144 \times 60 = 8{,}640$ lb. per sq. ft.
$W = P(V_2 - V_1) = 8{,}640 \times (8 - 5) = 25{,}920$ ft.-lb. Ans.

(*c*) By formula **2**, Art. **4**, and remembering that $P_2 = P_1$, $E_2 - E_1 = \frac{P(V_2 - V_1)}{k - 1} = \frac{8{,}640 \times (8 - 5)}{1.405 - 1} = 64{,}000$ ft.-lb., nearly. Ans.

(*d*) By formula **3**, $JQ = E_2 - E_1 + W = 64{,}000 + 25{,}920 = 89{,}920$ ft.-lb., and $Q = \frac{89{,}920}{778} = 115.58$ B. T. U. Ans.

ISOTHERMAL EXPANSION

7. **In isothermal expansion**, gas expands at constant temperature; then, in the general formula $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, $T_1 = T_2$, and in consequence $P_1 V_1 = P_2 V_2$; that is, the gas in expanding follows Boyle's law (see *Pneumatics*). The graphic representation of this expansion is shown in Fig. 3. As the expansion progresses, the volume of the gas increases and the pressure falls; hence, starting from the point A, which represents the initial pressure and volume, as in Fig. 2, the moving point recedes from OP as the volume increases, and approaches OV as the pressure decreases. As explained in *Pneumatics*, the curve traversed must be such that the product of the perpendicular distances

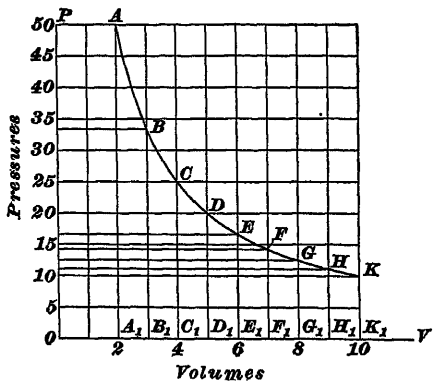


FIG. 3

of any point from OP and OV is the same as the corresponding product for any other point. Hence, as the volume and pressure are known for any state, the pressure may be found for any assumed volume or the volume for any pressure. This enables one to plot the curve by finding points that indicate the different pressures and volumes.

8. Equilateral Hyperbola.—The curve shown in Fig. 3 is called the **isothermal expansion curve**, or the **expansion curve of constant temperature.**

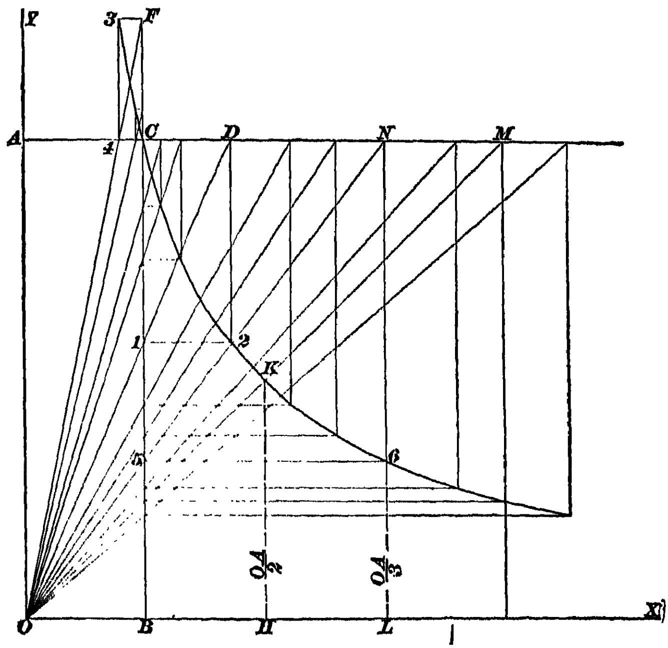


FIG. 4

It is known in mathematics as the **equilateral hyperbola**, and, hence, when used on indicator diagrams, is sometimes called the **hyperbolic expansion curve.** If the initial volume, pressure, and final volume are known, the curve may be constructed graphically without calculating the different points, as was done in Fig. 3. Thus, in Fig. 4, let OY and OX be two lines at right angles to each other. These lines are

known in mathematics as the **coordinate axes**, the line OY being called the **axis of ordinates**, or **axis of Y**, and the line OX, the **axis of abscissas**, or **axis of X**. Let OA represent the absolute initial pressure and OB the initial volume. Through A draw the indefinite straight line AM parallel to the axis OX, and through B draw the indefinite straight line BF parallel to the axis OY. The point C, where these two lines meet, is the point where the expansion is to begin; consequently. it is one point on the curve. Through the point O, called the **origin**, which is the point of no volume and no pressure, draw a number of lines, OF, OD, ON, OM, etc., cutting BF at F, *1*, *5*, etc., and AM at *4*, D, N, etc. Through the points F, *1*, *5*, etc. draw lines parallel to the axis OX, and through *4*, D, N, etc. draw lines parallel to the axis OY. These lines intersect in the points *3*, *2*, *6*, etc., which are points on the required isothermal expansion line. To prove this, lay off BH equal to OB, and draw HK parallel to the axis OY, intersecting the curve in K. Now, if K is a point on the isothermal expansion line, HK must be equal in length to one-half of OA, since, when the volume is twice as great, the pressure is only half as great. Similarly, if $HL = BH = OB$, $L6$ must be one-third as long as OA. By measurement, this will be found to be the case. This curve and this method of constructing it are much used in "working up" indicator diagrams.

9. External Work.—The **external work** done during the isothermal expansion is represented by the area $A A_1 K_1 K$, Fig. 3, under the curve AK. The following formula, which is developed by the use of higher mathematics, represents the work done during this isothermal expansion:

$$W = P_1 V_1 \log_e \frac{V_2}{V_1} \qquad (1)$$

in which V_1 and V_2 are the initial and final volumes, P_1 is the initial pressure in pounds per square foot, and $\log_e$ denotes the *hyperbolic logarithm*. The hyperbolic logarithm differs from the common logarithm, in that the base 2.71828+ is used instead of 10. It is apparent therefore, that the

logarithms of the same number in these two systems must have a definite relation to each other. This relation is expressed by the factor 2.3026, by which the common logarithm, explained in *Logarithms*, must be multiplied to obtain the hyperbolic logarithm. For work not requiring great accuracy, the factor 2.3 is often used. In formulas containing a logarithm derived by means of the higher mathematics, the hyperbolic logarithm is generally used. When it is desired to use the common logarithm, therefore, *2.3026 log* must be substituted for log_e. Formula **1** is then written

$$W = 2.3026\ P_1\ V_1 \log \frac{V_2}{V_1} \qquad (2)$$

In the following formulas, the common logarithm will be used. When the hyperbolic logarithm is used, it is always indicated by the subscript e, as shown in formula **1**.

EXAMPLE 1.—Find, by the exact method, formula **2**, the external work done in the expansion represented in Fig. 3.

SOLUTION.— $P_1 = 144\ p_1 = 144 \times 50 = 7{,}200$; $V_1 = 2$, $V_2 = 10$, and $\frac{V_2}{V_1} = 5$.

$$W = 2.3026 \times 7{,}200 \times 2 \times \log 5 = 2.3026 \times 7{,}200 \times 2 \times .69897$$
$$= 23{,}176 \text{ ft.-lb.} \quad \text{Ans.}$$

Since $P_1\ V_1 = P_2\ V_2$, $\frac{V_2}{V_1} = \frac{P_1}{P_2}$, and formula **2** may be written in the form

$$W = 2.3026\ P_1\ V_1 \log \frac{P_1}{P_2} \qquad (3)$$

EXAMPLE 2.—Air having an absolute pressure of 44 pounds per square inch and a volume of 3.75 cubic feet expands isothermally until the pressure reaches that of the atmosphere, 14.7 pounds per square inch; what is the external work?

SOLUTION.— $W = 2.3026 \times 44 \times 144 \times 3.75 \times \log \frac{44}{14.7}$. In the quotient $\frac{P_1}{P_2}$, pressures in pounds per square inch may be used for $\frac{P_1}{P_2} = \frac{144\,p_1}{144\,p_2} = \frac{p_1}{p_2}$. To obtain $\log \frac{44}{14.7}$, it is most convenient to take the logarithms of the numerator and denominator separately; thus,

$$\log \frac{44}{14.7} = \log 44 - \log 14.7 = 1.64345 - 1.16732 = .47613,$$

and $W = 2.3026 \times 44 \times 144 \times 3.75 \times .47613 = 26{,}049$ ft.-lb. Ans.

10. Change of Energy and Heat Imparted.—During isothermal expansion, there is no change in the energy of the gas; for, from formula **1**, Art. **4**, $E_2 - E_1 = Jc_v G(t_2 - t_1)$; and as the temperature remains the same, $t_2 - t_1 = 0$, and $E_2 - E_1 = 0$.

The heat added during the expansion is obtained by the general formula in *Heat*, Part 1; or, $JQ = (E_2 - E_1) + W = 0 + W = W$; that is to say, *the heat imparted to the gas is equivalent to the external work.* That this must be true is also evident from the general principles stated in *Heat*, Part 1. Of all the heat supplied to the gas, there is none needed for vibration work, because there is no rise in temperature; none is needed for disgregation work, because the substance is a gas; therefore, the whole must be expended in the performance of external work.

EXAMPLE.—In example 2 of Art. **9**, what amount of heat is imparted to the gas during the expansion?

SOLUTION.— $JQ = W = 26{,}049$ ft.-lb.

$$Q = \frac{W}{J} = \frac{26{,}049}{778} = 33.48 \text{ B. T. U.} \quad \text{Ans.}$$

ADIABATIC EXPANSION

11. Suppose a quantity of gas to be confined in the cylinder, Fig. 1, and that its pressure is greater than the atmospheric pressure on the upper side of the piston; and suppose further that the cylinder and piston are made of some non-conducting material, so that no heat can be imparted to or can escape from the contained gas. Because the upward pressure against the piston is greater than the downward pressure, the piston will rise, the gas will expand, and in so doing will perform external work. An expansion of this kind, in which the gas neither receives heat from nor gives up heat to an external body, but does external work, is called an **adiabatic expansion.**

12. Change of Energy and Work Performed.—Let the general equation of energy, $JQ = E_2 - E_1 + W$, be applied to the case of an adiabatic expansion. As no heat

is imparted to or abstracted from the gas, $Q = 0$; hence,

$$0 = E_2 - E_1 + W, \text{ or } W = -(E_2 - E_1) = E_1 - E_2$$

This expression shows that during an adiabatic expansion the energy decreases from an initial value E_1 to a final value E_2, and that the external work done is equal to this decrease of energy.

Using the expressions for $E_2 - E_1$ given in formulas **1** and **2**, Art. **4**,

$$W = -J c_v G (T_2 - T_1) = J c_v G (T_1 - T_2) \qquad \textbf{(1)}$$

and
$$W = -\frac{P_2 V_2 - P_1 V_1}{k-1} = \frac{P_1 V_1 - P_2 V_2}{k-1} \qquad \textbf{(2)}$$

Since the gas gives up energy while expanding, it follows that its temperature falls as the expansion progresses and that T_1 is greater than T_2.

EXAMPLE.—A mass of confined air weighing .84 pound and having a temperature of 100° F. expands adiabatically until the temperature drops to 30° F. (*a*) What is the external work? (*b*) What is the loss of energy?

SOLUTION.— $T_1 - T_2 = t_1 - t_2 = 100 - 30 = 70$. Using formula **1**,

(*a*) $W = 778 \times .16902 \times .84 \times 70 = 7{,}732.1$ ft.-lb. Ans.

(*b*) $E_1 - E_2 = W = 7{,}732.1$ ft.-lb. Ans.

13. Relation Between Pressure and Volume.—Suppose that there are two cylinders, each containing 1 pound of air under the same conditions as regards pressure, volume, and temperature. The state of the gas as regards pressure and volume is indicated by the point A, Fig. 5; $O A_1$ represents the initial volume V_1, and $A_1 A$ the initial pressure.

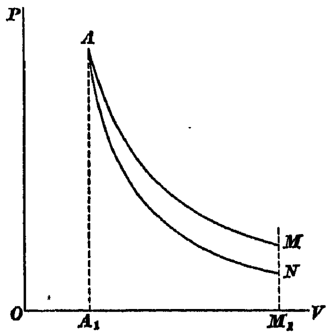


FIG. 5

Let the gas in one of the cylinders expand isothermally. The expansion then follows the law $P_1 V_1 = P_2 V_2 = P_3 V_3$, etc., and the final pressure $M_1 M$ is obtained from the general equation $P_2 V_2 = R T_2 = R T_1$, since $T_2 = T_1$; hence, $P_2 = \frac{R T_1}{V_2}$. Let the gas in the other cylinder expand adiabatically from the initial volume V_1 to the final volume V_2.

The final temperature T_2 is less than the initial T_1; therefore, the final pressure $M_1 N$, which is $P_2 = \frac{R\,T_2}{V_2}$, must be less than the final pressure, $P_2 = \frac{R\,T_1}{V_2}$, attained in the isothermal expansion. Starting from the same point A, the curve representing adiabatic expansion therefore lies wholly below that representing isothermal expansion.

For the isothermal expansion, the pressure varies inversely as the volume. For the adiabatic expansion, a different law is followed; the pressure varies inversely as some power of the volume. It can be shown by higher mathematics that the power is $k = \frac{c_p}{c_v}$, which, by Art. **3**, is equal to 1.405 for a perfect gas.

Then, $$P_1 : P_2 = V_2^k : V_1^k$$

and $$P_1 V_1^k = P_2 V_2^k \qquad (1)$$

or $$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^k \qquad (2)$$

As $\frac{P_1}{P_2} = \frac{p_1}{p_2}$, formula **2** may be written $\left(\frac{V_2}{V_1}\right)^k = \frac{p_1}{p_2}$, whence

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} \qquad (3)$$

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} \qquad (4)$$

These formulas may be used to compute the final pressure or volume, as illustrated by the following examples:

EXAMPLE 1.—A mass of confined air having a volume of 20 cubic feet and a pressure of 80 pounds per square inch, absolute, expands adiabatically until the pressure has fallen to 38 pounds per square inch, absolute. (*a*) What is the final volume? (*b*) Calculate the external work done.

SOLUTION.—(*a*) From formula **4**, $V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.405}}$. The calculation evidently must be made with the aid of logarithms.

Then, $$\log V_2 = \log\left[V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.405}}\right] = \log V_1 + \log\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.405}}$$

$$= \log V_1 + \frac{\log p_1 - \log p_2}{1.405} = \log 20 + \frac{\log 80 - \log 38}{1.405}$$

$$= 1.30103 + \frac{1.90309 - 1.57978}{1.405} = 1.53114$$

The number whose logarithm is 1.53114 is 33.97; therefore, V_2 = 33.97 cu. ft. Ans.

(*b*) Using formula **2**, Art. **12**,

$$W = \frac{144(80 \times 20 - 38 \times 33.97)}{1.405 - 1} = 109{,}916 \text{ ft.-lb. Ans.}$$

EXAMPLE. 2—If the air in example 1 expands until the final volume is 60 cubic feet, what is the final pressure?

SOLUTION.—By formula **2**, $\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^k$, from which $p_2 = p_1 \left(\frac{V_1}{V_2}\right)^k$ $= 80\left(\frac{20}{60}\right)^{1.405}$. Using logarithms,

$$\log p_2 = \log 80 + 1.405(\log 20 - \log 60)$$
$$= 1.90309 + 1.405(1.30103 - 1.77815) = 1.23274.$$

Hence, p_2 = 17.1 lb. per sq. in., nearly. Ans.

14. Relation Between Volume and Temperature. The general equation, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ (see *Pneumatics*), holds good for all expansions and changes of state. Clearing it of fractions,

$$P_1 V_1 T_2 = P_2 V_2 T_1 \qquad (1)$$

But

$$P_1 V_1^k = P_2 V_2^k \qquad (2)$$

in the case of adiabatic expansion. Dividing the second equation by the first, $\frac{V_1^k}{V_1 T_2} = \frac{V_2^k}{V_2 T_1}$, or $\frac{V_1^{k-1}}{T_2} = \frac{V_2^{k-1}}{T_1}$

whence,

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \left(\frac{V_2}{V_1}\right)^{.405} \qquad (3)$$

EXAMPLE.—Air at a temperature of 120° F. expands adiabatically from a volume of 20 cubic feet to a volume of 30 cubic feet; what is the temperature at the end of the expansion?

SOLUTION.—From formula **3**, $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{.405}$. $T_1 = 460 + 120 = 580$, and $\left(\frac{V_1}{V_2}\right)^{.405} = \left(\frac{20}{30}\right)^{.405}$. Then, $T_2 = 580\left(\frac{20}{30}\right)^{.405}$. Using logarithms,

$$\log T_2 = \log 580 + .405(\log 20 - \log 30)$$
$$= 2.76343 + .405(1.30103 - 1.47712) = 2.69211.$$
$$T_2 = 492.17° \text{ and } t_2 = 492.17 - 460 = 32.17° \text{ F. Ans.}$$

15. Relation Between Pressure and Temperature. As in Art. **14**,

$$P_1 V_1 T_2 = P_2 V_2 T_1 \qquad (1)$$

and for adiabatic expansion,

$$P_1 V_1^k = P_2 V_2^k \qquad (2)$$

Taking the kth root of both members of formula **2**,

$$(P_1)^{\frac{1}{k}} V_1 = (P_2)^{\frac{1}{k}} V_2 \qquad (3)$$

Dividing formula **1** by formula **3**, member by member,

$$\frac{P_1 T_2}{(P_1)^{\frac{1}{k}}} = \frac{P_2 T_1}{(P_2)^{\frac{1}{k}}}, \text{ or } (P_1)^{1-\frac{1}{k}} T_2 = (P_2)^{1-\frac{1}{k}} T_1$$

and from this

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{1-\frac{1}{k}} = \left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} \qquad (4)$$

Substituting for k its value, 1.405, formula **4** becomes,

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{.405}{1.405}} = \left(\frac{P_1}{P_2}\right)^{.2883} \qquad (5)$$

or,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{.2883} \qquad (6)$$

Formulas **4**, **5**, and **6** are homogeneous, and the pressures may, therefore, be taken either in pounds per square foot or in pounds per square inch.

EXAMPLE.—A mass of confined air at a pressure of 60 pounds per square inch, absolute, and a temperature of 140° F. expands adiabatically until the pressure falls to 20 pounds per square inch; calculate the final temperature.

SOLUTION.—By formula **6**, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{.2883}$, and $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{.2883}$. $T_1 = 460 + 140 = 600$. Therefore, $T_2 = 600\left(\frac{20}{60}\right)^{.2883}$. Log T_2 $= \log 600 + .2883(\log 20 - \log 60) = 2.64060$. $T_2 = 437.12$, and $t_2 = 437.12 - 460 = -22.88°$ F. Ans.

EXPANSION ACCORDING TO THE LAW PV^n = A CONSTANT

16. The isothermal and adiabatic expansions are special or limiting cases of the more general expansion, in which the relation between pressure and volume as the expansion progresses follows the law PV^n = a constant, or $P_1 V_1^n = P_2 V_2^n = P_3 V_3^n$, etc.

For $n = 1$, the formula gives the isothermal case, in which sufficient heat is supplied to the gas to do the external work and there is no change in the energy. For $n = k = 1.405$, it gives the adiabatic case, in which there is no heat supplied

to the gas and the external work is done wholly at the expense of the energy of the gas.

Between these extremes, there are any number of expansions depending on the amount of heat supplied to the gas. Thus, if the heat imparted is enough to do one half the external work, the gas must give up enough energy to do the other half of the work; and, while the temperature falls, it does not fall as much as in the adiabatic case, in which the gas must give up enough energy to do all the external work. For this case, therefore, the curve representing the expansion would lie between the adiabatic and isothermal, Fig. 5, and the exponent n would lie between 1 and 1.405.

It is possible to imagine cases in which n does not lie between 1 and 1.405, but these rarely occur in engineering practice. Thus, if the heat imparted is more than sufficient to do the external work, n will be less than 1, and the curve of the expansion will rise above the isothermal curve; on the other hand, if the gas gives up more energy than enough to do the external work, heat is abstracted from it, n is greater than 1.405, and the curve lies below the adiabatic.

17. External Work.—The following general formula for the external work done when the expansion follows the law $P_1 V_1^n = P_2 V_2^n$ is derived by the use of higher mathematics:

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1}.$$

This formula is true for any expansion in which the value of n is greater than 1. It will be seen that it differs from the work formula for adiabatic expansion, formula **2**, Art. **12**, only in that n is substituted for k.

18. Heat Added to a Gas.—The change of energy is expressed by the formulas

$$JQ = (P_1 V_1 - P_2 V_2)\left(\frac{1}{n-1} - \frac{1}{k-1}\right) \qquad (1)$$

and

$$Q = G c_v \frac{n-k}{n-1}(T_2 - T_1) \qquad (2)$$

For the derivation of formulas **1** and **2**, see Appendix IV.

Formulas **1** and **2** must, of course, give the same result. In cases arising in practice, it is usually more convenient to apply formula **1**, since the pressures and volumes are usually known rather than the temperatures and the weight of the gas.

In *Heat*, Part 1, the heat imparted to any substance is shown to be

$$Q = G s (T_2 - T_1)$$

in which T_1 and T_2 = the initial and final absolute temperatures;

s = specific heat;

G = weight.

Comparing this formula with formula **2**, it is seen that

$$s = c_v \frac{n - k}{n - 1} \qquad (3)$$

Formula **3**, therefore, gives the specific heat of a gas expanding according to the law $p v^n$ = a constant.

For values of n lying between 1 and k, that is, between 1 and 1.405, s is negative. Thus, suppose $n = 1.2$; $s = c_v \frac{1.2 - 1.405}{1.2 - 1} = -1.025\, c_v$. For air, $c_v = .16902$ and $s = -1.025 \times .16902 = -.17325$. The negative specific heat simply signifies that the temperature falls rather than rises as heat is added.

19. Ratio of Change of Energy to External Work. For a given value of n, the change of energy of the gas is some definite percentage of the external work. Since there is usually a decrease of energy during expansion, let the decrease equal C times the external work, that is

$$E_1 - E_2 = C W \qquad (1)$$

Inserting the values of $E_1 - E_2$ and W from formula **2**, Art. **4**, and the formula of Art. **17**, respectively,

$$\frac{P_1 V_1 - P_2 V_2}{k - 1} = C \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

whence

$$C = \frac{n - 1}{k - 1} \qquad (2)$$

and

$$n = C (k - 1) + 1 \qquad (3)$$

EXAMPLE 1.—Air expanding according to the law $PV^{1.25}$ = a constant, performs 13,500 foot-pounds of external work. (*a*) What amount of energy is given up by the air? (*b*) How much heat is imparted to the air during the expansion?

SOLUTION.— $C = \frac{n-1}{k-1} = \frac{1.25-1}{1.405-1} = .6173.$

(*a*) $E_1 - E_2 = CW = .6173 \times 13{,}500 = 8{,}333.6$ ft.-lb. Ans.

(*b*) The energy imparted is equal to the difference between the work done and the energy given up by the air, or, 13,500 − 8,333.6 = 5,166.4 ft.-lb. Then,

$$JQ = 5{,}166.4, \text{ and } Q = \frac{5{,}166.4}{778} = 6.64 \text{ B. T. U. Ans.}$$

EXAMPLE 2.—Air expands in such a way that two-thirds of the external work is done by the energy given up, and one-third by the heat imparted to the gas; what is the law of the expansion?

SOLUTION.—Here $C = \frac{2}{3}$. Using formula **3**,

$$n = C(k-1) + 1 = \tfrac{2}{3} \times .405 + 1 = 1.27$$

Hence, the air expands according to the law $PV^{1.27}$ = a constant, or $P_1 V_1^{1.27} = P_2 V_2^{1.27}$. Ans.

Referring to formula **2**, it is seen that when $n = 1$, the isothermal case, $C = 0$, showing that there is no change in energy; when $n = k$, the adiabatic case, $C = 1$, showing that all the work is furnished by the decrease of energy.

20. Relations Between Volume and Temperature, and Pressure and Temperature.—The formulas giving the ratio of temperatures for a given ratio of volumes or of pressures have the same form as formula **3** of Art. **14**, and formula **4** of Art. **15**; the only change is the substitution of n for k. Thus

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1} = \left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}$$

21. Illustrative Example.—The solution of the following example illustrates the principles developed in the preceding articles:

EXAMPLE.—A mass of confined air has a pressure of 60 pounds per square inch, absolute, a volume of 8 cubic feet, and a temperature of 72° F.; it expands according to the law $PV^{1.2}$ = a constant, until the final pressure reaches 35 pounds per square inch. Find: (*a*) the final volume; (*b*) the final temperature; (*c*) the external work done; (*d*) the change of energy; (*e*) the heat imparted during the expansion.

SOLUTION.—(*a*) From the law of the expansion $P_1 V_1^{1.2} = P_2 V_2^{1.2}$,
$V_2^{1.2} = V_1^{1.2}\left(\frac{P_1}{P_2}\right)$, or $V_2 = V_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{1.2}} = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.2}}$, and
$\log V_2 = \log V_1 + \frac{1}{1.2}(\log p_1 - \log p_2) = \log 8 + \frac{\log 60 - \log 35}{1.2} = 1.09816$;
hence, $V_2 = 12.536$ cu. ft. Ans.

(*b*) From the formula in Art. **20**, $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$. $T_1 = 460 + 72 = 532$, and $n = 1.2$; hence, $T_2 = 532\left(\frac{35}{60}\right)^{\frac{.2}{1.2}}$. Log $T_2 = \log 532 + \frac{.2}{1.2}(\log 35 - \log 60) = 2.68690$. $T_2 = 486.3°$ and $t_2 = 486.3 - 460 = 26.3°$. Ans.

(*c*) To find the work, use the formula in Art. **17**,

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{144(60 \times 8 - 35 \times 12.536)}{1.2 - 1} = 29{,}693 \text{ ft.-lb.}$$

Ans.

(*d*) From formula **2** of Art. **19**, $C = \frac{n-1}{k-1} = \frac{1.2-1}{1.405-1} = \frac{.2}{.405}$; and by formula **1** of Art. **19**,

$$E_1 - E_2 = C\,W = \frac{.2}{.405} \times 29{,}693 = 14{,}663 \text{ ft.-lb.} \quad \text{Ans.}$$

(*e*) From the general formula,

$$J\,Q = E_2 - E_1 + W = W - (E_1 - E_2) = 29{,}693 - 14{,}663$$
$$= 15{,}030 \text{ ft.-lb.};$$

hence, $Q = 15{,}030 \div 778 = 19.32$ B. T. U. Ans.

COMPRESSION OF GASES

22. The behavior of a gas when compressed according to some law is precisely the reverse of its behavior when expanding according to the same law. Let a given weight of gas expand from some initial state represented by the point A, Fig. 6, to some second state represented by the point B. The form of the curve $A\,B$ is determined by the law of the expansion, as expressed by the formula $P_1 V_1^n = P_2 V_2^n$, in which n lies between 1 and k. The characteristics of an expansion following this law are here given:

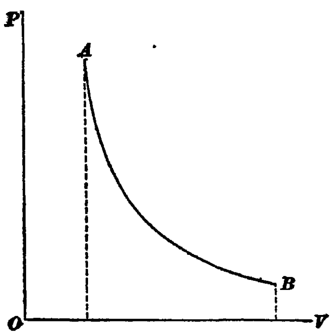


FIG. 6

The pressure falls as the volume increases.

The temperature falls.

External work is done by the gas.

The gas loses part of its energy.

Heat is imparted to the gas.

If the gas is compressed from the state B back to the state A along the same curve, that is, according to the same law, the following take place:

The volume decreases and the pressure rises.

The temperature rises.

External work is done on the gas.

The energy of the gas is increased.

Heat is abstracted from the gas.

In the case of the expansion, the initial state is represented by the point A; the initial conditions are distinguished by the subscript *1*, thus: P_1, V_1, and T_1. The final state is represented by B, and the pressure, volume, and temperature at this state are distinguished by the subscript *2*, thus: P_2, V_2, T_2.

In compressing the gas, B is taken as the initial state and A as the final state; the initial state B is distinguished by the subscript *1* and the final state A by the subscript *2*. Then all the formulas derived for the expansion of the gas hold good also for the compression, and the results will have proper signs. For example, the formula for external work, in Art. **17**, is

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1};$$

for expansion, P_1 and V_1 refer to the state A, and P_2 and V_2 to the state B, and if n is greater than 1, the product for A is the larger; that is, $P_1 V_1$ is greater than $P_2 V_2$, and the work W is positive. For compression, P_1 and V_1 refer to the state B, and P_2 and V_2 to the state A; hence, $P_2 V_2$ is greater than $P_1 V_1$, $P_1 V_1 - P_2 V_2$ is negative, and the result obtained for W is therefore negative. This is as it should be, for if the work done by the air during expansion is considered positive, the work done on the air during compression must be considered negative.

23. Compression of Air.—Probably the most important application of the principles of the thermodynamics of gases is found in the compression of air. The action of the ordinary air compressor is described in *Pneumatics*, which description should now be reviewed. In Fig. 7 is shown the compression cylinder with the piston at one end of the stroke. The cylinder is filled with free air, that is, air at atmospheric pressure. The volume of the air, which is equal to the volume swept through by the piston, is denoted by V_1, and the initial pressure, in pounds per square foot, by P_1. The state of the gas as regards pressure and volume is represented, with reference to the axes OP and OV, by the point A. OF is the length of the stroke of the piston, and may be conveniently taken to represent the volume V_1, and FA represents to some scale the pressure P_1.

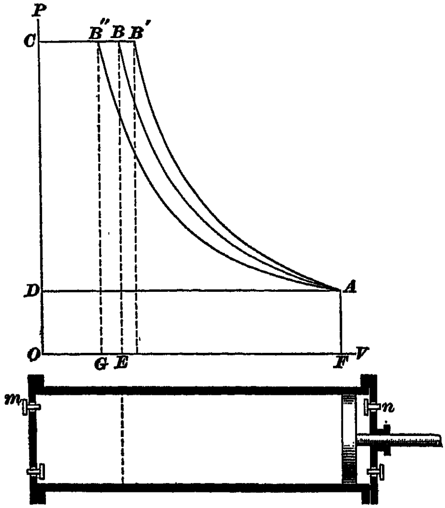


FIG. 7

As the piston moves to the left, the air is compressed and the pressure rises. If corresponding pressures and volumes are laid off from OV and OP, the points representing the successive states of the gas during the compression will lie on a curve AB. When the pressure EB is reached, the outlet valve m is forced open and the air is discharged into a receiver. The receiver is so large that the pressure is not raised appreciably by the addition of this air; hence, the pressure remains practically the same while the air is being forced from the cylinder. The line BC represents this operation. At B, the air has the volume CB and the pressure EB; since this pressure remains the same, the point representing the state of the gas must

move from B parallel to OV and approach OP. The volume CB is denoted by V_2, and the pressure EB by P_2.

The work of compressing the air from the state A to the state B is represented by the area $ABEF$ under the curve AB; and the work of pushing the air into the receiver after it has reached the state B is represented by the area of the rectangle $BCOE$ under BC. But as soon as the piston begins to move, air enters the cylinder through the valve n and exerts a pressure on the right-hand side of the piston and does work on the piston. The pressure of the air is P_1, represented by AF, and the work done on the piston is the product of this pressure by the volume swept through by the piston; hence, the work is represented by the area $ADOF$. The total work done is the sum of these three parts, each taken with its proper sign. According to Art. **22**, work done by the air on the piston is considered positive and that done by the piston on the air is negative.

The compression represented by the curve AB follows the law PV^n = a constant. From the formula of Art. **17**, the work of compression from A to B is

$$W_{AB} = \frac{P_1 V_1 - P_2 V_2}{n - 1} \qquad (1)$$

As explained in the preceding article, the work given by this formula will always have the proper sign. The work of expelling the air is

$$W_{BC} = EB \times BC = P_2 V_2 \qquad (2)$$

As this work is done by the piston on the air, it must be given the negative sign. The work done on the piston by the air in the right end of the cylinder is

$$W_{AD} = FA \times OF = P_1 V_1 \qquad (3)$$

This work is given the positive sign. The total work per stroke is therefore

$$W = W_{AB} - W_{BC} + W_{AD} \qquad (4)$$

which equals

$$\frac{P_1 V_1 - P_2 V_2}{n - 1} - P_2 V_2 + P_1 V_1 = (P_1 V_1 - P_2 V_2)\left(\frac{1}{n - 1} + 1\right)$$

or

$$W = \frac{n}{n - 1}(P_1 V_1 - P_2 V_2) \qquad (5)$$

EXAMPLE.—The initial volume of the free air in the cylinder is 16 cubic feet, and the air is compressed from atmospheric pressure to a pressure of 54 pounds per square inch gauge—that is, above the pressure of the atmosphere—according to the law $P V^{1.3}$ = a constant. (*a*) What is the work done per stroke? (*b*) If the compressor makes eighty working strokes per minute, what is the net horsepower required to drive it? (*c*) How much heat is abstracted from the air during the compression from V_1 to V_2?

SOLUTION.—First, the volume V_2 must be found. Since $p_1 V_1^n = p_2 V_2^n$, $\left(\frac{V_2}{V_1}\right)^n = \frac{p_1}{p_2}$, or $V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$. $V_1 = 16$ cu. ft.; $p_1 = 14.7$ lb.; $p_2 = 54$ lb.; gauge $= 68.7$ lb., absolute. Then, $V_2 = 16 \left(\frac{14.7}{68.7}\right)^{\frac{1}{1.3}}$. Log $V_2 = \log 16 + \frac{1}{1.3} (\log 14.7 - \log 68.7) = .68901$. Hence, $V_2 = 4.887$ cu. ft.

(*a*) From formula **5**,

$$W = \frac{n}{n-1}(P_1 V_1 - P_2 V_2) = \frac{1.3}{.3} \times 144 \times (14.7 \times 16 - 68.7 \times 4.887)$$
$$= -62{,}735 \text{ ft.-lb. Ans.}$$

(*b*) There are eighty working strokes per minute, and since the work found in (*a*) is the work of one stroke,

$$\text{H. P.} = \frac{80 \times 62{,}735}{33{,}000} = 152. \text{ Ans.}$$

(*c*) To determine the heat abstracted during the compression, formula **1** of Art. **18** may be used. Then,

$$JQ = 144\ (14.7 \times 16 - 68.7 \times 4.887)\left(\frac{1}{1.3-1} - \frac{1}{1.405-1}\right)$$
$$= -12{,}511 \text{ ft.-lb.}$$
$$Q = \frac{-12{,}511}{778} = -16.08 \text{ B. T. U. Ans.}$$

The result is negative, as it should be; for heat imparted to the gas has been considered as positive, and heat abstracted should therefore be negative.

24. Adiabatic and Isothermal Compression.—The fundamental formula, $JQ = E_2 - E_1 + W$, may now be considered in connection with the air-compression process. The work W being done on the gas is negative and, in practice, JQ is also negative; that is, heat is abstracted from the gas. The change of energy, however, is positive; that is, it increases, for during compression the temperature rises. Writing the formula with these signs,

$$-JQ = (E_2 - E_1) - W, \text{ or } W = E_2 - E_1 + JQ$$

It is to be noted that the W in this formula is the work of compression merely, that is, the work represented by the area $ABEF$, Fig. 7, and not the total work per stroke.

For adiabatic compression, $JQ = 0$ and $W = E_2 - E_1$. The entire work of compressing the air from A to B has been expended in increasing the energy of the air. If the compression is isothermal, there is no change of energy, $E_2 - E_1 = 0$ and $W = JQ$. The work of compression is taken away, as fast as it is performed, by the water-jacket. If the air were used as soon as it is compressed, the energy stored in it by adiabatic compression would be utilized and adiabatic compression would be as efficient as isothermal compression. As a matter of fact, the air is usually carried in mains perhaps several miles long, and in transmission cools to the temperature of the outside air and thus loses the energy due to the rise in temperature during compression.

Referring now to Fig. 7, suppose the compression to be adiabatic; then the compression curve is AB'. If it were isothermal, the compression curve would be AB'', lying below and to the left of AB'. That the adiabatic must lie to the right of the isothermal is evident; for at the final pressure, P_2, the final temperature is higher in the adiabatic case, and, therefore, the final volume CB' is greater than the final volume CB'' in the isothermal case. As has been seen, the net work per stroke is represented by the area $AB'CDA$. If the compressor is provided with a water-jacket effective enough to prevent the temperature from rising during compression, the work per stroke would be $AB''CDA$ and the work $B'AB''$ would be saved. In practice, the water-jacket is not so effective, and the actual compression curve AB lies between the adiabatic AB' and the isothermal AB''. The work saved is represented by the area $B'AB$. The following conclusions should now be evident:

The work that may be eventually derived from air at a given pressure is the same, whether it is compressed adiabatically or isothermally. The extra energy imparted to the air by raising its temperature in adiabatic compression is lost by radiation. The work of the compressor piston per

stroke is smaller the lower the final temperature is kept by the action of the water-jacket. Hence, the compression should be as nearly isothermal as possible.

25. Formula **5**, Art. **23**, holds good for any value of n except 1; that is, for any case but the isothermal. In that case, the work per stroke is represented by the area $AB''CDA$, Fig. 7, which is made up of the areas $AB''GF$, $B''COG$, and $ADOF$, as described in Art. **23**. By formula **2**, Art. **9**, the work represented by the area $AB''GF$ is $2.3026\, P_1 V_1 \log \frac{V_2}{V_1}$, and the remaining areas are represented by $P_1 V_1$ and $P_2 V_2$, as in Art. **23**. Then the total work $W = 2.3026\, P_1 V_1 \log \frac{V_2}{V_1} + P_1 V_1 - P_2 V_2$. But in isothermal compression or expansion $P_1 V_1 = P_2 V_2$; hence,

$$W = 2.3026\, P_1 V_1 \log \frac{V_2}{V_1} = 2.3026\, P_1 V_1 \log \frac{P_1}{P_2}.$$

Hence, the total work, represented by the area $AB''CDA$, is the same as the work of compressing the air from A to B'', that is, from V_1 to V_2, which is represented by the area $AB''GF$.

EXAMPLE.—In the example of Art. **23**, what would be the work per stroke if the compression were isothermal, and what would be the percentage saved?

SOLUTION.—Here $P_1 = 144 \times 14.7$, $V_1 = 16$, and $P_2 = 144 \times (54 + 14.7)$; hence,

$$W = 2.3026 \times 144 \times 14.7 \times 16 \times \log \frac{14.7}{68.7} = -52{,}223 \text{ ft.-lb., nearly.}$$

Ans.

In the previous case, the work was 62,735 ft.-lb. Hence, the percentage saved is

$$100 \times \frac{62{,}735 - 52{,}223}{62{,}735} = 16.76 \text{ per cent.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. If 5.68 cubic feet of air having a temperature of 50° F. is compressed adiabatically to a volume of 1.3 cubic feet, what is the final temperature? Ans. 466.7° F.

2. In example 1, if the initial pressure is 14.7 pounds per square inch, absolute, what is the final pressure?

Ans. 116.7 lb. per sq. in., absolute

3. With the same data as in examples 1 and 2, calculate the work required to compress the air when the compression is adiabatic.

Ans. 24,254 ft.-lb.

4. With the conditions the same as in example 3, calculate the work required when the compression is isothermal?

Ans. 17,730 ft.-lb.

5. Confined air having a volume of .8 cubic foot at a temperature of 120° and a pressure of 45 pounds per square inch, absolute, expands adiabatically to the pressure of the atmosphere. What is: (*a*) the final volume? (*b*) the final temperature? (*c*) the work done during expansion?

Ans. (*a*) 1.774 cu. ft. (*b*) − 39.9° F. (*c*) 3,527.9 ft.-lb.

THERMODYNAMICS OF CLOSED CYCLES

DEFINITIONS AND PRINCIPLES

26. Cycle of Changes of State.—Thus far only those changes of state that follow some one law have been considered. **Cycles** of changes of state will now be taken up, in which there is a series of processes and in which the substance changes its state according to a succession of different laws.

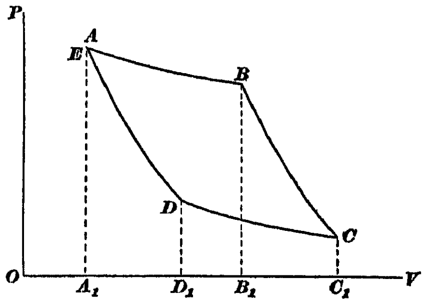


Fig. 8

A cycle of operations in which a substance, after passing through the several changes of state, is brought back to its initial state is called a **closed cycle**; otherwise it would be an **open cycle**. A closed cycle is represented graphically by a series of lines enclosing an area, as in Fig. 8. Thus, suppose the substance to start from the initial state represented by the point A, and to be subjected, in turn, to four changes of state represented in the pressure and volume

coordinates OP and OV, commonly called $P–V$ coordinates, by the curves AB, BC, CD, and DE, respectively; as the substance is brought back to the initial state, the end E of the last curve must coincide with A, and the four curves must therefore form a closed figure.

The cycle of the fluid used in a heat engine consists usually of four changes of state and is represented by four curves. If the alternate curves are of the same kind, it is said to be a **simple cycle.**

27. Relation Between Heat Used and External Work.—Let W_{AB} denote the external work during the change from A to B, Fig. 8, and Q_{AB} the heat imparted during the change; and similarly for the other changes of state; also, let E_A, E_B, etc. denote the intrinsic energy of the substance in the states A, B, etc. Applying the general energy equation to each change of state, then

$$\text{for } AB,\ JQ_{AB} = E_B - E_A + W_{AB}$$
$$\text{for } BC,\ JQ_{BC} = E_C - E_B + W_{BC}$$
$$\text{for } CD,\ JQ_{CD} = E_D - E_C + W_{CD}$$
$$\text{for } DA,\ JQ_{DA} = E_A - E_D + W_{DA}$$

Adding the members of the four equations,

$$J(Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}) = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

That is, *the total external work is the equivalent of the total heat imparted.*

In any closed cycle, work must be done by the fluid during part of the cycle, and on the fluid during the remainder; that is, part of the work is positive and part negative. Referring to Fig. 8, assume that the cycle is made in the direction $A–B$, $B–C$, etc.; then from A to B and from B to C work is done by the substance, while from C to D and from D to A work is done on the substance. Therefore,

$$W_{AB} = +\text{ area } A_1ABB_1$$
$$W_{BC} = +\text{ area } B_1BCC_1$$
$$W_{CD} = -\text{ area } C_1CDD_1$$
$$W_{DA} = -\text{ area } D_1DAA_1$$

Adding, $W_{AB} + W_{BC} + W_{CD} + W_{DA} = A_1ABB_1 + B_1BCC_1 - C_1CDD_1 - D_1DAA_1 = ABCD$. That is, *the net work*

done by the substance during the cycle is represented by the area enclosed by the curves representing the successive changes of state.

EXAMPLE.—A given weight of air goes through a closed cycle of changes and in so doing has imparted to it 148 B. T. U. and gives up 123 B. T. U.; what is the net external work done?

SOLUTION.—The net heat imparted is 148 − 123 = 25 B. T. U.
Then, $W = JQ = 778 \times 25 = 19{,}450$ B. T. U. Ans.

28. Heat Engine.—A **heat engine** is a machine or motor by which heat is transformed into work. The engine is supplied with some substance, called the **working fluid**, that is capable of receiving heat freely and of giving it up freely. This fluid receives heat from some external body, called the *source* or *hot body*, and gives up a smaller quantity of heat to another body, called the *refrigerator* or *cold body*. The heat not delivered to the refrigerator is transformed into work. In any heat engine, the working fluid goes through continuous cycles in which the original conditions are periodically repeated. In the ordinary reciprocating engine, there is a cycle for every revolution.

CARNOT'S CYCLE

29. Carnot's Heat Engine.—In 1824, Sadi Carnot, a French engineer, described an ideal engine having a simple cycle composed of isothermal and adiabatic changes of state. The conditions required by this engine cannot be complied with in practice and the engine cannot be actually constructed. Notwithstanding this fact, the study of the Carnot engine is of the first importance because it represents the limit of engine economy and is a standard by which engines may be compared with each other. The efficiency of an actual engine is always less than that of the ideal Carnot engine working between the same source of heat and the same refrigerator.

In Fig. 9, *c* represents the cylinder of a Carnot engine. Its walls are supposed to be perfectly non-conducting and its head a perfect conductor. The piston is also supposed to be a non-conductor of heat and to move in the cylinder

without friction. There is a source of heat s that is maintained at a constant temperature T_1, absolute, and a refrigerator r maintained at a constant temperature T_2, absolute; there is also a stand f that is a perfect non-conductor.

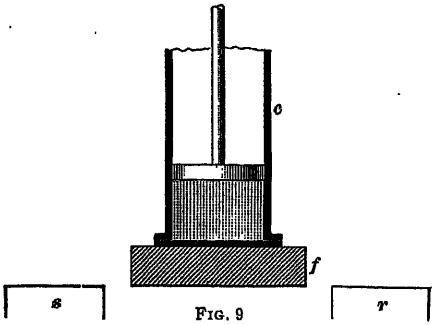


FIG. 9

The action of the engine is described as follows:

1. Let the fluid in the cylinder have the temperature T_1 of the source of heat, and let the cylinder be placed on the stand f. The piston is permitted to rise and the fluid expands adiabatically, as no heat can pass through the non-conducting walls. This expansion proceeds until the temperature drops to the temperature of the refrigerator r. Let A, Fig. 10, represent the initial state of the fluid as regards pressure and volume; then the adiabatic expansion is represented by the curve $A\,B$.

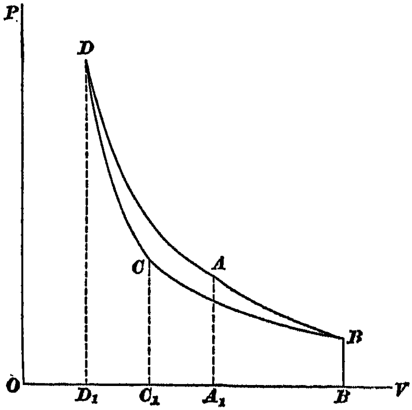


FIG. 10

2. Next, the cylinder is placed on the refrigerator r and the fluid is compressed slowly. Heat passes through the conducting head into the refrigerator and if the compression is sufficiently slow, the passage from cylinder to refrigerator may be accomplished without any rise of temperature; that is, the compression is isothermal. This compression is represented by the curve $B\,C$.

3. The cylinder is now placed on the non-conducting stand f and the fluid is still further compressed; since no

heat can escape through the non-conducting walls, this compression is adiabatic and the temperature of the fluid rises. Let the adiabatic compression be continued until the temperature of the fluid is T_1, the same as the temperature of the source s. The curve CD represents this third operation.

4. For the next operation, the cylinder is placed on the source s and the fluid is permitted to expand. The source s supplies heat during the expansion and keeps the temperature of the fluid constantly at T_1; hence, the expansion is isothermal. Let the expansion proceed until the fluid attains the initial state A.

The fluid has now passed through a closed cycle of operations consisting of four changes of state, namely, adiabatic expansion, isothermal compression, adiabatic compression, and isothermal expansion. This cycle is called **Carnot's cycle.**

30. Efficiency of Carnot's Engine.—The efficiency of a heat engine is the ratio of the heat transformed into work to the whole heat supplied from the source. Thus, if an engine receives 1,000 B. T. U. from the source, transforms 150 B. T. U. into external work, and gives up the remaining 850 B. T. U. to the refrigerator, its efficiency is $\frac{150}{1000} = .15$, or 15 per cent.

Let Q_1 = heat absorbed by working fluid from source;
Q_2 = heat given up by working fluid to refrigerator;
e = efficiency.

The heat transformed into work is evidently $Q_1 - Q_2$; hence,

$$e = \frac{Q_1 - Q_2}{Q_1} \qquad (1)$$

Since in a perfect gas $Q = sT$, in which s is the specific heat of the gas and T the absolute temperature, $\frac{Q_1 - Q_2}{Q_1}$ $= \frac{sT_1 - sT_2}{sT_1} = \frac{T_1 - T_2}{T_1}$, and

$$e = \frac{T_1 - T_2}{T_1} \qquad (2)$$

That is, *the efficiency of a Carnot engine working with a source at a temperature T_1 and a refrigerator at a temperature T_2*

is the ratio of the temperature range $T_1 - T_2$ to the temperature T_1 of the source.

31. The Reversed Heat Engine.—Carnot's heat engine is reversible; that is, it may run as described in Art. **29**, or it may traverse the cycle in the reverse order. The action of the reversed engine is as follows: Starting with the state D, Fig. 10, the cylinder is placed on the non-conducting stand and the fluid expands adiabatically to the state C. The cylinder is then placed on the refrigerator and the fluid expands isothermally as shown by the curve CB, at the same time receiving heat Q_2 from the refrigerator. Next, the cylinder is placed on the non-conducting stand and the fluid is compressed adiabatically to the state A. Finally, the cylinder is placed on the hot body and the fluid is compressed isothermally, as shown by AD. During this compression, the quantity of heat Q_1 is given up to the hot body by the fluid.

In the reversed engine, it will be observed that the total work done on the fluid during the compressions BA and AD is greater than the work done by the fluid during the expansions DC and CB; hence, the enclosed area $ADCB$ represents the net work done on the fluid by the piston. This external work is the equivalent of the difference $Q_1 - Q_2$ between the heat given to the hot body and that received from the cold body. The reversed engine, therefore, draws a certain quantity of heat Q_2 from the refrigerator, transforms a certain quantity of work W into heat, and delivers the sum $Q_2 + \frac{W}{J} = Q_1$ to the hot body.

32. Carnot's Principle.—*Of engines working between the same source and the same refrigerator, no engine can have a greater efficiency than Carnot's ideal reversible engine.* For, if another engine A should have a higher efficiency, it will do a larger amount of work than the Carnot engine for an equal expenditure of heat, both working between the same source of heat and the same refrigerator. It has been seen that when the Carnot engine is reversed, it

will require the same amount of work to drive it that it develops when running forwards. Suppose, now, that the two engines are connected together, so as to work in opposition to each other. Since engine *A* is capable of doing more work, the Carnot engine must run backwards. It, therefore, takes heat from the refrigerator and restores it to the hot body, as described in Art. **31**, while engine *A* takes heat from the hot body and gives up heat to the cold body. Assuming that there is no friction, engine *A* does just enough work to drive the Carnot engine, and the work of the one is equal to that of the other. But since *A* is more efficient, it takes less heat from the hot body than the Carnot engine would under the same conditions, and as the Carnot engine, when reversed, restores to the hot body the same amount of heat that it would take when running forwards, it follows that it must return to the hot body more heat than is drawn off by engine *A*. Heat would thus be delivered to a hot body from a cold body without outside aid.

Experience shows that this result cannot be attained. Heat of itself never passes from a body to a hotter body unless work is expended from without; and the supposition that a motor may run at the expense of the refrigerator leads to the conclusion that all the heat may be abstracted from the refrigerator, a result clearly impossible. Therefore, no engine can be more efficient than an ideal reversible Carnot engine for a given source and refrigerator.

33. The Second Law of Thermodynamics.—The formal statement of Carnot's principle constitutes the second fundamental law of thermodynamics. This law is stated in various ways, but each statement involves the same principle.

1. *Heat cannot, unaided by external agency, pass from a colder to a hotter body.*

2. *It is impossible to obtain work by cooling any portion of matter below the temperature of the coldest of surrounding objects.*

3. *The efficiency of the Carnot engine depends only on the temperatures of the source and refrigerator, and not on the nature*

of the working fluid; hence, all Carnot engines working between the same source and refrigerator have the same efficiency.

Carnot's principle, as shown in Art. **32**, is a direct consequence of either the first or the second statement of the law. The second statement is particularly suggestive. The atmosphere possesses an almost unlimited store of heat energy, and if some means could be found for utilizing this energy, motors could be driven without fuel. To thus utilize the heat energy of the atmosphere has been the dream of many inventors, but according to the second law it is not possible to do it. The temperature of the atmosphere may be regarded as a sort of sea level of temperature—the temperature to which all bodies either hotter or colder will ultimately attain if left to themselves. To obtain work from a hydraulic motor or waterwheel, a head of water is required; that is, there must be a fall from an altitude above the sea level. Similarly, to obtain work from a heat motor, there must be a fall of temperature, and this requires a source at a temperature higher than the temperature of the atmosphere, or, using the atmosphere as a source, a refrigerator at a temperature lower than that of the atmosphere. The last alternative is impossible, for there cannot be found in nature a portion of matter permanently colder than the atmosphere that can be used as a refrigerator.

34. Consequences of the Second Law.—According to the first law of thermodynamics, heat and work are mutually convertible; work may be transformed into heat, and vice versa. The questions now arise: With a given quantity of work, can the whole of the work or only a fraction of it be transformed into heat? and, conversely, with a given quantity of heat, can the whole or only a fraction of the heat be transformed into work? The entire quantity of work can be, and in fact usually is, transformed into heat; for example, the entire work done by an engine running with a friction brake is expended in overcoming friction and is converted into heat. On the other hand, only a fraction of the heat available can be converted into work. A quantity Q_1 is

taken from a source of heat, a smaller quantity Q_2 must be given up to a refrigerator of lower temperature, and only the difference $Q_1 - Q_2$ is transformed into work. To convert the whole of Q_1 into work, a refrigerator with a temperature at absolute zero must be found.

The next question is, what is the maximum value of the fraction $\frac{Q_1 - Q_2}{Q_1}$ of the heat Q_1 that can be converted into work? A Carnot engine converts the fraction $\frac{T_1 - T_2}{T_1}$ of the heat Q_1 into work, and according to the second law no other device can do more; hence, $\frac{T_1 - T_2}{T_1}$ is the maximum value sought. Evidently, the value of this fraction may be increased by lowering the temperature T_2 of the refrigerator. The value of the fraction $\frac{T_1 - T_2}{T_1}$ may also be increased by increasing the higher temperature T_1.

EXAMPLE.—A good ordinary steam engine requires the consumption of $2\frac{1}{2}$ pounds of coal per hour for each horsepower; taking the heating value of a pound of coal as 13,700 B. T. U., what fraction of the total heat liberated by the combustion is converted into work?

SOLUTION.—The heat resulting from the combustion is $13{,}700 \times 2\frac{1}{2} = 34{,}250$ B. T. U.; 1 H. P. is the performance of 33,000 ft.-lb. of work per min., or $33{,}000 \times 60 = 1{,}980{,}000$ ft.-lb. in 1 hr. The heat equivalent of this work is $Q = \frac{W}{J} = \frac{1{,}980{,}000}{778} = 2{,}545$ B. T. U. Of the 34,250 B. T. U. supplied, only 2,545 B. T. U. is ultimately converted into work. The efficiency is therefore $\frac{2545}{34250} = .0743$; that is, 7.43 per cent. of the total heat appears as work. Ans.

35. It must not be supposed that a heat engine is a poor and inefficient contrivance because it utilizes but a small part of the total heat supplied to it. A large part of that heat is absolutely unavailable for conversion into work, and the efficiency of the engine should be based on the ratio of the heat utilized to the available heat, rather than to the total heat. A simple hydraulic analogy will illustrate this point. Suppose that a supply of water in the Rocky Mountains is at an elevation of 12,000 feet above the sea level,

HEAT

(PART 2)

APPENDIX I

Derivation of formula 2, Art. 2

Let c_p denote the specific heat at constant pressure; then, since $\frac{K+W}{J}$ represents the heat imparted to the air, and $t_2 - t_1$ the rise in temperature,

$$c_p = \frac{K+W}{J(t_2 - t_1)} = \frac{K + R(T_2 - T_1)}{J(t_2 - t_1)}$$

The difference $T_2 - T_1$ is equal to the difference $t_2 - t_1$, the temperatures above zero; hence,

$$c_p = \frac{K}{J(t_2 - t_1)} + \frac{R(T_2 - T_1)}{J(T_2 - T_1)} = \frac{K}{J(t_2 - t_1)} + \frac{R}{J} \qquad (2)$$

APPENDIX II

Derivation of formula 2, Art. 4

Multiply and divide the second member of formula 1, Art. 4, by R; then,

$$E_2 - E_1 = \frac{J c_v G}{R}(R T_2 - R T_1) = \frac{J c_v}{R}(G R T_2 - G R T_1)$$

From the general equation, $P V = G R T$; hence,

$$G R T_2 = P_2 V_2 \text{ and } G R T_1 = P_1 V_1$$

Substituting these values

$$E_2 - E_1 = \frac{J c_v}{R}(P_2 V_2 - P_1 V_1)$$

Now, from Art. 3, $\frac{R}{J} = c_p - c_v$, and, $\frac{J}{R} = \frac{1}{c_p - c_v}$; hence,

$$E_2 - E_1 = \frac{c_v}{c_p - c_v}(P_2 V_2 - P_1 V_1)$$

Dividing numerator and denominator of the fraction by c_v,

$$E_2 - E_1 = \frac{1}{\frac{c_p}{c_v} - 1}(P_2 V_2 - P_1 V_1), \text{ or finally,}$$

$$E_2 - E_1 = \frac{1}{k-1}(P_2 V_2 - P_1 V_1) = \frac{P_2 V_2 - P_1 V_1}{k-1} \qquad (2)$$

APPENDIX III

Derivation of formula **3**, Art. **6**

By formula **2**, Art. **4**, the increase of energy is

$$E_2 - E_1 = \frac{P V_2 - P V_1}{k - 1}$$

and the external work done is

$$W = P V_2 - P V_1$$

From the energy equation, *Heat*, Part 1,

$$\begin{aligned} J Q = E_2 - E_1 + W &= \frac{P V_2 - P V_1}{k - 1} + P V_2 - P V_1 \\ &= \frac{P V_2 - P V_1}{k - 1} + \frac{(k - 1)\ (P V_2 - P V_1)}{k - 1} \\ &= \frac{k}{k - 1} (P V_2 - P V_1) \end{aligned}$$

APPENDIX IV

Derivation of formulas **1** and **2**, Art. **18**

The change of energy as given by formula **2**, Art. **4**, is

$$E_2 - E_1 = \frac{P_2 V_2 - P_1 V_1}{k - 1}$$

and from the formula in Art. **17**,

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

Then, applying the general equation,

$$J Q = E_2 - E_1 + W = \frac{P_2 V_2 - P_1 V_1}{k - 1} + \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

or $$J Q = (P_1 V_1 - P_2 V_2) \left(\frac{1}{n - 1} - \frac{1}{k - 1}\right) \qquad (1)$$

Since $P_1 V_1 = G R T_1$ and $P_2 V_2 = G R T_2$,

$$J Q = G R (T_1 - T_2) \left(\frac{1}{n - 1} - \frac{1}{k - 1}\right)$$

From the formula of Art. **3**,

$$R = J (c_p - c_v) = J c_v \left(\frac{c_p}{c_v} - 1\right) = J c_v (k - 1)$$

Substituting this value for R,

$$J Q = G J c_v (k - 1)\ (T_1 - T_2) \left(\frac{1}{n - 1} - \frac{1}{k - 1}\right)$$

whence, $$Q = G c_v (T_1 - T_2) \left(\frac{k - 1}{n - 1} - 1\right)$$

$$= G c_v \frac{k - n}{n - 1} (T_1 - T_2)$$

or, changing signs,

$$Q = G c_v \frac{n - k}{n - 1} (T_2 - T_1) \qquad (2)$$